Pricing Options via Double Auctions

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Pricing Options via Double Auctions

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Acronyms

AA Adaptive-Aggressive

ACE Agent-based Computational Economics

ATM At-The-Money

BIC Bayesian-Nash Incentive Compatible

CDA Continuous Double Auction

CS Consumer Surplus

DA Double Auction

DSIC Dominant Strategy Incentive Compatible

GBM Geometric Brownian Motion

GD Gjerstad-Dickhaut

ICE Iterative Combinatorial Exchange

IIKB Information-Inventory-Knowledge-Behaviour

IKB Information-Knowledge-Behaviour

ILP Integer Linear Programme

ITM In-The-Money

JD Jump-Diffusion

LMSR Logarithmic Market Scoring Rule

LOB Limit Order Book

LP Linear Programming

MAP Maximum A Posteriori

MGD Modified Gjerstad-Dickhaut

MLE Maximum Likelihood Estimator

OCC Options Clearing Corporation

OTM Out-of-The-Money

PDF Probability Density Function

PMF Probability Mass Function

PS Producer Surplus

QQ Quantile-Quantile

RB Red-Black

SDE Stochastic Differential Equation

SMA Simple Moving Average

SS Social Surplus

TBBL Tree-based Bidding Language

VCG Vickrey-Clarke-Groves

WDP Winner Determination Problem

ZI Zero Intelligence

ZIP Zero-Intelligence Plus

Abstract

This research develops and analyses a new set of agent-based models for pricing European options and option portfolios in the context of double auctions. After the financial crisis of 2008, it became obvious that the banking industry had been over-reliant on mathematical models such as Black-Scholes in pricing financial derivatives despite their major assumptions such as the efficiency of markets, the homogeneity and the risk-neutrality of traders, and the limitations in evaluating the risk itself. Although the Black-Scholes framework is regarded as the cornerstone of arbitrage-free pricing of financial derivatives, it does not involve market microstructure in forming option prices.

In this research, I add a simulated market component to the existing option pricing methodology through modelling automated option traders and running them on various auction-based mechanisms. My simulation model consists of three consecutive steps: asset pricing, automated traders, and market mechanisms. Firstly, I simulate asset prices beyond Black and Scholes' initial assumption which also involve fat-tailed distributions and mean-reverting aspects of risk-free interest rates that are common in most underlying markets.

Secondly, I design option traders according to my extended version of Information-Inventory-Knowledge-Behaviour (IIKB) framework. While the information and knowledge layers of the framework involve gathering and computing basic statistical parameters of the market, the behavioural layer is designed using three sublayers which are responsible for determining the option price, the quantity to bid/ask and the proxy trading algorithm. I also use Zero-Intelligence (ZI) and indifference pricing techniques along with the Black-Scholes formula to generate heterogeneous option prices. For proxy trading algorithms, I re-purposed well-known inventory- and information-based trading models to deal with options. I also use popular ZIP and GD trading algorithms to model the behaviour of speculative option traders.

Finally, in the third step, I feed the orders generated from automated traders to different mechanisms and analyse the obtained option prices. First, I consider direct double auction which has the Dominant Strategy Incentive Compatibility (DSIC) property, so that traders submit only truthful orders. I develop a multi-unit, revealed and simultaneous versions of a direct double auction and run different option pricing methods on them to evaluate the aggregated

option prices. I also analyse the allocative efficiency and budget-balance of the mechanism. Then, I run trading agents with proxy trading algorithms in an online double auction. I evaluate the obtained option prices and the performance of each proxy trading algorithm used.

Another important aspect of the research is the new perspective on pricing of compound financial contracts such as option portfolios using a combinatorial exchange. I explain the substitutability and complementarity of options in given option portfolios, and apply these concepts to the design of the combinatorial exchange for option portfolios. I also illustrate the expressiveness and flexibility of using combinatorial exchanges through a Tree-Based Bidding Language.

The main contributions of this research are the design and implementation of a direct double auction for multi-unit and atomic orders, revealed mechanisms for forecasting traders, inventory- and information-based option traders, Logarithmic Market Scoring Rule (LMSR) option pricing based on option portfolios, and the application of combinatorial exchanges to the realm of option pricing.

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To Dad, Mom and Gulmira

Chapter 1

Introduction

We have seen, in Financial Crash of 2008, the damages brought to large financial institutions like Lehman Brothers leaving the company in bankruptcy with more than $600 billion debt [109], the acquisition of Merrill Lynch & Co. by Bank of America for $50 billion saving it from potential bankruptcy [151], the sales of the most of Washington Mutual's assets to JPMorgan Chase & Co for $1.9 billion [101] and so forth. Along with the core reasons of the inflation of housing bubble and its ultimate burst such as subprime lending, overleveraging, easier conditions for mortgage and government deregulations [32], the key factor for its emergence was the creation of new financial instruments such as Mortgage Backed Securities (MBS), Collateral Debt Obligations (CDO) and a financial derivative called Credit Default Swaps (CDS). These financial instruments were the flesh and bones of a securitization process which involved packaging bonds, mortgages and other interest generating assets of different credit ratings into, so called, Asset-Backed Securities (ABS). These securities used waterfall method to prioritise the returns of different layers of the portfolio, called *tranches*. While the Senior tranches generally had AAA rating as they almost guaranteed to pay the interest and preserve the principal, the Junior tranches were mostly retained by the seller and absorbed the potential defaults within the portfolio of interest generating assets. The popularity of these securities in the market created more relaxed environment for mortgages, as banks can always sell out these obligations in the form of MBSs and thus transfer the risk to the market participants. Another motivation for subprime lending was the increased demand for housing due to more accessible mortgages, and thus the rise in house prices. Banks introduced the teaser rates like 6% for the first 3-4 years to even increase the number of applicants for mortgages to drive their profits up. However in 3-4 years when the teaser rates ended, many started to default, and the foreclosure returned the houses to the housing market. This caused a sharp decline in housing market and eventually led to complete collapse of the U.S. housing market. The main catalyst of the crisis was borrowers relationship with the lenders which replicated an *American put option* contract. Technically, borrowers had the right to sell the house to the lender at the principal amount they owed. And when the house prices went down, so they became less than the principal money owed, the borrowers had an incentive to sell the house at much higher price to the lender, and push this house into the market dragging the prices even lower [89]. American put option contract allows one to sell an underlying asset at an agreed price in future, and the above described situation with the housing market collapse was exactly the same process.

Nowadays it is not a secret that the financial markets are booming with fast algorithmic traders and automated markets which have already outpaced human traders in several magnitudes in their efficiency and speed. However these technological advantages bring with them more complexity and volatility which are hard to manage and predict. The increased complexity of the high-frequency trading mechanisms caused a glitch in Knight Capital Group in 2012 which costed the company about $440 million loss due to the use of an obsolete trading algorithm [137].

From above mentioned 2 crashes, one may conclude that options and algorithmic trading are crucial components of today's financial markets, and they require a particular attention in preventing such cases in future. For the former case, traders need to evaluate the risk of dealing with options correctly, so they take into account the heterogeneity of risk perception in markets, and combine all possible scenarios that may occur in the market while pricing options. Continuous asset prices that exclude potential jumps caused by market participants cannot be a valid way of assessing the risk involved with trading these assets. This, of course, involves taking into account the heterogeneous option pricing methods used by multiple traders. The simulation of these option pricing methods in a controlled environment and using the aggregated outcomes in making better trading decisions could be one way of dealing with the problem in future. The latter case would require the development of such mechanisms where traders does not employ complex trading algorithms that maximise the total gain, but truthfully reveal their valuations to the mechanism to achieve the same objective. This requires a re-engineering on the mechanism's side, because although current markets are highly efficient, they are not always direct and incentive compatible. In direct mechanisms, traders do not need to employ any trading strategy, as they can safely reveal their true type to the mechanism and still maximise their utility.

In this research, I propose a multi-agent system that can be used for pricing financial options through simulating a direct double auction. I use heterogeneous option pricing techniques to mimic the population of traders with diverse risk perception. I also test these trading agents in a continuous double auction mechanism which is widely used by established financial markets. Moreover, I propose some perspectives in the generalisation of the double auctions to a combinatorial exchange mechanism where the financial instruments such as options can be combined into more complex structures such as option portfolios, and can be priced truthfully as compound financial instruments.

1.1 Options And Markets

Financial derivatives are formal agreements that give certain rights and impose certain obligations to concerned parties in future depending on the situation in the market. The major role of such contracts is to provide some level of security from uncertainties associated with the future state of the underlying market. Traders can freely enter into such contracts and take the advantage of their guaranteed benefits. One of such contracts is *option* contract, which is the right of selling or buying some asset at future price. It is issued by market participants for an additional premium. Option is referred as a financial product that has its own price and can be traded in the market equally as good as, say, orange juice. However unlike orange juice, its value is derived from the value of some other asset. Therefore option market is closely connected to its underlying market.

Standard financial theory provides a number of methods for calculating option price based on the market performance of its underlying asset. But there are few models that take into account microstructural aspects of option market itself, and their role in forming the prices. It is commonly assumed that because option is a derivative product, its value is associated with the risk of buying or selling its underlying asset, and there is no impact of the option market itself to the evolution of the prices. Also, standard models do not take into account heterogeneity of risk evaluation methods, as most of the market participants use different sources to assess risk. It is commonly accepted that risk is measured by the volatility of asset price, and this is the key factor in calculating option prices. But in reality, traders may take other factors which affect their decision in evaluating option prices, and these factors ultimately determine their corresponding quotes.

In order to combine these heterogeneous approaches into one place, and use them to obtain option prices, we need to consider the actual computer simulation of the marketplaces with these traders. There have been multiple approaches proposed in designing market mechanisms, and one of them is the Double Auction (DA). However DAs are also further classified based on the overall protocol for trading, the types of orders submitted by traders, the matching and pricing rules, etc. For example, single/multi-unit DAs, multi-attribute DAs, Continuous Double Auction (CDA)s, prediction markets, markets without money, etc. Current financial markets, including option markets use CDA for facilitating the trade among participants. There are also additional market rules such as 'margin calls' that prevent traders from becoming default. However these mechanisms were actually used to clear the matching orders of the traders, but not used to price the actual traded goods.

1.2 Motivation

The financial crashes recently caused by misrepresenting the risk and sophisticated trading algorithms arise a need to review the existing methods and to provide viable solutions that could prevent such events in future. In fact, the option pricing that hardly relies on Black-Scholes formula also requires more careful attention as the mathematical models usually used to describe the markets are not sufficient to represent the heterogeneity of different beliefs. Professor of Mathematics at Warwick University, Ian Stewart said in his article for Guardian[153]:

> "Any mathematical model of reality relies on simplifications and assumptions. The Black-Scholes equation was based on arbitrage pricing theory, in which both drift and volatility are constant. This assumption is common in financial theory, but it is often false for real markets."
>
> ...
>
> "Despite its supposed expertise, the financial sector performs no

> better than random guesswork. The stock market has spent 20 years going nowhere. The system is too complex to be run on error-strewn hunches and gut feelings, but current mathematical models don't represent reality adequately. The entire system is poorly understood and dangerously unstable. The world economy desperately needs a radical overhaul and that requires more mathematics, not less. It may be rocket science, but magic it's not."

In my view, multi-agent simulation of financial markets can provide a better outlook for option pricing and help prevent financial crashes in future. This also aligns with the further thought of Professor Ian Stewart:

> "At the forefront of these efforts is complexity science, a new branch of mathematics that models the market as a collection of individuals interacting according to specified rules. These models reveal the damaging effects of the herd instinct: market traders copy other market traders. Virtually every financial crisis in the last century has been pushed over the edge by the herd instinct. It makes everything go belly-up at the same time."

One of the main reasons of doing this research is my quest in understanding important factors governing the markets, the ways of improving, simplifying and harnessing them in computing the option prices. As a fundamental mechanism of any market, option prices also emerge from the strategic interaction of traders. Unlike in spot markets where demand emerges from consumption, and supply from production, in derivatives market supply and demand emerge from the ratio of the risk and reward expected by different traders. However the trader beliefs about the risk can be different and therefore there is a possibility of trade that we usually see in financial markets. Hence there is a need for modelling option traders that use various option pricing techniques. This naturally leads to designing platforms for testing and running experiments on them to obtain valuable information about competitive option prices.

There are several reasons for the implementation of a simulation platform which could output option prices from heterogeneous trading behaviour. Firstly, we cannot control the real markets and cannot impose our own rules on them. We can pull the live data using APIs, or push our own bids and asks to the existing mechanisms, but we cannot control the behaviour of other agents. Hence it is impossible to set up a certain scenario in the underlying market in real world and measur the corresponding impact in option market. Moreover, live execution of trading agents in real markets is too expensive, and may cause losses. Backtesting subjects to historical bias, and lacks real-time interaction with the mechanism itself. This requires a testbed for the trading agents to perform their actions on various situations in given market environment. Secondly, we have no knowledge of real option traders, as we do not know what their returns are, which option trading strategies they are using, how they are evaluating options, what forecasting mechanism they use to predict underlying prices and other behavioural parameters that affect their decision making process. This information is hidden, and almost impossible to characterize. This would require us to encompass all possible types of trading behaviour in the market in order to achieve better results. Thirdly, most of today's financial markets are automated and involve less human interaction. For example, high-frequency trading is accomplished by software agents that are programmed to respond market changes in milliseconds and make valid decisions within such bounded context. This encourages us to use already adopted trading algorithms in simulating such markets to better understand what outcomes they can generate. Finally, the forth and the most important reason is the experimental curiosity in modifying the existing markets such as turning them into strategy-proof mechanisms to analyse resulted option prices emerging from the simulation heterogeneous traders.

Another motivation that drives us forward in developing this idea further is the use of combinatorial exchanges as generalised mechanism for mixing and matching different financial instruments and pricing them as a whole. It is important to note that modern financial markets lack the capability of expressing combinatorial preferences. Financial products such as MBSs and CDOs are, indeed, compound products consisting of a stack of several bonds linked to different riskiness profiles. Combinatorial exchanges would create such mechanism where such compound products can be truthfully priced in a market environment by clear interpretation of their content and valuation through the use of bidding languages. In the context of options, this idea would translate to option portfolios. This, as stated above, requires the design of a combinatorial exchange with corresponding bidding language for expressing option portfolios as combinatorial bids.

1.3 Key Assumptions

I state the key assumptions of this research in this section. I explain the main reasons why I decided to use European-style options and why such options are assumed on an underlying asset which does not yield any dividend. These factors directly affect the option price and therefore an appropriate justification is needed regarding their omission.

1.3.1 Why European-style Options?

Almost all exchange traded stocks, commodities, currencies and some indices have their American-style options. Quick summary from Options Clearing Corporation (OCC)[1] shows that more than 4 billion shares have been sold through American options in 2015. In comparison with European options which are mostly issued on some market indices such as SPX-500, NDX-100, FTSE-100, etc, American options are more popular and are commonly used by market participants. However, in this thesis, I will use the example of European options in every given model or simulation. I exclude other types of options including popular American options and more exotic Asian options, Binary options, Barrier options etc from the scope of this research. Besides narrowing down the scope of this research, there are other important elements associated with European options to be the best candidate for my research. I listed them below:

- European option is one of the most primitive option types. Similarly, there are other primitive options too, such as Arrow-Debreu contract [6], which pays $1 if the expected event happens. By saying primitive option type, I mean the mathematical simplicity of computation of its value given enough information about its terms and conditions. Therefore European or any other primitive option is mostly the starting point for any new methodology in pricing options. Many papers proposing a new approach in pricing options, besides the Black-Scholes solution, also use European options as an example [114, 80, 42, 24]. One of the key reasons for that is to isolate the important characteristics of the options in general and to focus on the key aspects of the proposed methodology itself dismissing the unnecessary complexity. I chose to use European

[1] `http://www.optionsclearing.com/`

option as an example for my model mainly because of a concrete formula for computing its final payoff on its known exercise date. In American option, for example, although the payoff formula is the same as European option, its exercise date is usually unknown and depends on the evaluation of optimality of its executing on a given date before its maturity. This adds additional complexity to the model, and distracts the reader from the main idea of my methodology which is mostly about including double auctions into the pricing process of options.

- Other pricing methodologies can be derived from pricing European options. We know that the Black-Scholes' framework suggests using risk-neutral probability measure in evaluating the expected payoff for given financial contract, and an option in particular. This was the key aspect of the framework, so it could be applied to nearly all financial derivatives no matter what their intrinsic payoffs are. Also, in Cox *et al.* [35], this approach has been discretised into a binomial lattice evolving into chain of probable events that may happen in the underlying market in future. By moving backward from the leaves of the constructed lattice tree, one can compute the expected value of the given financial contract, and continue this cycle until he reaches the origin. This proves that the methodology is universal, and is not restricted to European options only. Similarly, although I only use European option as an example in my model, it can further be extended to any other financial derivative given that the traders are equipped with the knowledge on the intrinsic value of the contract they are trading with.

- American call on an asset with no dividends costs same as European call. This is because by shorting asset at any time t before the expiry of held European call option, we can simulate the payoff model of an American call [87]. In simulations provided in this thesis, I use European call option whose value is equivalent to American call if the underlying asset does not yield any dividends. I also disregarded dividends in my asset pricing model due to their uncertainty. This is because it is never optimal to exercise such American option before its maturity, which effectively simulates the European call. However this is not true for an American put whether the asset yields dividend or not. Depending on the situation in the underlying market, trader might be better off exercising an American put earlier and make such option worth more than its European alternative. Although my simulation results are not symmetrical to American put options, we can still read the valuations obtained for European calls as the potential valuations for equivalent American calls.

- Put-call parity holds exactly in European options [89]. As it was mentioned earlier, this relationship holds exactly for European options, so the price of the put option can be computed exactly if one knows the price of its call equivalent. However this relationship does not hold for American options, and it only tells that the put price should greater or equal to computed value. For example, it is easier to evaluate the price of an option portfolio which includes both calls and puts by only knowing the price of its calls. This significantly simplifies the process of pricing an option portfolio. Also from my simulation results for call options, one can infer the put price too.

- European option has a closed-form solution for the risk-neutral values of both call and put, but there is no such closed-form solution for an American option. However there are some approximation models such as Roll-Geske-Whaley method [67] which constructs an American option using European compound options (i.e. options on option itself). This involves additional uncertainty in determining the expiration time (i.e. optimal execution time for American option) of compound option besides the parameters of the underlying option. I used Black-Scholes formula to obtain a benchmark price for the simulated option prices to analyse the discrepancies that arise from certain situations in the market, because it is a straightforward process to do. In fact, I even use risk-neutral benchmark to measure if the option is overpriced or underpriced, and based on this criteria determine the quantity the trading agent would require for particular option. In case of American options, the computation of the benchmark price would not be a straightforward task, as it would require additional assumptions on the optimal exercise time for the American option.

- Implied volatility can be solved given the price of a European option using the closed-form solution given by Black-Scholes. European option is monotonically increasing with respect to its implied volatility which makes the unknown implied volatility easily solvable using numerical schemes. Finding the price of an American option would employ numerical methods such as binomial trees. Numerically solving for an implied volatility using a function which by itself numerically computes the American option is computationally intensive task when it is done for thousands of historic option prices. I had to use the prices of historical European option on NDX-100 to compute the implied volatility and construct a volatility surface per each moneyness and time-to-expiry parameter of the option. Volatility surface is then used by volatility-based traders (referred with prefix `VOL-` in the thesis) to generate the risk-neutral prices for given option types [64].

- European options have applications outside financial derivatives domain. There are many cases in computer science and other relevant fields where European options were used as an intermediary instrument to evaluate the cost of uncertainties associated with future events. In auction theory, option-based mechanism is used to build an incentive compatible sequential auction for bidders with combinatorial preferences [93]. In market microstructure theory, the straddle option portfolio made of European put and call can be used to evaluate the expected loss from informed traders [33]. Options are also used in taking strategic decisions on investment projects, business opportunities, negotiations and resource allocation tasks in management. In fact, this type of options are called *real options*. Despite not being a financial derivative, similar techniques used to price a European option are also applied to evaluating real options [2].

1.3.2 Why No Dividends?

Computing European option prices on an underlying asset which yields dividends means simply decreasing the risk-free interest rate by the dividend rate and using the resulted rate as a risk-neutral rate. There is no other implication of introducing dividends into the model in the case of European options. Therefore given any pricing model for European options, it is a trivial task to adjust it to a corresponding dividend yielding underlying asset. Due to this triviality, I dropped this parameter from my model and denoted this fact with a single parameter r which may or may not represent the dividend adjusted risk-free investment rate depending on the value it is set to. Empirically speaking, this also introduces additional uncertainty into the model, because estimating the future dividends from all 100 companies included in chosen NASDAQ-100 index would require more effort on either fundamental analysis of the performance of these companies or chartist forecasting models that could reasonably replicate the behaviour of dividend yields based on historic data. Instead, I used either constant value taken from `www.treasury.gov` to set parameter r or modelled it as a mean-reverting stochastic process. Moreover, including dividends into my simulation model would also involve the implementation of a process monitoring the dividend payouts to each agent for each stock on a specified date, consequently changing the risk-neutral rate r and thus imposing additional complexity in analysing the resulted option prices.

Once again, the key assumption was that no matter what the dividend rates would be from the underlying stocks, at the end of the day they are subtracted from risk-free investment rate and would either be equal to some constant value or would follow a stochastic process with dividend adjusted mean. Of course, in case of American options, the inclusion of dividends into the model would arise an opportunity for the holders to exercise the option earlier and thus introduce significant changes into the model. I mentioned about this fact earlier too. However, in case of European options, the role of dividends is negligible, because it can be represented as part of risk-neutral rate.

1.4 Research Questions and Objectives

I draw the main questions and objectives of the research based on the motivating aspects of my work. There are several major goals that I will address in this research. Once of the main aspects of the research is to characterize a potential market-based framework which could be used for pricing options and option portfolios. This involves determining its main functional components, the way they interact with each other, and the details of their implementation. This can be summarised into following research questions:

1. Can European options be priced via double auctions?
2. What are the important components involved in pricing European option via double auctions?
3. What categories of trading agents are involved in option market and how are they designed for double auctions?
4. How is the direct double auction designed for trading options, and what is learned from its simulation?
5. How is the online double auction designed for trading options, and what

is learned from its simulation?

6. What are the benefits of using combinatorial exchanges for pricing option portfolios?

The answers to above stated questions will be limited to the proposed model and its core assumptions. The lessons learned from particular simulation case may not encompass the general behaviour of the model, and represents the knowledge obtained only under specific conditions.

Now let me draw research objectives that I aim to fulfil in this work. The research objectives are the tasks that should be accomplished in order to answer the posed questions:

1. *To study the constituent components of the option market.* This involves the extensive study of the option market as an interconnected complex system which identifies its constituent elements and the relationship between them. First, I should review how the underlying market is modelled in practice and what key parameters are used to define them. Second, I should consider the option traders and the methods they use to price options and submit their orders to the market. This also involves linking the data coming from the underlying market to the behavioural model of the traders. Third, I should review the economical aspects of the markets in general, and in particular, study the current mechanisms that are implemented in common option exchanges, and derive important properties of such mechanisms.

2. *To propose an abstract simulation model that includes all the constituent components of the option market.* I should define the main functionality and features of the simulation components, and the ways they interact with each other. I need to determine the mathematical models that can be used to describe different scenarios in underlying markets and the key parameters involved in controlling them. For trading agents, I need define an abstract framework which identifies the main aspects involved in implementing option trading agents in general. And finally, for the option market, I have to determine the protocol for running double auctions.

3. *To develop and test option trading agents.* Based on the framework proposed in simulation model, I need to determine the concrete possible implementations of the option trading agents. The traders should be

developed in a modular architecture so functional components can be replaced to change the behaviour of the trader. I should also run them under specific conditions to review their orders and compare them with analytical option prices.

4. *To develop and simulate direct double auction.* I need to determine the key properties of the direct mechanisms (i.e. a mechanism which runs one time and achieves the maximisation of its objective function, so the participants can truthfully reveal their types.) along with the requirements of option markets, and based on that knowledge develop the direct double auction which is suitable for option trading. I should run series of experimental cases with different option pricing methods to obtain the option prices and analyse their sensitivity to different market parameters.

5. *To develop and simulate online double auction.* I need to develop or use existing continuous double auction to experiment trading algorithms associated with options. CDA is the current implementation of modern financial markets, and it is important to evaluate the performance of proposed trading algorithms in this environment. The simulation should produce series of option prices and plots that analyse their sensitivity to different market parameters.

6. *To suggest perspectives in combinatorial exchange for option portfolios.* I should explain why combinatorial exchanges is the next step in pricing financial products (in the example of options) via double auctions and how they can be applied for trading compound financial products such as option portfolios. This provides my forward thoughts and future research objectives in the generalisation of financial markets into combinatorial exchanges.

I will address each of the objectives stated above in consecutive chapters and stress the importance of each chapter's outcome to continue to the next one.

1.5 Contributions

There are several contributions that can be drawn from the accomplished work. There have been a number of new ideas proposed, implemented and simulated throughout the course of the research. Most of my proposals can be considered as an extension, adaptation and revision of existing models. I also provided completely new perspectives of looking at pricing options and option portfolios. I summarised the main contributions of the research in the list below:

1. *Simulation model for pricing options via double auctions.* I provided high level simulation model addressing the research objective (2) which consists of the implementations of common asset pricing models such as GBM and jump-diffusion model. For the simulation of risk-free rates, I used Vasicek and Vasicek-Jump processes. For the option traders, I provided an abstract framework breaking down the relevant concepts into corresponding layers. I extended the existing Information-Knowledge-Behaviour (IKB) framework[160] with new Inventory layer into new Information-Inventory-Knowledge-Behaviour (IIKB) framework and adapted its functional components to option trading. Third, I provided the general simulation flow for direct and online double auctions, and explained the overall interaction of the mechanism with traders and the underlying market.

2. *Option Trading Agents.* I provided the implementation and the analysis of a range of different option trading agents built based on proposed IIKB framework and thus addressed the objective (3). I proposed modular architecture for designing the trader's behaviour which consists of three important modules: option pricing, choosing the quantities and proxy trading strategies. During the implementation of these modules, I developed new option pricing methods and proxy trading algorithms for options based on the recent literature. I listed these novelties below:

    - *Volatility-based option pricing.* Although this methodology has been previously described in financial engineering textbooks[64], my contribution can be seen as the agent-based approach to this concept. I separated the construction of volatility surface and pricing the actual option into two independent layers: knowledge and behaviour layers in proposed IIKB, and linked them through the feedback the trader gets from the market and uses it to update his knowledge base. The trader continuously adapts his volatility to the market's implied volatility and uses the resulted volatility surface to price different options.

    - *Risk-averse option pricing.* Using indifference pricing methods such as modelling the risk-averseness through exponential function. It is a comparatively new idea [24] in option pricing in contrary to standard Black-Scholes risk-neutral framework. In my model, I defined an agent-based architecture that uses this option pricing method to interact with the option market, and obtained simulation results from its execution in direct double auction.

- *LMSR option pricing.* This way of option pricing emerges from recently proposed design of prediction markets [26, 42]. It defines one of the cutting-edge approaches in looking at options as predictions and rewarding their owners according to the market scoring rule. I integrated this methodology into proposed IIKB framework and linked it to the option portfolios that agents may already hold while pricing the options. I obtained the LMSR option prices of agents holding common portfolios through simulating them in a direct double auction environment.

- *Inventory-based option dealers.* I adapted the algorithm proposed by Garman [63] to trading options based on the inventory levels of cash and options, and simulated it in an online double auction. I linked this algorithm with the option pricing modules and the inventory accounts within IIKB framework. Also I computed the probability of inventory-based option dealer's failure through modelling the intensity of order arrivals as linear functions.

- *Information-based option dealers.* I adapted the algorithm proposed by Copeland-Galai [33] to trading options based on the information available about informed traders. Taking advantage of the fact that Copeland-Galai's algorithm replicates the straddle option portfolio, I used Geske's [66] formulas to construct the straddle portfolio using compound options and used it to evaluate the expected loss from informed traders. I also proposed Bayesian updating method for learning about the proportion of informed traders in the market. The proposed algorithm's expected payoff has been analysed through the example of linear intensity of order arrivals. I also simulated this algorithm in online double auction environment, and closely observed the payoffs it generated along with the prices it submitted.

- *Option adapted ZIP and GD algorithms.* I used the ZIP algorithm in the context of proposed IIKB framework where options prices are computed continuously and used by the algorithm as the private price. For GD algorithm, I proposed the use of the ratio of evaluated option price and theoretical price (instead of raw option prices, because their values change with the time) in determining the probability of the success of the bid or ask. These algorithms have been used in online double auction environment to simulate the speculative trading behaviour.

3. *Direct multi-unit double auction with atomic orders.* I designed new multi-unit double auction which holds Dominant Strategy Incentive Compatible (DSIC), efficiency and individual rationality properties at the cost of uncontrolled budget-deficiency. This mechanism is used to simulate various option pricing methods in a market environment where the traders have no incentive in modifying their true valuations. I obtained the equilibrium option prices, and computed their sensitivity on different factors by simulating different populations of option traders. Besides that, I also proposed the use of revealed mechanisms for certain types of traders who can reveal their private forecasts about future asset prices. I ran them to collect their aggregated forecasts resulting from the changes in supply and demand of the market. This addressed the objective (4).

4. *Online double auction for option traders.* I implemented an instance of a continuous double auction and used it for running option traders with proxy trading algorithms. I implemented the LOB using binary trees to accept continuous orders from trading agents. The option prices obtained from the simulation of newly proposed inventory- and information-based dealers have been analysed along with ZIP and GD traders. Besides that I also reviewed the performance of each type of trading agent in terms of its overall payoff and the way how its inventories change according to their bid-ask spread. This addressed the objective (5).

5. *New perspectives on pricing option portfolios using combinatorial exchange.* I provided several scenarios where the options can be considered as substitutes/complements for the trader wishing to take an option portfolio, and emphasised the need for pricing them as bundles in a combinatorial exchange. I applied the proposed design of combinatorial exchange for trading option portfolios and interpreted the option portfolios using Tree-based Bidding Language (TBBL). I also presented how the mechanism solves the Winner Determination Problem (WDP) and computes budget-balanced payments. I also provided a possible extension of TBBL

language for trading option portfolios, and presented a generic class of TBBL that can be reused to interpret different option portfolios. This provided simplification for the mechanism and the trader in communicating the preferences in a complete and succinct way. This addressed the final objective (6) of the research.

The outcome of the research helps us to understand how different option pricing methods when put into a market environment generate different option prices, how these prices are sensitive to the external factors and how the changes in the rules of markets themselves affect them. It gives an interesting insight on strategy-proof mechanisms and their role in simplifying the interaction between traders and the market. Also the proposed architecture is open and easily extendible, so one can implement his own option pricing, or proxy trading algorithm and use it to simulate the option market. Also the mechanism's allocation and clearing rules can be easily modified to test different experimental cases. This can be useful for the researchers and financial analysts to evaluate certain situations in option market, test their own trading agents or market rules in making any trading decision.

I have also published the key aspects of the research in papers listed below:

1. Sarvar Abdullaev, Peter McBurney, and Katarzyna Musial-Gabrys, Direct Exchange Mechanisms for Option Pricings, In Proc. of *12th European Conference on Multi-Agent Systems*, Springer LNAI Multi-Agent Systems, Prague, Czech Republic, December 2014

2. Sarvar Abdullaev, Peter McBurney, and Katarzyna Musial-Gabrys, Market-based Mechanism for Option Pricing, In Proc. of *16th International Workshop on Agent-Mediated Electronic Commerce and Trading Agents Design and Analysis*, Paris, France, May 2014

1.6 Research Methodology

I use computer simulation as my main methodology of investigating the role of market microstructure in forming the option prices. As automated trading continues to shape modern financial markets by forming a global complex network of software traders, I believe that multi-agent simulation with its ability to mimic this heterogeneity is the most effective methodology for studying derivative markets. Most of today's financial markets are organised as LOB with the broker acting as a facilitator in matching buyers and sellers. Option trading occurs at exchanges such as New York Stock Exchange (NYSE) or Chicago Board of Trade (CBOT) under highly standardised and regulated environments. These markets are also hierarchical so that players have to go through several levels of intermediaries to process their orders. Brokerage firms or clearinghouses are the ones which are engaged in verifying orders, maintaining investors' margin accounts, matching and settling the orders. Therefore it is practically impossible to control or manipulate such exchanges for experimental reasons to analyses the consequences of certain market policies or trading behaviour on option prices. It is impossible to model the mindset of each market participant and develop an algorithm which could replicate their decision making process. Neither can we control the information flow between market participants, nor can we isolate or fix some factors in the market to undertake *ceteris paribus* experiments. Option markets are complex systems not only governed by the intrinsic characteristics of the options traded such as strike price, expiration date, but also influenced by other factors such as traders' subjective beliefs and pricing methodologies applied, and more importantly by the events happening in underlying market.

Moreover, it is also impossible to fit option market into a universal mathematical model because of the extreme complexity and heterogeneity of its constituent elements. Standard models used for pricing options imply homogeneity of risk evaluation techniques, and based on this assumption derive their values. But in a complex scenario every trader is different in making decisions, and thus he is different in evaluating the risk. My method uses Monte-Carlo methodology in simulating the random behaviour of traders and imposing certain market rules on them to obtain simulated option prices. Monte-Carlo method is a handy technique which relies on the law of large numbers, and avoids complex combinatorial analysis through generating random numbers and inputting them into given model. Unlike traditional Monte-Carlo option pricing technique which simulates the possible paths of the asset prices with given stochastic model and then finds the expected value of the derivative, I design adaptive and heterogeneous option pricing and trading algorithms on top of it and use them to generate bids and asks for proposed mechanisms which could be controlled and modified for the experiment. Multi-agent simulation of financial markets provides conditions for a controlled experiment, and thus allows us to build cause and effect relationship between different factors. There have been many studies accomplished in past using multi-agent simulation of financial markets, and LeBaron provides a comprehensive review of the researches undertaken in this field [99].

I have planned the research lifecycle using an iterative methodology which involves gradual evolution of the research outcomes from continuous design, implementation and simulation of experiments loops. This methodology which takes its roots from agile software development proved to be a productive way of conducting a simulation-based research, because one could realise the technical issues and model's fundamental pitfalls during the implementation of automated traders and mechanisms.

I list the steps that I have undertaken to accomplish this research. Iterative approach involves standard steps which are common to most of the software development processes, and mostly used by agile methodologies [111]. Also it has been suggested as one of the key approaches of conducting a doctorate research [163]. It consists of initial stages such as posing research questions and objectives, literature review and theoretical model, and then proceeds to iterative development of experiments and analysis of obtained data. This approach proved to be very useful for our research because it enabled us to identify and correct technical issues while running the simulations. Especially, with the popularity of `IPython` console [2] in scientific community, it became very easy to modify the code, change specific parameters of the simulation and render comprehensive visualisations of the data right on the spot without major changes to the main code. The availability of fast and accurate libraries of Python's scientific stack such as `NumPy`[3], SciPy [4] and `Pandas` [5] enabled the minimisation of the amount of code required to build proposed simulation models. Tools such as `MatPlotLib` [6] and `Seaborn` [7] helped me to visualise the obtained data in no time in much comprehensive and effective way. To sum up, above technologies came hand in hand with the iterative research approach I have chosen, due to the short time and little effort required to build, run and modify the experiments.

Below diagram 1.1 highlights the key stages of my research. The downward arrows in the diagram indicate the material resulted from given stage, and the upward arrows indicate the issues identified from successive stage. Below I

[2] http://ipython.org/
[3] http://www.numpy.org/
[4] http://www.scipy.org/
[5] http://pandas.pydata.org/
[6] http://matplotlib.org/
[7] http://stanford.edu/ mwaskom/software/seaborn/

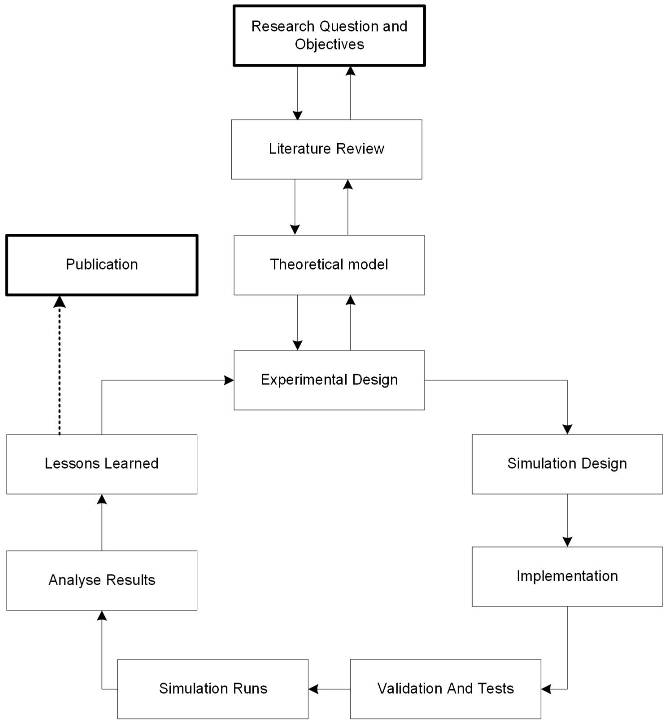


Figure 1.1: Research Lifecycle

provide description of each stage:

- *Research Questions and Research Objectives:* At this stage, I formulated my research questions and specify the list of research objectives that I have to accomplish.
- *Literature Review:* This stage involved the research of the published material on relevant aspects of the work. I reviewed works in areas such as option pricing, microeconomics and market microstructure, automated trading, mechanism design and auction theory.
- *Theoretical Model:* In this stage, I designed a conceptual model for trading agents, mechanisms and their mutual interactions. It involved the design of the trading agents framework, and definition of mechanisms used for pricing options.
- *Experimental Design:* In this state, I defined experiments necessary to conduct in order to reach the research objectives stated. I determined what is planned to test, what the control factors are, what factors are fixed and what they are respective values, what outcome I am expecting to get from the experiments. For example, in order to observe the impact of different traders and mechanisms on option prices, I have to use the

same path of asset prices every time. Hence I decided to fix underlying market, and set traders and mechanisms as control factors.

- *Simulation Design:* This stage determined how I organised the simulation to conduct the experiment. Here I decided how different components of the simulation interact together in terms of data they generate and what functionality each component must have. I also made sure that the simulation model conforms to proposed theoretical model while having the specifics determined for each simulation. I also designed the simulations to be reusable for multiple experiments at the same time.

- *Implementation:* In this stage, I decided which technology to use in order to implement the simulation model. Most of the simulation has been implemented using Python and its scientific stack. But I also used MATLAB for analysing certain aspects of trading agents. This is because MATLAB had good integration with computationally efficient mixed-integer programming tool Gurobi[8].

- *Validation and Testing:* Validation and testing of the simulation model has been interactively accomplished using `IPython` console where I tried different ranges of values for certain parameters, having other parameters fixed, and observed the results generated from them. I tested trivial cases such as if Monte Carlo pricing roughly approximates Black-Schole's method, and if the mechanism populated solely with Black-Scholes traders output the same Black-Scholes result, etc.

- *Simulation Runs:* This stage executes the scripts written for each simulation and save the output in CSV and Excel files. It tests wider range of parameters compared to Validation and Testing stage. Also there are non-trivial hybrid cases involved which are set by the experiment design.

- *Analyse Results:* This stage aggregates and interprets results using data visualisation tools such as `Matplotlib` and `Seaborn`. It also involves measuring the performance of the mechanisms through their allocative efficiency, traded volumes, trader participation and budget-balance. I analysed the option prices through Greeks.

- *Lessons Learned:* In this stage, I summarised findings drawing conclusions about the experiments I conducted based on the analysis of the results. Also I identified the limitations of proposed model in simulating

[8]http://www.gurobi.com/

the markets. These limitations may involve computational limitations as well. For example, I have also noticed that Monte Carlo option pricing with jump-diffusion models take more time than with standard Brownian model. Therefore I had to reduce the number of trials for jump-diffusion model. Also another limitation I observed is the complexity of solving certain problems such as mixed-integer problem.

- *Publication:* In this final stage, I published new findings from experimental study and bound them together into one thesis. Taking into account the limitations I encountered, I propose possible improvements and future plans to develop this research further.

I would also like to stress the backward moving nature of my research methodology. In the initial stages of the research, I have made repetitive modifications to stated research questions and goals based on new findings in literature review. Once covering enough material on topic, I devised a simulation model of trading agents and mechanisms. I went backward to literature review stage several times to justify the validity of the simulation model with published works of other authors. In the iterative experimental phase, I encountered difficulties in attaining certain objectives, so I had to go back to simulation model to adjust the initial architecture. For example, I realised that there is no continuous historical record of option prices for certain type of option, due to big variety of available option contracts in the market, and hence simply lack of open interest on each of them in a continuous time span. Hence I had to adopt historical volatility surface in order to approximate the continuous option prices for given option time.

1.7 Thesis Structure

This section briefly describes each chapter of the thesis.

- *Chapter 2:* In this chapter, I review relevant topics from financial and economical aspects of the research. In Section 2.1, I look into the definition of options, how they are priced, what are the factors that affect option price and some mathematical methods used for analysing their performance. I also illustrate various option trading strategies, and their corresponding profit functions. In Section 2.2, I review what an agent-based economics is and its outstanding implementations until today. Then I focus on the auction theory from the perspective of a computer scientist and discuss the works recently accomplished in this field. I also characterise them based on their basic properties.

- *Chapter 3:* In this chapter, I describe the abstract simulation model consisting of three layers: underlying market, trading agents and mechanisms. In Section 3.2, I describe different stochastic models used to simulate asset prices and interest rates. Section 3.3 proposes a multi-layered framework for designing option trading agents. Section 3.4 talks about the overall flow of the simulation for both direct and online double auctions.

- *Chapter 4:* This chapter proposes the implementation of various option trading agents based on the proposed multi-layered IIKB framework. In Section 4.2, I describe the implementation of the Knowledge layer through 2 cases: calibrating the volatility surface and learning about the informed traders in the market. Section 4.3 implements various option pricing methods and reviews their valuations for predefined configurations. In Section 4.4, I propose the methods used for choosing the quantities for bids and asks. In Section 4.5, I develop proxy trading algorithms for dealers and traders. I also evaluate their results in per-configured setup.

- *Chapter 5:* This chapter presents the design, implementation and simulation results for a direct double auction. In Section 5.2, I determine the key properties of direct mechanisms, and make some assumptions used throughout the chapter. Section 5.3 proposes the new design of a multi-unit double auction, and then extends it for revealed and simultaneous mechanisms for particular types of traders. Section 5.4 determines the instances of traders, options, asset prices and mechanisms to be used in the experiments. In Section 5.5, I verify the results from simulating a hypothetical scenario that has to generate Black-Scholes prices. In Section 5.6, I present the simulation results for various experimental cases and highlight the important aspects of the obtained results.

- *Chapter 6:* This chapter presents the design, implementation and simulation results for an online double auction. Section 6.2 provides the details of the design and implementation of continuous double auction for trading options. In Section 6.3, I define the instances of traders, options, asset prices and mechanisms that are used in different experimental scenarios. Section 6.4 tests the option prices in a hypothetical scenario which must generate Black-Scholes prices. Finally, Section 6.5 provides experimental results with different trader populations, and a close look at each type of trader's performance.

- *Chapter 7:* In this chapter, I suggest further generalisations in pricing option portfolios via combinatorial exchanges. In Section 7.2, I present few examples for potential scenarios where options can be considered as substitutes or complements. Then in Section 7.3, I propose the design of the combinatorial exchange for trading option portfolios. I use TBBL to represent the option portfolios. In Section 7.4, I suggest some improvements to TBBL that could facilitate the communication between traders and mechanism, and create more flexibility in trading option portfolios.
- *Chapter 8:* This chapter draws conclusion to the thesis by emphasising the key findings of the research. I also talk about the current limitations and drawbacks encountered throughout the research, and propose ideas in tackling them in future.

Chapter 2

Background

In this chapter, I review relevant literature starting from the concepts described in textbooks to recent researches written in academic papers. I organized this chapter into three major parts: Options Market, Microeconomical Aspects of Markets and Agent-Based Economics. In Section 2.1, I review basic notions about financial derivatives, and their particular type - options. I define types of options, main parameters associated with options, and the ways how they are priced. I also discuss the stochastic models that are commonly used to simulate asset prices and interest rates. This Section also covers the key aspects of options such as option Greeks that used to measure the performance of the option, and the option portfolios that are commonly taken by the traders in market to cut the losses and at the same time express bullish, bearish or neutral outlook to the market. In Section **??**, I cover the microeconomical principles of markets that are crucial in understanding the competitive behaviour among traders. I cover quasi-linear utility, demand and supply laws, and competitive equilibrium. Section 2.2 provides background and review on recent works in agent-based economics, and particular auctions and double auctions.

2.1 Option Market

In this section, I discuss about financial derivatives, particularly I focus on options, their main attributes, ways how they are priced, trading strategies involving them and tools used to analyse them.

Derivatives are financial instruments the value of which is associated with the value of another asset. Arnoldi [5] defines derivatives as virtual assets the value of which is calculated from the price volatility of its underlying asset. The marketplace where such derivative contracts are traded is called an exchange. Derivatives traded in the exchanges such as Chicago Board of Trade (CBOT)[1] and Chicago Mercantile Exchange (CME)[2] are highly standardized and regulated. Futures contract is one kind of derivatives traded in exchanges and it obliges concerning parties to sell or buy underlying assets at agreed future price. The exchanges strictly specify the terms and conditions of futures contracts traded. Typical futures contract should include exact amount, category or quality of an asset to be delivered, the agreed future price and delivery place [87]. Because both are taking the equal risk of losing from market fluctuations, it is free to enter a futures contract for both sides.

Financial derivatives are used mostly by three categories of traders. Firstly, these are *hedgers* who should mitigate the risk that they are exposed in future. Hedgers secure themselves with futures and options from possible losses that may happen due to future price movements in an underlying market. For example, if the company produces grain then it would like to freeze the next year price of the grain in order to secure itself from sharp price drops. Because the company must invest into the production of grain now, it is in company's interest to justify the costs associated with it. Futures or option contracts are good tools to lock the price in future, therefore company may enter such contract to secure itself from potential loss.

Second category of traders are *speculators* who take the risk of betting on price movements and make profit. Speculators are driven in the pursuit of profit, and they use derivatives in order to monetise their future predictions. For example, consider the case when speculator has private information that quotes for oil will increase next year. Speculator may take long position in futures contract and if his expectations are right next year, he can benefit from buying oil for less price. Economists Keynes and Hicks noticed that hedgers are likely to accept deals below the expected future price, if they tend to maintain their short position and speculators on the other hand try to keep their long position [94, 81]. This phenomenon occurs because speculators do not participate in futures agreement if there is no foreseen gain, however hedgers are subjected to risk and in order to minimize it, they agree to accept less profitable deals.

Finally, the *arbitrageurs* are third important group of derivative traders that

[1] `http://www.cbot.com`
[2] `http://www.cme.com`

look for an arbitrage opportunity by getting involved in two or more markets simultaneously. For example, asset is going to be sold at \$120 next year, and its current spot price is \$100. If interest rate is 10% annual, then arbitrager can borrow \$100 and buy a unit of the asset. Further he can take short position in futures market for this asset and sell it for \$120. Returning the bank loan and interest together as \$110, arbitrager makes risk-free profit of \$10 [87].

2.1.1 Options

I provide more background on European options only, as this is the only type of option used in the scope of this work. I use European option because of its comparatively simple definition and its benchmark role in option pricing theory. When I use the word 'option', I mean European option unless otherwise is stated. Also I mean the underlying asset, when I use the word 'asset'

Option is the type of financial derivative that enables its *holder* (i.e. owner) to buy or sell specified assets at certain future price to *writer* (i.e. issuer) of the option. Holder of the option buys or takes *long position* for an additional cost (i.e. option premium) determined by the market or the writer of the option. On the other hand, the writer of the option sells or takes *short position* by taking future obligation to trade assets if holder chooses to exercise his right to buy or sell. Option contract must specify the *underlying asset* to be traded, its *volume*, *exercise price* (i.e. strike price or future price) and expiration time of the contract. European options can be exercised only on their maturity date, while American options on any date until expiration. Options are classified to *Put* and *Call* options depending on rights and obligations that they bear. Put option gives its holder the right to sell underlying assets at agreed strike price whereas the writer has the liability to buy them when holder exercises his right. Call option gives its holder the right to buy at agreed strike price where the writer has the liability to sell. Options value depends on several parameters of the underlying market and the conditions written in the contract:

S_0 Asset's initial spot price

S_t Asset's spot price at time t

K Option's strike price

T Option's Time to maturity

r Risk-free interest rate

	OTM	ATM	ITM
CALL	$K > S_t$	$K = S_t$	$K < S_t$
PUT	$K < S_t$	$K = S_t$	$K > S_t$

Table 2.1: Options by Moneyness

σ Asset price volatility

The other parameter of the asset (if it is a company stock) is the dividend it yields annually. This is normally subtracted from the overall return the asset is likely to make, and in risk-neutral setting, the dividends are subtracted from the risk-free rate. The omission of dividends from the scope of this research has been discussed in Section 1.3.

Option belongs to different *moneyness* range depending on if its strike is greater or less than the current asset price. Put option is said to be if its strike price is below the market's price, In-The-Money (ITM) if it is above the market price and At-The-Money (ATM) if it is equal to the market price. Call option is said to be Out-of-The-Money (OTM) if its strike price is above the market's price, ITM if it is below the market price and ATM if it is equal to the market price. Table 2.1 summarises the options by moneyness.

I use continuous compound interest to discount or find the future value of the asset. Below is the asset price future value at time T is set to option's strike (2.1):

$$K = S_0 e^{rT} \tag{2.1}$$

In fact, (2.1) determines the strike price K for ATM calls and ATM puts [97]. Because holder has the right, not an obligation to exercise option, and writer has a potential obligation to fulfil his liabilities once option gets exercised, holder has to pay the premium to the writer. Writer's risk is normally associated with the option's expected payoff. If the option is ATM which means that its strike price K is equal to $S_0 e^{rT}$ and its intrinsic payoff is zero, there is a possibility that the option can end up ITM and yield positive payoff. It is for this reason, the expected price of the ATM option can be greater than zero.

The option holder's gain is the difference between underlying asset's spot price and strike price minus the option premium paid to the option writer. On the other hand, writer's gain is fixed in terms of option premium received. The call option holder makes profit if the spot price exceeds the strike price. Similarly, the holder of put option makes profit if the strike price is higher than spot price. It is known that the estimated future price of an underlying asset converges to the spot price, as option approaches its maturity. The further is the maturity date of the option, the greater is the uncertainty about the future price of its underlying asset. Thus it is good for options to have longer period to expire, as the strike price can significantly deviate from spot price and there is greater room for variation.

From this point onwards, I will use the words option price and option premium interchangeably. Option price can vary within its upper and lower bounds. Upper bound cannot be greater than the asset's spot price and lower bound is not less than the option's intrinsic value. Below formulas (2.2) (2.3) illustrate the intrinsic values for call c and put p options at time $t = 0$.

$$c = \max(S_0 - Ke^{-rT}, 0) \tag{2.2}$$

$$p = \max(Ke^{-rT} - S_0, 0) \tag{2.3}$$

Hence, we can determine the valid range for the option price as follows:

$$\max(S_0 - Ke^{-rT}, 0) \leq c \leq S_0 \tag{2.4}$$
$$\max(Ke^{-rT} - S_0, 0) \leq p \leq Ke^{-rT} \tag{2.5}$$

There is an established relationship between put and call options with the same strike price and maturity date. This relationship results from the possibility of buying the one and selling the other. Consider a case, when trader buys a call option at K strike price, and at the same time sells a put option with K strike price, and both have the same maturity T. In some sense, it seems that trader can compensate the cost of a call option he bought for with the premium he received for selling put. So on maturity date, S_T turns out to be higher than strike price K, so the trader can benefit profit as a difference of $S_T - K$. However if S_T appears to be less than K, then trader has a liability to fulfil the put option that he sold, so he incurs a loss of $K - S_T$. This market position actually simulates a forward contract which could be obtained for free. This type of contract is free because it involves future possible liability or profit at the same time, so the risk for both parties is even. Once the combination of put and call options can replicate the liabilities of a forward contract, the prices for put and call options must hold the *put-call parity* relationship given in (2.6). Using the put-call parity relationship, we can easily convert call prices to put prices, and vice versa [82].

$$c(K,T) - p(K,T) = S_T - Ke^{-rT} \tag{2.6}$$

2.1.2 Option Exchanges

In this section, I provide brief overview of real-life option exchanges, the way how they are organised and run. Then I stress out the differences of my simulation model from the existing real-life exchanges. I also provide some insights on the main advantages and disadvantages of both systems.

Option market is strictly regulated by exchanges and clearinghouses which apply robust mechanisms to enable the successful fulfilment of liabilities that arise from trading options. Trader playing in any exchange must open a *margin account* with the broker in order to leverage his initial investment. Margin account is an account used by a broker as a collateral to provide short-term loans to the trader to participate in the market. The funds in this account are normally used to cover potential losses that the trader may incur during the loan's term. After depositing the initial investment (also referred as initial margin), trader can submit orders the total value of which does not exceed the rate determined by corresponding clearinghouse. For example, if the initial margin is 20% of the requested loan, then trader has to deposit $20 for each $100 requested.

Margin account is necessary for the trader who issues the option, because he should maintain sufficient funds to cover his potentially unlimited liability in front of buyer of that option. As the prices fluctuate, so they fall below agreed strike price, the funds from option issuer's margin account are transferred to buyer's margin account. This is also referred as *marking to the market.* If the funds in margin account decrease below certain maintenance threshold, option issuer receives a maintenance call from the broker requesting him to top it up to an agreed initial margin. If trader defies the maintenance call, broker has the right to close the outstanding liability of the trader at the cost of funds remaining in the margin account. This may involve buying out an equivalent option which has been issued by the trader.

Traders may short naked or covered options. The former is an option that is not supported with an underlying asset bought beforehand to guarantee the delivery of the asset once option gets exercised. The latter is an option whose underlying asset is owned by the writer at the moment of sales. Trader is allowed to write a naked option if his initial margin is greater than the option premium received plus 20% of the underlying asset's price. If the price of an underlying asset is less volatile, then the option premium overhead can be decrease to 15%. With regard to covered call options, writer does not have to maintain the margin, as he can always sell owned assets for the price stated in the option. Similarly, margin account is also not generally applied to buyers of the options limiting their ability to leverage. Option buyer has to pay the full price of the option to be purchased beforehand.

There are three ways of closing positions in the option exchange. First way is the actual delivery of the asset at an agreed strike price. The second method is that the option issuer pays the positive difference between the spot price and the strike price at the maturity of the option contract. The last method is that the trader takes an opposite position in the market which eventually offsets his current open position. For example, the issuer of call option can buy an equivalent call option at any point before option's maturity to close his position for the sold option.

Above I briefly described the inner workings of real option exchanges, although most of these details will be ignored in my proposed model due to their irrelevance in a computer simulated environment. Instead, I will focus on more integral components of an option market such as the underlying market, the traders and their pricing and bidding strategies and the market itself. My model does not involve brokers or clearinghouses which stay in between traders and the market, and ascertain that the mutual liabilities are honoured. The proposed mechanism involves one agent who plays the role of a market-maker and matches the orders in a specified way. Below I summarises the key differences between the real option market and my proposed simulation model:

- *Brokers:* Unlike in real option exchanges, in my proposed model there are no brokers who would maintain traders' accounts and place orders on their behalf. However there is a mechanism which accepts the orders and matches them resulting in an efficient trade between participating traders. Traders can directly interact with the mechanism and get their orders cleared in the process. Also mechanism does not bear any responsibility in mitigating the potential losses that may occur due to unpaid liabilities. In fact, I assume that the traders always honour their liabilities, so the mechanism does not undertake any action upon preventing the trader's default and simply exercises the outstanding options on their maturity date by transferring required amounts between corresponding traders' accounts. Mechanism also provides basic functionality to submit or withdraw orders from the order book.

- *Margin Accounts:* The mechanism who partially plays the role of a clearinghouse does not maintain any accounts of the traders in my simulation model. The accounts are directly managed by the traders themselves. Also there is no opportunity for leverage for the traders, as they have to use their endowed cash fully for both buying options and paying the liabilities on sold options. This important distinction between real exchange and the simulated model is made due to my assumption that none of the automated traders will default, although most are assumed to have access to unlimited funds, some automated traders such as inventory-based trader should be restricted to the amounts in his inventory accounts.

- *Options:* As I mentioned already, the simulation model deals only with European options. Moreover, all European options traded in my simulation model will have the same maturity date and will be on the same underlying asset. This narrows down my simulation model only to the set of European options differing in strike price and option type. Of course, in real-life option exchanges there are myriad of different options varying in their maturity date, style, strike price, underlying asset and type, and they are all traded simultaneously on the same platform.

- *Underlying Market:* In contrast to real option exchange where traders can get involved in the underlying market to cover their outstanding liabilities in sold options, in my simulation model traders have no access to such market. In practice, the underlying market is by itself a real marketplace where the asset prices result from the ongoing trades. However in my simulation model, the underlying market is a stochastic model which simulates the price movements in the underlying market. Traders cannot be directly involved in buying or selling assets from the underlying market, because this would impose additional complexity on their corresponding impact on the formation of asset prices in the underlying market. Therefore, I assume that the option traders are simply price-taker towards the underlying market. I will discuss more about how I model the trajectory of the asset prices more in Section 2.1.4.

- *Risk-Free Market:* Similar to underlying market, the risk-free market is also another class of marketplace where the risk-free assets such as Treasury bills are traded. Their corresponding prices also result from the ongoing transactions in this market. However, in my simulation model this is either a constant value or a stochastic process which replicates the movements in this market. Similar to underlying assets, the option traders are price-takers towards this market, and simply use the emerged rates in computing their risk-neutral valuation of options. I will discuss about this process more in Section 2.1.4.
- *Market Frictions:* Unlike in real exchanges where every transaction bears some cost in the form of broker's commission, borrowing cost, taxes, insurance, etc, the simulation model does not involve such costs in the trading process. Also I assume that there are no frictions in the information flow across the marketplace. All of the information which is publicly available is set to be known to every agent. I talk about this fact more in Chapter 3.
- *Traders:* The simulation model involves only automated traders whereas the real option exchanges may involve both human and automated traders. The automated traders generally base their pricing and trading decisions on certain historic data and private beliefs embedded into the agent's internal structure, while human traders may also involve fundamental analysis of the markets through digesting news and forming subjective opinion from the performance of markets, companies and governments. I describe the formal methods that are commonly used in pricing options by both humans and automated traders in Section 2.1.5. The trading algorithms that are used in my model are described in Chapter 4.

2.1.3 Competitive Markets

In this section, I discuss about the basic economic principles of markets in general, the forces that form them and the way how economists study them. In some sense, it may contradict with options market, because options are virtual assets, and anyone can issue them in unlimited amounts. Also the demand and supply rules are different in financial markets, as the drop in price does necessarily increase demand, but can further push down the prices, or vice versa. In order to determine the demand or supply for option, I accept using the ratio of the market option price to its risk-neutral price. In this way, it is intuitive to suggest that when the option is overpriced (its ratio is more than one), then there is less demand for such option, and vice versa. Despite being a virtual asset, an option can also be considered as a limited resource, because it involves risk for the trader who issues it, and due to this risk there can be a scarcity of options in the market. Moreover multiplicity of variations of options in the market scatters the overall demand and supply across each causing some options being popular over others.

I describe how the clearing prices are achieved in competitive markets. Traders can both buy or sell options in any quantities, and in my simulation model, the aggregate demand and aggregate supply of the options ultimately affect the price of the option traded in the marketplace. For example, if trader needs to buy certain type of option to hedge his risk at some other market, but he cannot find that particular option in sufficient quantities, he might be forced to increase its price above its risk-neutral value to attract more sellers. This of course results in the rise of the option price. In order to present how the clearing option price is determined, let me define aggregate demand and supply functions.

Definition 2.1.1. Aggregate Demand:

$$x^D(p) = \sum_{i=1}^{D} x_i^D(p) \tag{2.7}$$

where D is the number of buyers, $x_i^D(p)$ is demand function for buyer i for given price p.

Definition 2.1.2. Aggregate Supply:

$$x^S(p) = \sum_{i=1}^{S} x_i^S(p) \tag{2.8}$$

where S is the number of sellers, x_i^S is supply function for seller i for given price p.

The allocation of the options in the market is x_i quantity of options to be bought by each buyer i and x_j quantity of options to be sold by each seller j. Any allocation is called feasible for given price p if x_i and x_j satisfy following conditions:

Definition 2.1.3. Feasible Allocation:

$$
\begin{aligned}
x^D(p) &= x^S(p) \\
\sum_{i=1}^{D} u_i^D + \sum_{j=1}^{S} u_j^S &= \sum_{i}^{D} W_i
\end{aligned} \tag{2.9}
$$

where D is the number of buyers, S is the number of sellers, $u_i^D = W_i - px_i$ utility of buyer i, $u_j^S = px_j$ utility of seller j, W_i initial wealth of buyers.

I assume that every trader is rational and therefore chooses best quantity for demand or supply based on given price p. Traders reach *competitive market equilibrium*, if there is a feasible allocation for given price $p*$ which maximises both aggregate demand and supply. The well-being of all buyers and sellers can be measured by the their cumulative utilities referred as Consumer Surplus (CS) and Producer Surplus (PS) respectively. Similarly the well-being of the market is measured using Social Surplus (SS) which is the difference between CS and PS.

Definition 2.1.4. Social Surplus:

$$SS(\mathbf{x}) = \sum_{i=1}^{D} u_i(x_i) - \sum_{j=1}^{S} u_j(x_j) \tag{2.10}$$

where $\mathbf{x}$ is the feasible allocation for both buyers and sellers.

The competitive markets maximise SS making the allocation Pareto optimal for all traders. Pareto optimality implies that items sold and bought are results of trader's individual rational decision making towards maximising utility, and no one is better off from other allocation. If the feasible allocation is Pareto optimal, it is said to be an efficient allocation. Although the traditional markets achieve such allocation through Adam Smith's 'Invisible Hand of The Market', my simulation model will find it by solving an Linear Programming (LP) constrained to the corresponding demand and supply in the market.

2.1.4 Underlying Market

In this section, I review the stochastic models that are commonly used to simulate the underlying assets market in option pricing. Although the underlying market itself is the result of the transactions happening inside a competitive market, classic finance views the asset prices as a random walk process. Unlike to deterministic functions, stochastic process implies that the current state of the system may proceed to several possible states with some probability. This can be translated to the underlying market where the next price of the asset is determined by the arrival of the next order to the market. In fact, Bachelier pioneered the study of stock markets using stochastic processes in 1900 [13]. The key element of a random walk model is that the state of the system become more and more uncertain as it goes further in time. The models described here are not the exhaustive list of asset pricing models used by financial analysts, but the most basic and important ones. I discuss more about specific types of stochastic processes that can help us to model the changing behaviour of asset prices and interest rates.

Geometric Brownian Motion Model

GBM is a stochastic process which moves from its current state to the next state with lognormal probability, and the magnitude of the change in states is measured by the time interval between the states. The bigger is the interval, the greater is the probability that the next state would significantly differ from the current state. Similarly in an infinitesimal timeframe, the changes are also infinitesimal, but random. The cumulative effect of this infinitesimal changes in infinitesimal time steps would result in the random walk path which could follow any direction from any given point, disregarding the previous states. This complies with the *efficient market hypothesis* where the next state of the market cannot be predicted from its previous states, and only depends on the current state of the market. This is also called the *Markov property* of the system. Hence it can be used to model underlying asset markets as an efficient markets, and simulate the asset prices using the GBM model. It can be written as follows[89]:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{2.11}$$

where

$dS(t)$ infinitesimal change in the asset price

dt infinitesimal change in time

$dW(t)$ is Wiener process, such that $W(t)$ is continuous over t, $W(0) = 0$ and $dW(t) \sim \mathcal{N}(0, dt)$

$S(t)$ asset price at t

μ mean drift, $\mu \in \mathbb{R}$

σ volatility, $\sigma \in \mathbb{R}^+$

.

We can see that the Equation (2.11) has two terms: deterministic term and the diffusion term. Deterministic term defines the drift of the function over time, and it is described as an exponential function with regard to drift rate μ. The diffusion term determines the uncertainty in the next step, and is scaled by the volatility σ. The greater is the volatility, the greater is the chance that the asset's next step can be a significant. Thus it creates room for greater loss on one hand, or greater profit on the other hand. Thus volatility can be referred as the measurement of risk in GBM model. This would mean that the option's price should be related to the risk of engaging the in the underlying market. $dS(t)$ represents the absolute changes in the asset price, but if one needs to find the change in return, the logarithmic change should be computed. It is given in below formula:

$$d \log S(t) = (\mu - 0.5\sigma^2)dt + \sigma dW(t) \tag{2.12}$$

where $d \log S(t)$ represents the change in return of the asset price.

In this way, the asset price is guaranteed not to fall below zero. This is important for the simulation of asset prices using GBM model. This way of modelling the asset prices has been used by Black and Scholes [18] in computing the risk-neutral option prices of which I shall discuss later in this chapter.

Jump-Diffusion Model

Jump-Diffusion (JD) model has three terms, two of which are the same as in GBM: drift and diffusion, and the third one is the jump. It models an asset price which can exhibit discrete jumps the arrival of which is modelled using Poisson process. The magnitude of the jump itself is modelled using Normal random variable. This results the distribution of the asset price returns to be fat-tailed. Merton applied this model to option pricing for the assets that exhibit sharp shocks [114]. The stochastic model is described below:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) + S(t)dJ(t) \tag{2.13}$$

where

$dJ(t) = (Y_{N(t)} - 1)dN(t)$ - jump process with $Y_{N(t)} \sim \log\mathcal{N}(\mu_Y, \sigma_Y)$ is the jump magnitude.

$N(t)$ - Poisson process $N(t)$ of an intensity λdt.

.

We can see in the third term of the Equation (2.13) that the jump process is modelled as a discrete term which makes $dJ(t)$ take either zero or one depending on the intensity of jumps λ over the period of T. When $dJ(t)$ takes one, the random variable $Y_{N(t)}$ is drawn from lognormal distribution, and used to determine the discrete change in the asset price. Otherwise when $dJ(t)$ is zero, the process is the same as GBM. Merton found an analytical solution for pricing options using this asset pricing model. I dropped this valuation from the scope of this research.

The absolute changes in asset price are converted into returns using logarithmic function:

$$d\log S(t) = \left(\mu - 0.5\sigma^2\right) dt + \sigma dW(t) + \log(Y_{N(t)})dN(t) \tag{2.14}$$

Vasicek Model

The interest rates usually exhibit a mean reverting behaviour in the market. This is the property when the process returns to a certain constant and fluctuates around it over time. This process involves autoregressive model AR(1) meaning that the previous state of the system determines the current state. The effect of the previous state to the current state is determined by a autocorrelation coefficient $|\alpha| < 1$. The AR(1) process is defined below:

$$r(t+1) = \mu + \alpha r(t) + \sigma\epsilon_{t+1} \Rightarrow \tag{2.15}$$

$$\Delta r(t+1) = (1-\alpha)(\frac{\mu}{1-\alpha} - r(t)) + \sigma\epsilon_{t+1} \tag{2.16}$$

where α is an autocorrelation coefficient, $\epsilon_{t+1} \sim \mathcal{N}(0,1)$ standard perturbation.

It can be seen in Equation (2.16) that the next value of the interest rate is partially autocorrelated to the previous state of the interest rate. AR(1) is defined over discrete t, but if t is set to be continuous, it replicates Vasicek's model[158] process with constant coefficients:

$$dr(t) = \alpha(\theta - r(t))dt + \sigma dW(t) \tag{2.17}$$

where α, θ and σ are all positive.

The limitation of the model is that it has positive probability of negative values, and in order to overcome this the Exponential Vasicek model is used. This can be reformulated into returns of the interest rate using logarithmic function:

$$dr(t) = \alpha r(t)(\theta + \frac{\sigma^2}{2\alpha} - \log r(t))dt + \sigma r(t)dW(t) \tag{2.18}$$

Vasicek-Jump Model

The Vasicek-Jump model, similar to jump-diffusion model, include a jump process defined by the Poisson random arrivals. It is used to simulate the interest rates that exhibit sudden jumps due to sudden changes caused by important news. It is defined as following stochastic process:

$$dr(t) = \alpha(\theta - r(t))dt + \sigma dW(t) + dJ(t) \tag{2.19}$$

where

$dJ(t) = Y_{N(t)}dN(t)$ - jump process determined by $N(t)$ Poisson process with λ intensity and $Y_{N(t)} \sim \mathcal{N}(\mu_Y, \sigma_Y)$ is normally distributed jump magnitude.

2.1.5 Option Pricing Models

It is an outstanding concern in financial theory to determine the fairest price for given option such that it could reflect the risk associated with buying or selling it. Assuming that markets are efficient and martingale [58], which suggests that every new information gets immediately embedded into the price of an asset and no previous records can be used to predict asset's future price, it is impossible to tell the future price of an asset. This uncertainty emerges risk which needs to be properly evaluated. The option pricing models discussed below take following assumptions on the assets market:

- All traders are risk-neutral. It means that the expected return from trading options is the risk free interest rate r for each trader.
- Risk-free interest rate (r) is constant.
- Risk -free interest rate (r) is the same for lending and borrowing money.

- Variance (σ^2) of the asset price is constant.
- There is no arbitrage opportunity in the market.
- There is no additional transaction cost associated with the trade.

The key assumption that is common for all methods is the absence of any kind of arbitrage which refers to a profit higher than risk-free profit. This enables us to tie pricing methods only with risk associated by isolating them from other factors. In other words, the greater is the risk, the greater will be the price, and vice versa. Also practice shows that arbitrage opportunity is unlikely to last long in contemporary markets, where information flow is fast and highly interconnected. Once investors notice arbitrage opportunity, all of them will try to gain from it, which eventually result in its dissolution [108].

In this section, first I define what the arbitrage-free assumption means, and then provide two methods commonly used for option pricing under this assumption. First is binomial option pricing model, which directly applies the arbitrage-free assumption to option pricing by spanning out a binomial tree for possible asset price changes, and calculating the expected gain from given scenarios. This model is the key for Monte-Carlo simulation of option prices. Secondly, Black-Scholes model which extends this concept further taking into account all possible cases of asset price changes under lognormal distribution, and then calculates the expected gain for given option.

No Arbitrage Assumption

The crux of the *no-arbitrage assumption* means that one cannot make riskless profit higher than the risk-free interest rate by simultaneously engaging in different positions in the market. If there is such riskless profit, then one can take advantage of this arbitrage opportunity. There are 2 types of arbitrage opportunities that may arise in the market:

- *Type A arbitrage:* A security or a portfolio that produces immediate payoff at $t = 0$, and has non-negative value at $t = 1$. In other words, $v_0 < 0$ and $v_1 \geq 0$. This would mean that the trader receives certain positive payoff for a security now, and in future he, at least, has no obligation to pay back. In simpler terms, this can be described as someone who found dollar bill in the street.
- *Type B arbitrage:* A security or a portfolio that produces at least zero

payoff at $t = 0$, and has at least one guaranteed payoff in future. In other words, $v_0 \leq 0$ and $v_t \geq 0, \forall t > 0, \exists t' > 0; v_{t'} > 0$. This would mean that the trader pays at most zero for the security, but there is a chance in future that the value of the asset is strictly greater than zero, yielding a riskless payoff. This can also be described as finding a lottery ticket in the street.

Let me give an example on how the arbitrage situation can occur in the asset market. For example, let us assume that the asset price has a positive drift $\mu > 0$, and the risk-free rate is $r > 0$, and $t = 0$. So the asset's expected return at time $t = 1$ is $\mu + 1$. Let the maximum possible decrease in asset price is measured by return $1 + \mu - \epsilon$, for $0 < \epsilon < \mu + 1$, and the maximum possible increase in asset price is $1+\mu+\epsilon$. Then if $(1+r) < 1+\mu-\epsilon < 1+\mu+\epsilon$, there is a type B arbitrage for the trader. Trader can borrow money at r, and invest it in stock. At $t = 1$, trader sells the stock, and realises more than he has to pay for the borrowed money in any case. Trader takes the difference between asset price at $t = 1$ and the interest payable, and makes profit with no risk. The opposite situation where $(1+r) > 1+\mu+\epsilon > 1+\mu-\epsilon$ can also be exploited as an arbitrage opportunity. Therefore the arbitrage free boundary must set the asset price drift and volatility into given inequality $1+\mu+\epsilon > r+1 > 1+\mu-\epsilon$.

Binomial Option Pricing Model

This option pricing method uses binomial tree (i.e. tree-like graph which has a root, n leaves, and $\binom{n}{k}$ paths from root to its kth leaf) for projecting the two possible cases for the asset prices in discrete time steps, and based on these projections computes the expected option's payoff. It has been first proposed by Cox-Ross-Rubinstein in 1979 [35]. Figure 2.1 illustrates one-step binomial tree structure. This represents two possible directions for asset price, one when price goes up by factor $u \geq 1$ and the other when price goes down by factor $0 < d \leq 1$. This complies with the assumption of the asset price as a random-walk process, but in this scenario asset has only 2 directions to follow: S_0u or S_0d. Now let us consider a trader who sells a call option for f amount, and invests this money to buy Δ shares of an asset. Hence his current portfolio is $S_0\Delta - f$. In order to conform with the arbitrage free argument, we must ensure that in all two possible cases of an asset price at time $t = 1$, the trader's portfolio should be equal to the risk-free investment of his current portfolio.

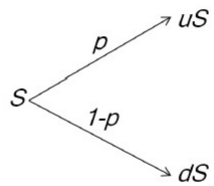


Figure 2.1: One-step Binomial Tree

So:

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT} \tag{2.20}$$
$$S_0\Delta - f = (S_0 d\Delta - f_d)e^{-rT} \tag{2.21}$$

Solving the above equation for Δ gives:

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d} \tag{2.22}$$

And if we replace Δ with corresponding term:

$$f = \frac{f_u(1 - de^{-rT}) + f_d(ue^{-rT} - 1)}{u - d} \tag{2.23}$$
$$= e^{-rT}(pf_u + (1-p)f_d) \tag{2.24}$$

where $p = \frac{e^{rT}-d}{u-d}$ is the risk-neutral probability of upward movement.

Note that the option pricing formula takes into account neither the real drift of the asset price μ, or the real probability of asset price moving upwards or downward. So this would mean that even if there the real probability of asset price moving upwards is $q = 0.99999$ and downwards is $1 - q = 0.00001$, this would make no difference in pricing the option's value. This is also referred as the change of the probability measure of an asset price from historical probability to risk-neutral probability.

This simple model can be further generalised into multiple steps as shown in Figure 2.2. In this case, the leaf nodes of the tree are computed as $S_{i,j} = u^j d^{i-j} S_0$ where i denotes the time and j denotes the potential state of an asset price at time t. If there are n time steps, then we can also see that there are $\binom{n}{j}$ paths to the jth leaf node. This would mean that the asset price is more likely to end up around the middle leaf nodes, rather than top and bottom nodes. We can use the binomial distribution of final states j of the asset price, to iteratively compute the option price f. Below formula summarises the option

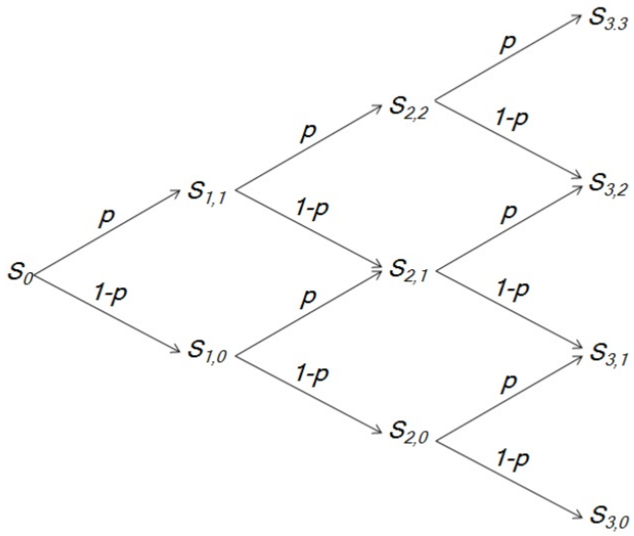


Figure 2.2: Multi-step Binomial Tree

price in a multi-step lattice.

$$f = e^{-rT} \sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \max(S_0 u^j d^{n-j} - K, 0) \tag{2.25}$$

Black-Scholes Model

In 1973, Black and Scholes proposed an analytical solution for pricing the options with a risk-neutral measure[18]. According to Black-Scholes model the asset prices are viewed as GBM with a drift $\mu = r$ to address arbitrage-free assumption. We can also see from binomial approximation of the option price that the asset price drift is not used in projecting the future asset price. In fact, the Equation (2.25) can be generalised into a Black-Scholes analytical formula if the number of steps $n \to \infty$, and the discrete binomial distribution converges into continuous normal distribution. However from binomial option pricing formula, we can see that the upward or the downward movement of the asset price plays a key role in determining option's value. This idea is generalised into standard deviation, or in other words, volatility. Black-Scholes model also shows how option prices can be calculated based on the volatility of an asset price. Let $f(t, S(t); \sigma, r)$ denotes option price function based on the asset price $S(t)$ and parametrised by its volatility σ and risk-free interest rate r. The asset price $S(t)$ is GBM function, and t is changing variable, and the other parameters such as σ and r are assumed constants. Then the derivative of this function is equal to 2nd order Taylor expansion according to Ito's lemma

[91]. This results in following generic stochastic differential equation [122, 52]:

$$rf = \frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \tag{2.26}$$

The solution of Equation (2.26) gives a closed form function f for computing the option price based on r, σ and t. $S(t)$ is not passed as an argument because it is actually the function of other given parameters.

Knowing the payoff functions for call $\max(S_T - K, 0)$ and put $\max(K - S_T, 0)$, the analytical solution for $f(t, S(t))$ can be found by taking the partial derivatives. So the solution for call (c) and put (p) can be computed as follows:

$$c = (S_0)N(d_1) - Ke^{-rT}N(d_2) \tag{2.27}$$

$$p = Ke^{-rT}N(-d_2) - (S_0)N(-d_1) \tag{2.28}$$

where

$d_1 = \frac{\ln\frac{S_0}{K} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$

$d_2 = \frac{\ln\frac{S_0}{K} + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$

$N(\cdot)$ standard normal distribution

2.1.6 Option Portfolios

In this section, I review different types of option portfolios used in practice. Traders can take different positions with options of different moneyness and create option portfolios which can align with their forecast and at same time limit their loss in case if their forecast is not true. Cohen counts more than 40 trading strategies and classifies them based on their market direction (i.e. bullishness or bearishness), volatility level, riskiness and gain [31]. I will not review all of them, but consider only the ones that I use in the scope of this work.

Option Portfolios with Single Option

Traders can cut their loss or gain fixed amount using one of the following option portfolios:

1. *Short Call:* Trader buys Δ asset at S_0 and shorts call option at c with strike price K. Long position in asset will cover the trader's liability if there is sharp rise in asset price, thus trader makes fixed profit. If asset price goes down, short call will not be exercised by its holder, however trader will loose in his long position. Nonetheless there is an interval between S_0 and K which could minimize the risk of loss. Formally trader's profit P can be written as follows:

$$P = \begin{cases} K + c - \Delta S_0 & \text{if } S_T \geq K \\ S_T + c - \Delta S_0 & \text{if } S_T < K \end{cases} \tag{2.29}$$

2. *Long Call:* Trader takes short position by selling Δ asset at S_0 and buys call option at c with strike price K. If asset price goes up, trader caps the risk of loosing infinitely using his call option. If the price goes down, trader makes profit in his short position without call option. Trader's profit P is below:

$$P = \begin{cases} \Delta S_0 - c - K & \text{if } S_T \geq K \\ \Delta S_0 - c - S_T & \text{if } S_T < K \end{cases} \tag{2.30}$$

3. *Long Put:* Trader buys put option at p with strike price K and at the same time buys Δ asset at S_0. If asset price goes up, trader makes profit in his long position. If asset price goes down, he can sell asset at K using his put option, thus saving himself from infinitely loss. Trader's profit P is written below:

$$P = \begin{cases} S_T - \Delta S_0 - p & \text{if } S_T \geq K \\ K - \Delta S_0 - p & \text{if } S_T < K \end{cases} \tag{2.31}$$

4. *Short Put:* Trader takes short position by selling Δ asset at S_0 and writes put option for p with strike price K. If asset price goes up, he looses infinitely in his short position. If asset price goes down, he wins in his short position but it will be offset by short position in put option, thus it makes only fixed profit.

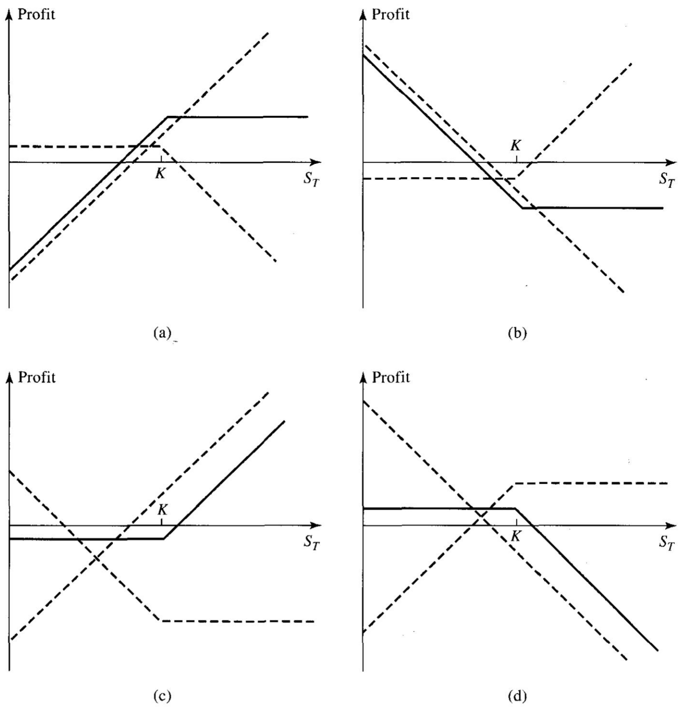


Figure 2.3: Option Portfolios with Single Option[1]

$$P = \begin{cases} \Delta S_0 + p - S_T & \text{if } S_T \geq K \\ \Delta S_0 + p - K & \text{if } S_T < K \end{cases} \tag{2.32}$$

Figure 2.3 summarizes four main option portfolios using single option discussed above.

Spreads

A spread is another type of option portfolio which involves taking a position in two or more options of the same kind, but different moneyness. Below I summarize important spreads:

1. *Bull Spread:* Bull trader hopes that the asset price will increase. Therefore he buys call option at c_1 with strike price K_1 and sells another call option c_2 with higher strike price K_2. Both options have same expiration date. Because K_2 is higher than K_1, c_2 should be lower than c_1. If the asset price becomes less than K_1, trader will loose $c_2 - c_1$. If the asset price falls between K_1 and K_2, then trader's payoff is the difference between current price S_T and K_1 minus the difference in option prices.

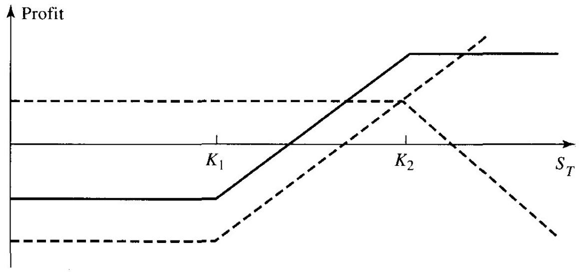


Figure 2.4: Bull Spread with Call Options.[1]

Bullish trader wants the asset price to go up, so he can make fixed profit of $K_2 - K_1$ minus the difference in option prices. The same bull spread can be made with put options as well, so the trader buys put with low strike price and sells another put with high strike price. Payoff for bull spread with call options is written below (2.33):

$$P = \begin{cases} c_1 - c_2 & \text{if } S_T \leq K_1 \\ S_T - K_1 - (c_1 - c_2) & \text{if } K_1 < S_T < K_2 \\ K_2 - K_1 - (c_1 - c_2) & \text{if } S_T \geq K_2 \end{cases} \tag{2.33}$$

where $c_1 > c_2$ and $K_1 < K_2$

Figure 2.4 illustrates bull spread with call options.

2. *Bear Spread:* Bearish trader hopes that the asset price will decrease. Thus he buys call option at c_2 with high strike price K_2 and sells another call option at c_1 with low strike price K_1. Similarly, c_1 should be greater than c_2. Both options have same expiration date. If the asset price turns to be greater than K_2, the trader looses fixed amount in his short position. If the asset price is less than K_1 then trader makes fixed profit as the difference of $c_1 - c_2$. If the asset price falls between K_1 and K_2, then the profit is the difference of option prices minus the difference of S_T and K_1. Traders can enter bear spread with put options too. Below is the formula (2.34) that shows the profit for bear spread of call options:

$$P = \begin{cases} c_1 - c_2 & \text{if } S_T \leq K_1 \\ (c_1 - c_2) - (S_T - K_1) & \text{if } K_1 < S_T < K_2 \\ (c_1 - c_2) - (K_2 - K_1) & \text{if } S_T \geq K_2 \end{cases} \tag{2.34}$$

[1] Source: Hull et al, Options, Futures and Other Derivatives, 5th Ed, 2001 [87]

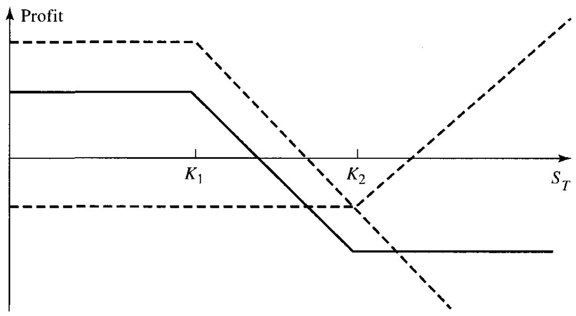


Figure 2.5: Bear Spread with Call Options.[1]

where $c_1 > c_2$ and $K_1 < K_2$

Figure 2.5 illustrates bear spread with call options.

3. *Butterfly Spread:* This type of spread involves taking positions in options with three different strike prices. In *long butterfly spread* trader has an estimate that the price is not going to change sharply, so he buys 2 call options: one with low K_1 and another one with high K_3. At the same time, he sells 2 call options with K_2, where K_2 is halfway between the range of K_1 and K_3. This spread leads to a profit if the asset price is deviating around K_2. It will incur in fixed loss if the asset price changes sharply in either direction. Butterfly spread can be used for put options as well. If trader hopes that the asset price changes sharply, he can enter *short butterfly spread* which involves selling 2 options at K_1 and K_3, and buying 2 options at K_2, where $K_1 < K_2 < K_3$ condition holds. Below formula (2.35) shows the profit for long butterfly spread with call options:

$$P = \begin{cases} 2c_2 - c_1 - c_3 & \text{if } S_T \leq K_1 \\ (S_T - K_1) + 2c_2 - c_1 - c_3 & \text{if } K_1 < S_T < K_2 \\ (K_3 - S_T) + 2c_2 - c_1 - c_3 & \text{if } K_2 < S_T < K_3 \\ 2c_2 - c_1 - c_3 & \text{if } S_T \geq K_3 \end{cases} \tag{2.35}$$

where $c_1 > c_2 > c_3$ and $K_1 < K_2 < K_3$

Figure 2.6 illustrates long butterfly spread with call options.

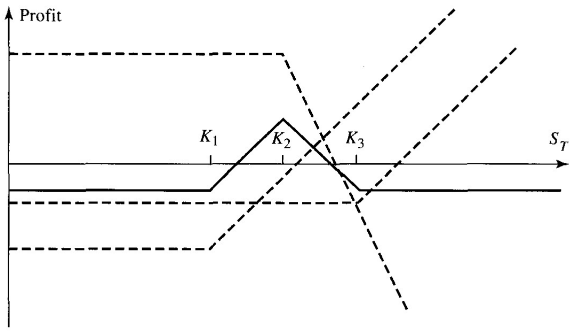


Figure 2.6: Butterfly Spread with Call Options.[1]

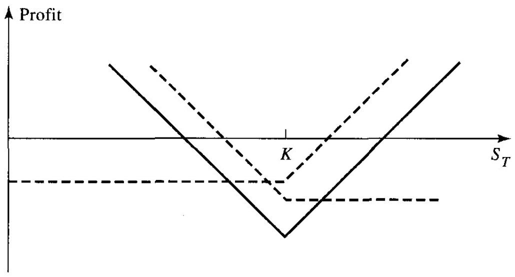


Figure 2.7: Long Straddle Combination.[1]

Combinations

Combination is an option portfolio that makes the use of both calls and puts of different moneyness. Below I summarize the main types of combinations.

1. *Straddle:* If the trader is sure that the asset price is going to change sharply in either directions, he can create straddle combination which involves buying call c and put p options with the same strike price K. Below formula (2.36) shows the profit pattern for straddle portfolio:

$$P = \begin{cases} K - S_T - c - p & \text{if } S_T \leq K \\ S_T - K - c - p & \text{if } S_T > K \end{cases} \tag{2.36}$$

   where $c = p$

   Figure 2.7 illustrates long straddle combination.

2. *Strips and Straps:* Trader can change the slope of either side his straddle combination legs based on his estimate about the direction of price movement. If trader buys one call option and two put options, he expects asset prices to drop significantly so he can make twice more profit in his short position. *Strips* is the combination when trader is more in short

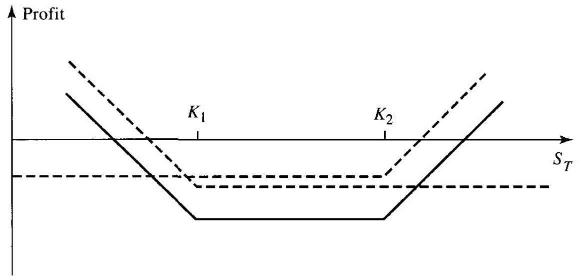


Figure 2.8: Long Strangles Combination. [1]

position, and *straps* is the combination when trader is more in long position. The profit pattern for these combinations is the same as straddle but it is multiplied by the number of corresponding options if the price follows the expected direction.

3. *Strangles:* This combination involves put and call options with the same expiry date, but with different strike prices. Trader buys call option c with high strike price K_2, and put option p with low strike price K_1. It is similar to straddle combination, but the asset price should change more dramatically than straddle combination expects for the trader to make profit. On the other hand, trader pays less for acquiring options in strangle compared to straddle. Formula (2.37) below is the profit function for strangles combination:

$$P = \begin{cases} K_1 - S_T - c - p & \text{if } S_T \leq K_1 \\ -c - p & \text{if } K_1 < S_T < K_2 \\ S_T - K_2 - c - p & \text{if } S_T \geq K_2 \end{cases} \tag{2.37}$$

where $c < p$ and $K_1 < K_2$

Figure 2.8 illustrates long strangles combination.

2.1.7 Greeks Analysis

Greeks analysis provides set of measurements for evaluating the sensitivity of option price on different factors in the market. They have huge role in hedging portfolios and evaluating the volatility of the asset prices. It also allows us to create options synthetically using a *replicating portfolio* described in binomial pricing model. For example, one can create a portfolio of cash and stock, where cash has a deterministic risk-free return in future, and stock has uncertain return in future. The replicating portfolio would consist of $\Delta S_0 + c$ at time $t = 0$, where Δ is the option's change in values with respect to the change in asset price. Further in this section we will discuss more about various ways of hedging using Greeks analysis. Replicating portfolio simulates the option's payoff, and synthetically creates such financial instrument.

1. *Delta* (Δ): It measures the rate of change of the option price with respect to the change in price of the underlying asset. For example, if the delta of the call option is 0.4, then it means that if there is small change in the underlying asset's price, there will be a change in call options price in 0.4 of that amount. This technique can be used in hedging, as we know how many assets we have to buy in order to cover our short position in options. For example, if investor writes off 20 call options that gives the right to buy 2000 shares, then for the investor it is enough to buy 800 shares to cover losses he may incur in future due to price changes in shares. Delta is defined as the partial derivative of option's price function with respect to underlying asset price [87, 15].

   Another meaning of delta is the probability of an option ending up in-the-money, assumed the price moves under GBM. For example, if the delta of out-of-the-money call option is 0.2, then it means that there is 20% chance option will expire in-the-money.

$$\text{Delta's definition:} \qquad \Delta = \frac{\partial c}{\partial S} \tag{2.38}$$

$$\text{Black-Scholes solution of call delta:} \qquad \Delta_c = N(d_1) \tag{2.39}$$

$$\text{Black-Scholes solution of put delta:} \qquad \Delta_p = (N(d_1) - 1) \tag{2.40}$$

2. *Gamma* Γ: It is the sensitivity of option's delta to the asset price. It is defined as the partial derivative of delta with respect to the asset price, hence it is the second partial derivative of option price to asset price. Gamma for both call and put options is the same, and it reaches its peak when the option is ATM. If trader buys the option, it is positive, and if the trader sells the option it is negative. Trader hedging the portfolio using the delta-hedging rule is more secure for the wider range of price movements, if his portfolio's gamma is neutralised. In this way, the portfolio is less sensitive to asset price movements. Below is the

definition of gamma [87, 121]:

$$\text{Gamma's definition:} \qquad \Gamma = \frac{\partial^2 c}{\partial S^2} \tag{2.41}$$

$$\text{Black-Scholes solution of call/put gamma:} \qquad \Gamma = \frac{N(d_1)}{\sigma S \sqrt{T}} \tag{2.42}$$

3. *Theta* Θ: It can be defined as the sensitivity of the value of the option to the passage of time, or 'time decay'. Hence its value is always negative. In other words, theta is the rate of change in option price with respect to change in time. Normally theta is calculated in terms of years, but once divided to 365, it will be the daily rate of change in value of an option. For example, if the theta of call option is -15, then its daily rate is about -0.05, so we can assume that everyday our call option will loose its value by 0.05. Below is the formal definition of theta [87, 121]:

$$\text{Theta's definition: } \Theta = -\frac{\partial c}{\partial \tau} \tag{2.43}$$

Black-Scholes solution of call theta:

$$\Theta_c = -\frac{S_0 \phi(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(-d_2) \tag{2.44}$$

Black-Scholes solution of put theta:

$$\Theta_p = -\frac{S_0 \phi(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2) \tag{2.45}$$

where $\phi(\cdot)$ is normal Probability Density Function (PDF)

4. *Vega:* This parameter measures the change in the option price with respect to the change in volatility of an underlying asset price. So far we assumed that the volatility, or in other words, the standard deviation of underlying asset price is constant, but in fact it is not constant, and can also change by time. Vega measures how the price of an option is affected per σ change of its underlying asset's price volatility. We can interpret it in percentage by dividing it to 100. For example, if the vega of call option is 66, then 1% change in underlying asset's volatility will result in 0.66 increase in option value. Formally vega can be defined as

follows [87, 121]:

$$\text{Vega's definition: } V = \frac{\partial c}{\partial \sigma} \tag{2.46}$$

Black-Scholes solution of call/put vega:

$$V = S_0\sqrt{T}\phi(d_1) \tag{2.47}$$

Above I reviewed some of the Greeks that are commonly used in analysing the sensitivity of the option price to different external factors. However these are not the all of them. For example there other Greeks indicators such as Rho which measures the option price's sensitivity to changes in interest rate, Vanna which measures the delta's sensitivity to changes in volatility, or Charm that measures the delta's sensitivity to changes in expiration time. Although these indicators are very important in analysing the option prices, I have to omit them from the scope of this research in order to reduce its volume.

2.2 Agent-Based Computational Economics

One of the vivid implementations of agent-based models can be found in the field Economics, particularly in modelling the marketplaces. Tesfatsion coined the term Agent-based Computational Economics (ACE) as a methodology which is used for studying the behaviour of economic processes modelled in dynamic systems of interacting agents [157]. One of the important topics that ACE studies is the way of providing microfoundations in the form of optimized agent behaviour to explain traditional macroeconomic phenomena. For example, we cannot directly manipulate supply and demand curve, but we can engineer the decision making mechanism of an individual agent and observe emerging macroeconomic patterns in larger scale [139]. Hodgson noted that not only agent's individual behaviour but their interactions with each other should also be studied to understand emerging social phenomena [84]. There are a number of aspects that should be examined while designing ACE system. Epstein suggests some of them [53, 54]:

- *Heterogeneity*: Unlike analytical models which try to reduce the differences between individuals into universal model, in computational models the heterogeneity of agents are considered natural and easy to implement. For example, agent may vary in their preferences, endowments, social contacts and abilities in making decisions, and this should be taken into account. In my simulation model, the agents employ various option pricing and trading methodologies which make them different in their preferences and making trading decisions.

- *Explicit Space*: Individuals often differ in the physical place where they are located, and they are likely to share the same culture, mindset and social contacts within this neighbourhood. In this case, agents can be categorised into explicit groups sharing certain behaviour. In my simulation model, the groups of agents with similar pricing and trading behaviour are populated. For example, I employ Monte-Carlo traders who use the same technique to price options, although their final prices can be different due to randomization.

- *Local interaction*: Analytical models usually assume global interactions happening between entities, but in agent-based models interaction may happen locally between finite number of agents. This enforces the asymmetry of information between different agents. Although, in my simulation model, there is no asymmetry in the publicly shared information such as asset prices, some agents may have better knowledge about the market in CDA environment, because the time one's order submitted differs from other's giving the last trader an opportunity to learn more about submitted orders throughout the day.

- *Bounded rationality*: Analytical models can tell us the optimal behaviour in given scenario, however in agent-based models make rational decisions based on the agent's own beliefs and desires, and the limited information that is available to them. In my simulation model, especially in CDA environment, traders have to submit orders continuously which bounds their rationality to the timeframe and the listed orders in the market. Also in general traders have to make pricing decisions on options without knowing what will happen to the underlying asset in the future.

Beinhocker compared ACE with traditional economics and pointed out number of differences based on its dynamics, agents involved, networks, emergence and evolution [17]. Table 2.2 summarised these differences.

In the scope of this research, I consider auctions as a main instrument for building ACE. I review different types of auctions that are used in my simulation model along with popular trading algorithms.

	ACE	Traditional Economics
Dynamics	Open; dynamic; non-linear systems	Closed; static; linear systems
Agents	Modelled individually; use heuristic rules; deal with incomplete information; prone to errors and biases; can learn and adapt automatically	Modelled collectively; use deductive and analytical reasoning; deal with complete information; no errors, irrationality or bias assumed; no learning required
Networks	Explicit interactions between agents, form networks that can change over time	Indirect interaction with the system; approximated model for interactions between large groups
Emergence	Fusion of macro- and microeconomics; Patterns emerging on macro level are the direct result of microeconomic behaviour	Macro- and microeconomics are separate fields
Evolution	Agents and mechanism can evolve into new systems	Agents and mechanisms are not endogenously improved

Table 2.2: ACE and Traditional Economics

2.2.1 Auctions

The ultimate goal of any auction is the allocation of scarce resources to agents willing to buy it[165]. It is usually done because the resources are scarce, the agents who would like to have them are many. Auctions are organized between the *auctioneer* (also specialist, mechanism) and *bidders* (also buyer, seller, trader). The job of an auctioneer is to decide to who the item should be given and at which price, while the bidders have an incentive in buying the item at lowest possible price. So one of the most common auctions is considered as the English auction where the item is given to an agent who values it the most, and the price paid is the same as the agent's bid. This allocation mechanism is also called *first-price* auction, and it, indeed, maximises the social surplus of the auction. This is because the seller receives the best price which is offered from bidders. However the incentive of the bidder is not to tell his truthful valuation to the seller, but to decrease it just enough to beat the second highest bidder. However there is a mechanism which results in the same revenue to the seller, maximises the social surplus and also makes truthful bidding a dominant strategy for each trader. This is called a *second-price auction* where winner is the one with the highest bid, but he pays only the second highest price. In this mechanism, traders can directly submit their true valuations in sealed-bid fashion. It is also referred as *Vickrey auction* [11], and it simplifies the auctioning process by making the traders more open to the auctioneer to achieve the same outcome as in English auction.

Based on the protocol of holding auctions, they can vary in several different ways. For example, in *open cry* auctions every bidder has an access to the bids of others, whereas *sealed-bid* auctions do not disclose the latest bids until all bids are collected. Also auctions can be *ascending* or *descending* based on direction of price after bids. English auction is first-price, open cry, ascending auction. However Vickrey auction is sealed-bid, second-price, ascending auction. Parsons describes more than 30 variations of auctions based on their following properties [134]:

- *Single- Or Multi-dimensional*: In single dimensional auctions, traders bid only on one parameter of the offered item, which is normally price. In multi-dimensional auctions, other parameters of the item such as quality, delivery time, expiration date can be bid as well.
- *One- Or Two-sided*: In one-sided auction, bidders are either buyers or seller and the auctioneer decides who is the winner. However in two-sided auction bidders can be buyers and sellers, and the job of the auctioneer is to match them together.
- *Open-cry or Sealed-bid*: As it is mentioned above, in open-cry auctions bidders know all bids submitted, in sealed-bid no one knows each other's bid.
- *First- or kth-price*: In first-price, winner of the auction pays the price he bid, in kth price he pays the kth highest price bid.
- *Single- or Multi-unit*: In single unit auction, bidder can bid on only one unit of offered item, while in multi-unit auction bidder can bid on several units.
- *Single- or Multi-item*: In single item auction, there is only one item is auctioned, however in multi-item auctions the bundle of different items can be bid.

Each of these attributes described above add more complexity to the design of an auction. For example, there is an inherent problem with organising a multi-unit auctions because the identical goods can be priced differently, even if the single-item auctions are run in parallel, let alone subsequently. Empirical evidence from Sotheby and Ashenfelter [152, 8, 9] shows that the price for the identical good starts to decline as the number of allocated goods increases. This can be the reason for bidders to review their bidding strategies, and use methods such as demand reduction to influence the price of the good. In classic Vickrey auction demand reduction results in the use of lower bid to clear the auction if winning bidders reduce their demands. Other mechanisms such as Simultaneous Ascending Auctions [38] propose a mechanism where goods are priced in simultaneous auctions through a number of rounds each having the standing high bidder to win the good. Crampton also introduces an activity rule to prevent sniping strategy where bidder can submit insignificant bids in all auctions, and post his significant bid on final round of an auction where the good is the cheapest. In this activity rule, the bidder cannot increase his demand as the auction proceeds forward, meaning that it is unlikely to see demand rise when the price increases. This method borrows its origins from *Walrasian tatonnement* process where supply and demand are matched by adjusting the price.

Besides the characterisation of auctions by the attributes of their protocols, there are desired properties of auctions which emerge from their actual execution. These properties are listed below [129]:

- *Allocative efficiency:* The auction should optimise its allocative objective function while determining the winners, or matches between sellers and buyers. The most commonly used objective function is the *surplus maximising* objective function which maximises the overall welfare of the auction. In two-sided auctions, it also leads to a competitive equilibrium among buyers and sellers. There are many other objective functions can be used as well, for example, *liquidity maximising* objective function maximises the participation of traders in the market. Or else, *volume maximising* objective function maximises the quantity of items traded in the market.

- *Budget-Balance:* In a strong notion of budget-balance, the mechanism should neither gain, nor lose from facilitating the trade. While in a weak notion, the mechanism should never lose from facilitating the trade. This property may hold in 3 conditions described below:

    1. Ex ante: The expected gain of the mechanism is either zero (in strong budget-balance) or greater than zero (in weak budget-balance).

2. Interim: The conditional expectation of mechanism's gain for agent i is either zero (in strong budget-balance) or greater than zero (in weak budget-balance).

3. Ex post: The mechanism is always budget-balanced.

- *Individual Rationality:* The trader is individual rational if participating in trade is at least as good as not participating in trade. Mechanism is individual rational if all of its participants are individual rational. Similar to budget-balance, individual rationality may hold in 3 conditions:

  1. Ex ante: The expected utility from participating in the mechanism is greater or equal to zero.

  2. Interim: The conditional expectation of agent i's utility from participating in the mechanism is greater or equal to zero.

  3. Ex post: The mechanism is always incentive compatible.

- *Incentive Compatibility:* The trader is incentive compatible if revealing his true type maximises his gain while not revealing his true type does not. Mechanism is said to be incentive compatible if all traders are incentive compatible. There 2 main types of incentive compatibility: DSIC and Bayesian-Nash Incentive Compatible (BIC). In the former one, truthfulness is the dominant strategy for every trader, so every rational trader truthfully reveals his type to the mechanism. Such mechanism is also referred as *strategyproof* mechanism. In the latter case of incentive compatibility, the trader is ex ante better off with truthful strategy. I define all 3 conditions of incentive compatibility below:

  1. Ex ante: Truthful strategy maximises the expected utility.

  2. Interim: Truthful strategy maximises the conditional expectation of agent i's utility.

  3. Ex post: Truthful strategy always maximises the agent's utility.

- *Tractability:* The mechanism is computationally tractable if it can resolve allocation of orders, and their clearing prices in polynomial time.

2.2.2 Double Auctions

is the type of a two-sided auction where multiple buyers and sellers submit their orders to the mechanism which matches and clears them. Double Auction (DA)s can vary based on their overall protocol of accepting and clearing orders, the matching rule, and the clearing rule. Myerson *et al.* was one of the first researchers to study the bilateral trading relationships between buyers and sellers [119]. The main results such as the monotonicity of an allocation rule, and the critical value as the payment rule were established to assure the strategyproofness of mechanism. Myerson and Satterthwaite also proved the impossibility of having allocative efficiency, individual rationality and budget-balance along with BIC which is even weaker condition than DSIC. This theorem is assumed while designing my direct DA which drops the budget-balance property in favour of incentive compatibility.

The early designs of DAs have been proposed by Satterthwaite and Williams [146, 145] which involved single-unit DA that is asymptotically incentive compatible for buyers given the strategic behaviour of sellers is dismissed. This also gave an asymptotic efficiency and budget-balance for the mechanism. Then McAfee proposed a single-item DA where both buyers and sellers were allowed to exhibit strategic behaviour [113] and still maintain incentive compatibility of the mechanism. Although mechanism was asymptotically efficient, individual rational and DSIC, it required the mechanism to absorb the potential gains to sustain the mechanism's weak budget-balance. I will use McAfee's mechanism as a starting point while designing the direct DA for trading options.

There have been other proposals of double-auction design, particularly the ones with multi-unit orders. Barbera and Jackson proposed the design of a multi-unit, strategyproof exchange where every trader could be either buyer or seller at the same time [16]. However their mechanism was not asymptotically efficient and required an additional agent to specify the clearing prices beforehand. Also more recently, Huang *et al.* proposed a new design of a multi-unit DA which is asymptotically efficient, but this is done at the cost of spreading the burden of excess demand or supply among other traders [86]. In other words, the mechanism forces either buyers or sellers to cut their demand or supply to ensure the balance. This mechanism supports DSIC, individual rationality, weak budget-balance and asymptotic efficiency. There have been also further extensions on making multi-unit DA mixed, so the traders can also simultaneously engage in buying or selling multiple types of items [68].

In multi-unit DA, demand (supply) reduction can be exploited to obtained lower (higher) prices for certain goods. However, demand (supply) reduction cannot significantly affect the prices if the buyer (seller) has insignificant demand (supply) compared to the market volume. It is unlikely that in a highly competitive market buyer (seller) could set the price, and therefore it is almost a dominant strategy for the trader to report truthfully [117]. In the context of option markets, although options are intangible, traders are restricted in supplying significant volumes of options due to the exposed risk that should be covered by taking long/short position in underlying market.

I will present an implementation of direct DA for pricing options based on the techniques used previously to design direct DAs. Although there are certain similarities in the implementation, the overall properties of my mechanism versus other mechanisms differ. Below Table 2.3 summarises these key differences:

Properties	**Proposed Mechanism**	**Satterthwaite and Williams** [145]	**McAfee** [113]	**Barbera and Jackson** [16]	**Huang** *et al.* [86]
Incentive compatible	Buyers and Sellers	Only buyers	Buyers and Sellers	Buyers and Sellers	Buyers and Sellers
Individual rational	Buyers and Sellers	Buyers and Sellers	Buyers and Sellers	Buyers and Sellers	Buyers and Sellers
Efficiency	Theoretical	Asymptotic	Asymptotic	Not efficient	Asymptotic
Budget balance	Not supported	Strong	Weak	Weak	Weak
Mechanism as agent	Trading agent	None	Non-trading agent	Non-trading agent	Non-trading agent
Maximum order size	Capped	1	1	Infinite	Infinite
Tractability	Polynomial	Polynomial	Polynomial	Polynomial	Polynomial

Table 2.3: Proposed Direct DA and Other DAs

Vytelingum describes mechanism involving an agent (i.e. auctioneer) who decides how to allocate and price items in order to maximise certain objective function as a *centralised market* and the direct DAs described above fall into this category. However the second type of the market is described as *decentralised market* where orders are matched as they arrive to the market [161]. These systems are referred as CDAs, so they allow traders continuously submit their bids and asks to the system. Traders can see and accept any order submitted to the CDA without the help of an auctioneer. This type of protocol is used in financial markets through its LOB implementation. According to this implementation, there are 2 types orders:

- *Market order:* This order does not quote the price, but indicates the quantity the trader wants to buy or sell. The order is cleared at the current outstanding price in the LOB. For example, if the trader submits market bid on 10 items, the item is cleared at the lowest ask in the LOB. If there are multiple asks required to cover the trader's demand, then different clearing prices corresponding to asks posted will be used.
- *Limit order:* This order quotes both the price and quantity to buy or sell the good. Once it arrives to the market, the specialist checks if it has a matching order in his LOB, and if yes, he clears it. If not he adds the limit order to his LOB and retains it until it gets either cancelled or matched to new orders.

The researches in the field of CDA were basically focused on simulating different trading agents and analysing their results. The initial structure of CDA for simulating these agents has been proposed by Smith [150]. According to this protocol, agents submit orders for homogeneous goods, and get cleared as soon as their orders are matched with another order in the market. The clearing price is computed as a weighting average between matched bid and ask. Friedman and Rust were among the first who studied the real-world exchanges in the format of a CDA [60]. The established CDAs were not strategyproof due to the uncertainty about incoming orders, and therefore agents simulated had a bounded rationality towards the information available to the traders at the time he makes buying or selling decision. Bounded rationality of agents emerges from behavioural economics which advocates the irrationality of agents due to the factors such as lack of full information about given deal, risk-averseness, and other psychological aspects that drive their decision making process. One of seminal works studying an irrational behaviour of traders has been accomplished by Gode and Sunders where they put zero-intelligence traders who submitted random bids and asks to test in CDA, and observed that the market can still converge to equilibrium prices[74]. It has been also shown that the human traders perform approximately the same as zero-intelligence traders constrained with a private value[73]. Gjerstad and Dickhaut has proposed using agents with Bayesian updating rule to form beliefs about the successful bids and asks based on the history of submitted orders. It has been shown that such traders reach equilibrium prices much quicker than zero-intelligence traders[69]. Cliff proposed an adaptive rule for zero-intelligence traders to modify their private constraints according to the current bids and asks submitted in the market, and named it as 'ZIP' traders [29]. Such automated traders could outperform the human traders in many settings by employing simple rules such as limiting the losses and continually converging to a target price uniformly chosen from a finite range around current price [105, 104].

Since then, automated traders have been significantly improved in their ability to learn and adapt to dynamic market conditions. Das and Tesauro developed Modified Gjerstad-Dickhaut (MGD), tested it against ZIP,GD and Kaplan's sniping algorithm [142] in CDA environment and showed that it outperformed these algorithms [39]. Vytelingum *et al.* proposed an Adaptive-Aggressive (AA) trading algorithm which splits the agents trading activity into short-term and long-term goals where in former case learning algorithm regulates the agents aggressiveness towards making profit, and in latter case the algorithm monitors and controls the overall success of certain aggressive trading routine based on historic data [161]. In other words, short-term learning algorithm is necessary to make an immediate trading decision based on adaptive aggressiveness of the agent, while the long-term learning is used to evaluate the overall profitability of the short-term trading algorithm. Cliff has extended his original ZIP algorithm which requires 8 parameters to be set to a more advanced version of the algorithms with 60 parameters to be set using a genetic algorithm. The ZIP60 algorithm adapts to a new market environment by calibrating multiple parameters (from 8 to 60 control factors), and choosing the best set that optimally fits to given mechanism. Moreover genetic algorithms are involved not only calibrating the parameters of the trading algorithm, but automatically generating fundamentally new mechanisms that best fit for given market conditions[30]. One of the other approaches at re-engineering the CDAs such that they hold certain properties like BIC has been accomplished by Parkes *et al.* who proposed the design of a truthful online mechanism with an asymptotic efficiency[131, 132]. Friedman *et al.* showed the issues of using truthful online mechanisms in the example of pricing the Wi-Fi in Starbucks [61].

I will also present the implementation of a CDA in the scope of this research which implements standard LOB approach. Although most of the implementation is similar to previously developed models, my version has couple of distinctions from previous mechanisms. Below I summarised them:

- LOB is implemented using red-black binary trees which balances ask and bid sides of the order book efficiently, and helps to match outstanding orders in constant time. Most of the standard implementations of LOB are done using priority queues which when naively implemented takes $O(n)$ to rearrange the orders after successful match. This is also efficient, but my implementation slightly improves the data structure for storing order entries, so as a self-balancing binary tree it takes $O(\log n)$ to find the outstanding orders after successful match.

- Only limit orders are allowed in my CDA because trading algorithms post option prices resulted from pricing options and other intrinsic trading behaviour. Trading algorithms I implemented do not have the sophistication to decide whether it is optimal to submit a market order and bear the cost of immediacy resulted from that. In real-life CDAs traders can submit both limit and market orders, and some of the automated trading algorithms have the ability to evaluate the costs associated with posting market orders and their overall benefit in longer terms.

- Order submission is invoked my the mechanism, not by traders. The mechanism controls who submits orders when in randomized fashion, instead of traders deciding when to enter the market and when to exit. Every trader has one call to submit an order per day giving the last trader full information about the quotes of all traders. However, being last trader to post order also means missing an opportunity to get best deals throughout the day. In real-life CDAs, traders should have the intelligence to detect the signals on when to enter or exit the market by themselves, but in my simulated version this is done by the mechanism.

- CDA also simulates the underlying market. Traders in CDA obtain daily updates on the new prices from simulated underlying market and make corresponding decisions on option prices. This would mean that the option price will change daily not only due to transactions happening in the mechanism, but due to changes in the underlying asset. This also implies that traders withdraw their obsolete orders from the market as soon as new information about the underlying asset price arrives. The same is true for risk-free rates when they are simulated as mean-reverting process. CDAs used in most researches do not support the simulation of underlying asset market and solely provide a platform for traders to submit and match orders.

- Options are cleared on their maturity date. My CDA also takes the responsibility of clearing the options at the end of their maturity date, while commonly used CDAs do not have this feature.

- Analytical parameters specific to options are computed. The proposed CDA provides important characteristics of the options traded such as Greeks which are discussed in Section 2.1.7. It can run ad-hoc simulations to observe the sensitivity of the options by fixing the other parameters of options such as strike price, time-to-maturity, asset price, etc. Commonly used CDAs do not support this feature, because they are not specialized in trading options.

2.2.3 Combinatorial Auctions

In *combinatorial auctions* there are multiple heterogeneous items sold to many bidders who can buy bundles of these items. The important concern while designing the combinatorial auctions is the relationship among the goods traded in the combinatorial auction. In Vickrey auction, there is only one item traded and the rules for finding the allocation and payment are simple. Let us define $Z = \{z_1, z_2, \ldots, z_m\}$ to be the set of items auctioned. Any agent i has a valuation function v_i that can tell the value of any single item or the subset of items from Z. A valuation function is said to be normalized, if $v_i(\emptyset) = 0$, and if $Z_1 \subseteq Z_2$ implies $v_1(Z_1) \leq v_2(Z_2)$.

$$v_i : \mathbf{2}^Z \to \mathbb{R} \tag{2.48}$$

In this way, traders can see the goods in one of these two perspectives:

- *Goods are substitutes:* The goods are substitutes to agents if their combined value is not greater than the sum of their separate values. Formally, it is given as below:

$$v_i(\{z_1, z_2\}) \leq v_i(\{z_1\}) + v_i(\{z_2\}) \quad \forall i \tag{2.49}$$

- *Goods are complements:* The goods are complements to agents if their combined value is not less than the sum of their separate values. It is given below:

$$v_i(\{z_1, z_2\}) \geq v_i(\{z_1\}) + v_i(\{z_2\}) \quad \forall i \tag{2.50}$$

In this way, in combinatorial auctions traders might have different valuations for the bundles of the items traded rather than having them separately. This emerges two important issues associated with combinatorial auctions. One is the preference elicitation and second is the winner determination. Both are explained in more detail below.

Preference Elicitation

In this section, I review the importance of bidding languages in designing mechanism and their different types suggest by other researchers. I will use a specific bidding language, TBBL, to describe the implementation of a combinatorial exchange for trading option portfolios, and therefore it is worthwhile to review the basic principles behind designing a bidding language for a combinatorial auction or a combinatorial exchange. E In combinatorial mechanisms there is an issue in expressing trader preferences, as the valuations to different bundles of goods can be combinatorially large. The possible combinations of bundles increases at an exponential rate as the number of traded goods increases. If, say, set Z consists of 100 items, then $\mathbf{2}^Z$ has 2^{100} subsets that should be evaluated. If we assume that we have modern 2 GHz computer and each evaluation is one computer operation, then it will take roughly 2^{91} seconds, which is billions of times longer than the time past since 'Big Bang'. Hence this would require succinct and complete way of interpreting the preferences of the agents without doing all these exponential operations. This is the problem of preference representation and elicitation where agent need to use some bidding language to communicate their preferences to the auctioneer.

Because of the exponential complexity of determining bids, traders have to use a *bidding language*. Specifying the bidding language for traders, and the protocol for the mechanism to elicit the preferences to solve the allocation and payment problems are the core aspects of any combinatorial exchange design, and they should be addressed by any mechanism designer considering its implementation. The literature suggests many kinds of bidding languages, starting from simple atomic bids, `OR`, `XOR` to more complex `XOR-of-ORs`, `OR*`, L_{GB} etc [124]. For example, `OR` bid connects atomic bids in form of tuples (S_i, p) using `OR` operator where S_i is the subset of m goods and p is the corresponding price for it. In this case, valuation for m goods can be described with an expression consisting of m tuples joined using `OR`. However, the limitation is that trader has no choice to state how many of m goods he wants to buy at minimum, and maximum. Additionally, it may cause exposure problem if elements of one bundle are connected in separate atomic bids. `XOR` language, on the other hand, allows traders to buy only one specific subset out of many other subsets submitted. The problem with this is that trader cannot specify several subsets at the same time that he would like to buy. It will take $2^m - 1$ tuples to express valuations for all possible possible subsets, which makes this language highly verbose.

Other languages such as `XOR-of-ORs` and `OR-of-XORs` use the combination of logical operators to express trader preferences. For example, `OR-of-XORs` combines subsets of `XOR` bids with `OR` operator, so that each trader can submit arbitrary number of `XOR` bids. This is good for specifying bids with a downward slopping valuation, where items to be purchase can be prioritised in form of a diminishing price quotes for subsequent subsets of goods. However it is unknown how many bids the trader finally gets. Also in extremal cases, it may cost $2^{m/2+1}$ tuples to interpret certain valuations such as monochromatic valuation. One of the power languages of this kind is L_{GB} which involves operators `OR, XOR, AND` to formulate a bid [22]. In this language individual bids can be bound together using any of these operators in any hierarchy while specifying the maximum price for each subsets joint. There has been WD problem formulated by Boutilier as a mixed-integer problem (MIP) for L_{GB} [21].

There have been other proposals that extend existing logical operators with additional parameters. For example `OR*` language introduces dummy items for defining specific constraint in bidding. Dummy items will not have any intrinsic value, but they can also be used in forming bundles [62, 124]. This language can simulate any of previously mentioned languages. For example, $(S_1 \cup d_i, p_1) \vee (S_2 \cup d_i, p_2)$ is equivalent to $(S_1, p_1) \oplus (S_2, p_2)$, where d_i is a dummy item of bidder i which is used for marking subsets that are mutually exclusive. Trader can have as many dummies as needed, and his dummies are unique to him only, and can be used to identify the special needs of each trader. Although it can represent any logically structured bidding language, majority of them requires the size at least $\binom{m}{m/2}$ which is quite big for large m. Another commonly discussed marriage of logical operator and a parameter is a `k-of-OR` bidding language. This language allows bidder to specify how many tuples at most he would like to have in an `OR` joined expression. This extends the notion of `XOR` where $k = 1$ and for WD it picks maximum k subsets.

We can summarise the desirable properties of bidding language in following way [25]:

- *expressiveness:* Determines if bidding language is complete, or in other words if it can specify every possible combination of bids. If it is not complete, then to what extent it can specify bidder valuations out of all possibilities.
- *compactness:* Determines how verbose is the language, and if all valuations can be expressed concisely. It should not require a bid to grow exponentially.
- *simplicity:* Determines if it is simple to understand for bidders and the market-maker.
- *computational tractability:* Determines if it is possible to formulate a WD problem using this language, and how complex is the solution to this problem. It should consider if it is possible to implement computationally tractable WD algorithm for it.

Winner Determination

In combinatorial auctions, auctioneer faces the problem of finding the surplus maximising allocation of goods. This problem is also named as WDP in the context of combinatorial auctions. With regard to auctioneer's problem of optimally allocating items, either heuristic methods or linear programming methods can be used. Andersson *et al.* describes a linear programming method for finding the optimal allocation allocation through using the sum of bundle valuations as an objective function constrained under feasibility conditions [3]. Sandholm *et al.* proposed greedy allocation rule to assign auctioned bundles to bidder who value them most until the bundles are fully allocated[144]. There is also an iterative approach proposed by Ausubel *et al.* for finding the optimal allocation and determining the prices of the bundles. It uses multi-round clock-proxy method for better price discovery process among trading agents and eliminating the exposure problem[37]. Clock phase is used to determine the demand of each bidder for particular item in the bundle for given price, while the proxy phase enables traders to submit their valuations for bundles. The auctioneer knowing the price and demand for each item in the bundle can efficiently clear the bundles in proxy phase by assigning them to those who highly values each individual item. There have been other methods proposed by Lehmann[100] and Ausiello[10] to approximate the solution of the winner determination problem in much tractable way.

Now let us formulate the WDP itself. Let $S \subseteq Z$ be the bundle of goods wanted by the trader and $p_i(S)$ he quoted for that bundle. Also each trader provides set of bundles he wants in the form of combinatorial bid linked through XOR, $B_i \subseteq \mathbf{2}^Z$. Then the winner determination problem for this combinatorial auction is defined as follows [129]:

$$\max_{x_i(S)} \sum_{S \in B_i} \sum_i x_i(S) p_i(S) \tag{2.51}$$

$$\text{s.t.} \sum_{S \in B_i} x_i(S) \leq 1 \quad \forall i \tag{2.52}$$

$$\sum_{S \in B_i, S \ni j} \sum_i x_i(S) \leq 1 \quad \forall j \tag{2.53}$$

$$x_i(S) \in \{0, 1\} \quad \forall i \tag{2.54}$$

where $x_i(S)$ is the mechanism's decision whether to allocate given bundle S to trader i.

It can be seen from the LP problem formulation (2.51), set packing problem can be reduced to WDP[140, 12]. It is well known that set packing problem belongs to NP-hard class.

Once these 2 major issues, namely preference elicitation and WDP, of the combinatorial auctions is sorted, the mechanism has to determine the payments collected from the winning traders. Let us denote the allocation derived from the WDP as X, and the allocation derived from the auction where the trader i does not participate as X_{-i}. Then the payment scheme named after Vickrey, Clarke and Groves should be applied in order to get DSIC outcome. In other words, the monotonicity of WDP is necessary and Vickrey-Clarke-Groves (VCG) is a sufficient condition to guarantee DSIC property of a mechanism according to Myerson's lemma [119]. Also let $p_i(B_i)$ be the price vector indicating the prices of each S set of goods included in XOR combinatorial bid, and $x_i(B_i)$ be the mechanisms decision which of sets inside B_i to satisfy. The VCG scheme is defined below:

$$\rho_i = \sum_{k \neq i, x_k \in X_{-i}} p_k(B_k)^\intercal x_k(B_k) - \sum_{k \neq i, x_k \in X} p_k(B_k)^\intercal x_k(B_k) \tag{2.55}$$

In VCG scheme every winning trader pays the social cost that he imposed on the mechanism, while the rejected traders do not pay or receive anything from the mechanism. In this way, VCG scheme generalizes the payment rule of Vickery auction, where the winning agent pays the second-highest bid. In other words, the winning bidder in Vickery auction pays the best price that could have emerged, should he not participate in given auction. VCG uses the same technique in computing the social cost of a traded bundle. Some limitations of VCG include that it is susceptible for collusion directed at lowering (increasing if seller-side) the price of the item traded, and in some cases it may yield a budget-deficit [148].

2.2.4 Combinatorial Exchanges

The concept of DAs can be further extended to *combinatorial exchange.* In fact, combinatorial exchanges can be considered as two-sided combinatorial auctions where multiple items traded by buyers and sellers. There are several aspects emerges when the WDP of a combinatorial exchange is defined:

- *Sourcing:* If mechanism can disassemble and re-assemble bundles of items, which may involve buy-side aggregation, sell-side aggregation or both. If bids are not allowed to be disassembled, then each bid should be matched to exactly one ask.
- *Divisibility:* If bids can be satisfied partially, or it needs to be all-or-nothing.
- *Homogeneous/Heterogeneous Goods:* If goods traded are same, or they are different. Multiplicity of goods emerges substitutability and complementarity cases. In this case, bidders can express their possible alternative substitutes in their bids and specify them in combination with different complements.

I will look through WDPs of two specific cases: DA and combinatorial exchange. Let us denote $x \in \mathcal{F}$ as the set of feasible allocations. In former case, consider set of bids B and set of asks A. Each bid $b_i \in B$ includes price p_i and quantity q_i, and each ask $a_i \in A$ includes price p_j and quantity q_j. Bearing in mind that multi-sourcing is allowed, we can specify for any b_i a subset $A_i \subseteq A$ that satisfies the bid, and the same for any a_i ask, there is $B_i \subseteq B$. Also let $0 \leq x_{ij} \leq 1$ denote the fraction of the demand q_i to asked q_j. WDP can be re-formulated for DA as follows:

Definition 2.2.1. Winner Determination Problem for Double Auction

$$x = \arg\max_{x \in \mathcal{F}} \sum_{i \in A} \sum_{j \in B_j} (p_i - p_j) q_i x_{ij} \tag{2.56}$$

subjects to:

1. Each asked item is allocated once:

$$\sum_{i \in B_j} q_i x_{ij} \leq q_j, \forall j \in A \tag{2.57}$$

2. At most one subset for every trader

$$\sum_{j \in A_i} x_{ij} \leq 1, \forall i \in B \tag{2.58}$$

3. Multi-sourcing allowed

$$0 \leq x_{ij} \leq 1, \forall i, j \tag{2.59}$$

In case if bids are not allowed for multi-sourcing, then $x_{ij} = \{0, 1\}$ needs to be discrete. This problem is usually referred as matching problem, and can be solved using LP. However constraint (2.58) makes it an assignment problem which known to be NP-hard [110].

In case of combinatorial exchange, where bids and asks of heterogeneous items allowed. Although it is not the case for combinatorial exchange, in this formulation, we define agents to be exclusively either seller or buyer. This is done to distinguish the subsets wanted from the subsets offered, so the surplus maximising objective can find optimal allocation. So, let B denote buyers and A denote sellers. Each seller can submit asks for multiple bundles $S \subseteq Z$ with price $m_i(S)$. Similarly, we allow buyers to submit bids for multiple bundles, and the price is $p_i(S)$. Let agents $i \in B \cup A$, and their corresponding bids defined as $C_i \subseteq 2^Z$. Also $x_i(S) = 1$ would mean that bid on bundle S from buyer i is accepted, and $y_i(S) = 1$ would indicate that the ask on bundle S from seller i is accepted. Given above, we can formulate WDP for a combinatorial exchange:

Definition 2.2.2. Winner Determination Problem for Combinatorial Exchange

$$\max_{(x_i, y_i) \in \mathcal{F}} \sum_{S \in C_i} \sum_{i \in B \cup A} (x_i(S) p_i(S) - y_i(S) m_i(S)) \tag{2.60}$$

subjects to:

1. At most one subset for every buyer:

$$\sum_{S \in C_i} x_i(S) \leq 1, \forall i \in B \tag{2.61}$$

2. At most one subset for every seller

$$\sum_{S \in C_i} y_i(S) \leq 1, \forall i \in A \tag{2.62}$$

3. Free-disposal assumption

$$\sum_{S \in C_i, S \ni j} \sum_{i \in B \cup A} (y_i(S) - x_i(S)) \geq 0, \forall j \tag{2.63}$$

4. Discrete Allocation

$$x_i(S) \in \{0, 1\}, y_i(S) \in \{0, 1\}, \tag{2.64}$$

One way of computing the corresponding payments for both buyers and sellers is running VCG scheme on both sides separately for each allocation result as it is done by Duetting *et al.* [47]. Duetting showed that such mechanism can achieve DSIC, individual rationality, approximated efficiency and weak budget-balance given there is a composition rule which accepts buyer-seller pair if and only if its trade surplus exceeds certain threshold t. However, I will follow Parkes *et al.* approach who suggested applying the *threshold rule* such that the discount $\Delta_{thresh,i}$ minimises the distance between VCG discount $\Delta_{vcg,i}$[133]. Below is given how it works only for buyers side, as the sellers is symmetric. The VCG payment given in equation (2.55) can also be written as follows:

$$\rho_{vcg,i} = p_i(B_i)^\intercal x_i(B_i) - \Delta_{vcg,i} \tag{2.65}$$

$$\Delta_{vcg,i} = \left(\sum_{j, x_j \in X} p_j(B_j)^\intercal x_j(B_j) - \sum_{j \neq i, x_j \in X_{-i}} p_j(B_j)^\intercal x_j(B_j) \right) \tag{2.66}$$

Then we can find such $\Delta_{thresh,i}$ which solves following LP.

$$\mathbf{\Delta}^*_{thresh} = \underset{\mathbf{\Delta}_{thresh}}{\arg\min}\, \epsilon \tag{2.67}$$

$$\text{s.t. } \Delta_{vcg,i} - \Delta_{thresh,i} \leq \epsilon \quad \forall i \tag{2.68}$$

$$\Delta_{vcg,i} - \Delta_{thresh,i} \geq 0 \quad \forall i \tag{2.69}$$

$$\sum_i \Delta_{thresh,i} \leq \sum_{i,x_i \in X} p_i(B_i)^\intercal x_i(B_i) \tag{2.70}$$

The solution $\mathbf{\Delta}^*_{thresh}$ can be used to compute the budget-balanced payments for buyers as given in formula below:

$$\rho_{thresh,i} = p_i(B_i)^\intercal x_i(B_i) - \Delta_{thresh,i} \tag{2.71}$$

I will use similar payment rule while designing a combinatorial exchange for trading option portfolios in Chapter 7 and refer to above formula.

2.2.5 Existing Implementations

One of the earliest implementations of market mechanisms is the Santa Fe Institute Artificial Stock Market (SFI-ASM) built in 1989 and outlined by Arthur et al [7, 99]. It combines well defined market structure and inductive learning methods. In SFI-ASM framework, myopic agents have to decide on making their assets portfolio out of risk-free bond and risky stochastic dividend paying asset. The risk-free bond is in infinite supply and pays a constant interest rate of r. Also dividend paying asset is in infinite supply, however the dividends are paid based on the outcome of a stochastic process. Traders use risk aversion utility function which makes them more reluctant to take risk if the payoff is uncertain. Agents use Holland's classifier algorithm [85] to forecast the future prices and dividends. Agents also apply genetic algorithms to evolve their portfolio selection mechanism continuously throughout the trading process.

Another, more 'up-to-date' implementation of auctions is Java Auction Simulation API (JASA)[3] developed by Phelps *et al.* [136]. It allows researchers to test their trading algorithms in a number of auction protocols. The platform provides base classes for trading agents that can be further extended with customized trading algorithms. JCAT[4] is the slight modification of JASA which

[3] `http://sourceforge.net/projects/jasa/`

[4] `http://jcat.sourceforge.net`

is used in Trading Agent Competition (TAC) [164, 23]. JCAT is the implementation of CDA where traders can do both buy and sell simultaneously in continuous market environment. Another important feature of JCAT is that it supports multiplicity of specialists, which means that there are many markets where agents can operate at the same time. Specialist have different policies for attracting more traders to their market. For example, they may have low charging policy which takes less fees for registration, transaction, information etc. They can also attract traders with their efficient matching functions that pair buyers and sellers together and determine the price at which their orders should be cleared. JCAT runs on several rounds, and comes with 4 types of benchmarking trading agents.

There have been several researches done in implementation of auctions for derivatives market. One of the earliest researches that simulate the trade of future contracts in auctions is accomplished by deMaza *et al.* which uses genetic algorithms to evolve simple trading strategies [41]. It assumes a futures market that trades future contracts which expire in one day. Agents use prices coming from previous 10 days to forecast their future price, and based on this forecasts enter to futures contract. King *et al.* described multi-agent model for options market where agents use Black-Scholes model for option pricing [95]. In this model, there are two types of traders with a requirement to buy or sell underlying asset in future. They have to generate a list of actions based on the present and estimated future values of their constrained portfolios. Streltchenko *et al.* described a reference architecture for multi-agent simulation of financial markets [155, 156]. They use Gaia methodology which views the system as a collection of roles that interact with each other. In this architecture, there are 3 types of agents: Underlying Market Agent, Broker Agent and Investor Agent. The role of the first Underlying Market Agent is to generate price quotes using stochastic processes while the role of the Broker Agent in derivatives market is to broadcast the quotes obtained from Underlying Market Agent and also inform Investor Agents about the current quotes on corresponding derivatives. Broker Agent also registers the orders submitted by investors and clears them. There is no limit on the quantity of assets traded on this platform. However, the simulation model developed by Espinosa imposes a constraint on the number of assets traded, as he uses it for efficient allocation of resources between agents [56].

The research of particular interest to this topic has been accomplished by Ecca [50]. She studied the impact of the option market on the underlying stock market using multi-agent simulation. Especially she tested how options affect the volatility in spot market and proposed a reference architecture for modelling options market. She also provided simulation results for different kinds experimental setups [51]. Although Ecca's work shares a lot with my research, my priorities are somehow different. The main purpose of this research is to price options using DAs, and it does consider its influence on underlying markets.

There are other types of auctions implemented for testing computer traders against humans. Luca and Cliff developed a CDA platform named as Open Exchange (OpEx) for testing the performance of trading agents in comparison to human traders [104]. The results of the competitions has been published by UK's Government Office for Science [106].

There are two common types of combinatorial auction models implemented in practice: *parametric model* and *abstract process model*. The first model is proposed by Wurman et al.[167], the developer of Michigan Internet AuctionBot [166], who identified a range of parameters that describe variety of auctions. These parameters are grouped into 3 parts:

1. *Common auction characteristics:* There are 3 main activities common to any auction: receiving bids and asks, clearing (i.e. matching bidders) and revealing information about traders (eg. their quotes, valuation mechanism etc). Important point here is to identify if the allocations are discrete or continuous, or in other words, whether traders quote in indivisible units of money, and buy or sell indivisible units of stock, or the quotes and amounts are described in continuous variables.

2. *Auction parameter space:* These parameters specify the rules of trading in auctions, such as whether traders may bid or asks, or do both, the bidding language they use (eg. if they can submit OR-bids, XOR-bids etc), acceptance rules (eg. in ascending auctions only those bids that beat specified/previous bid can be accepted), activity rules that oblige traders to participate in the trade constantly and thus avoid last-minute sniping problem. Another parameter is whether specialists send identical anonymous quotes to every trader or tailor specific discriminatory quote for given trader. Auctions may also be configured to reveal the *order book* which lists all unsatisfied bids and asks to some or all traders. Some specialists may reveal information about past trades. Some auctions settings may specify additional costs for obtaining such information from specialists or even having the right to obtain discriminatory quotes

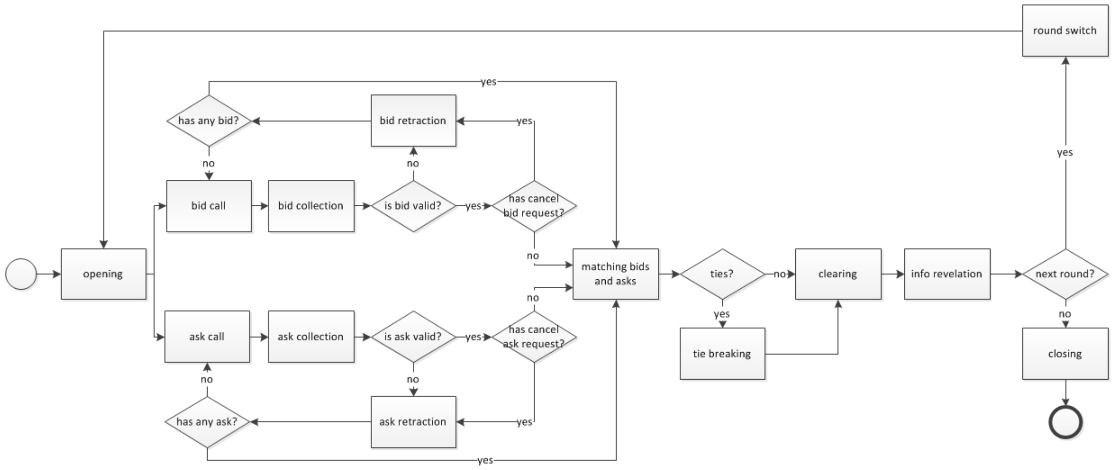


Figure 2.9: Auction's process flow diagram

specially tailored to them.

3. *Matching Functions:* These are the functions that determine which agents can trade based on the offers they made. One approach for pairing quotes can be the minimization of the difference between bid and ask quotes, and thus matching the highest ask and lowest bid. In contrast, maximizing this gap between asks and bids leads to better control on price for specialists. The core outcome of any matching function is the trade price which is applied for both buyers and sellers. The price is said to be uniform if it is applied for all traders equally, whereas discriminatory if it is applied only for specific pair of buyer and seller. Specialists once they have a control on price, can encourage buyers or sellers by manipulating the price between ask and buy quotes. Below formula shows how specialist can draw the price:

$$p = kp_{buyer} + (1-k)p_{seller} \qquad \text{where } k \in [0,1] \tag{2.72}$$

This process can be further extended to all buyers and sellers, where specialist picks n bids that are above p_{buyer} and acceptable to at least one seller, select m asks below p_{seller} and acceptable to at least one buyer. Then find $l = min(n, m)$ winning bids and compute a uniform price for them based on above formula.

The second common auction model, the abstract process model, breaks down the auction into an order of subprocesses [134]. Figure 2.9 is the action flow diagram which illustrates the processes involved in any auction with the description of each process below. The arrows in this diagram represent the direction of the action flow.

- *Bid/Ask Call*: Specialist calls for bids/asks. The call for bid/ask can be accompanied with reserved price if it is sealed-bid auction, or with the previous quote if it is an open-cry auction.
- *Bid/Ask Collection*: Specialist collects and validates orders.
- *Bid/Ask Retraction*: Specialists may allow traders to cancel their orders until specified time. If the auction is open-cry, then specialist needs to repeat call for bids/asks with the next highest/lowest bid/ask. For this purpose, it is the responsibility of the specialist to keep the track of all bids/asks.
- *Winner Determination*: Specialist needs to calculate the price based on supply and demand, and then match bids and asks together.
- *Clearing*: Once the winners are identified, specialist needs to notify them and clear their orders.
- *Information Revelation:* Specialist can reveal different types of information to traders. This may include bids/asks of other traders, winning bids/asks, overall turnover etc.
- *Tie Breaking:* Specialist is also responsible for breaking ties if there are conflicts between two or more buyers against a single seller, or vice versa.
- *Round Switch:* If the auction is multi-round, then specialist has announce the round's end, and switch to the next round which may involve changing the auction protocol, reserved price for goods etc.
- *Closing:* Specialist is responsible for closing auction, or indeed postpone the closing to prevent last-minute sniping.

Chapter 3

Simulation Model

3.1 Overview

In this chapter, I describe high-level architecture of my simulation model. I explain three important stages of the simulation, namely: a) Underlying Market, b) Option Trading Agents and c) Auctions. The first stage highlights the ways how I generate uncertain market data such as the asset prices and interest rates. I use stochastic models generating continuous/discontinuous and stationary/non-stationary time series. Non-stationary time series are used to simulate asset prices, while stationary time series model is good way of simulating interest rates in the underlying market. I use historical market prices and interest rates to calibrate my computer simulated data. In the second stage, I propose a generic architecture of option trading agents and stress the roles of each of its constituent components. Namely, I model trading agents through four layers: Information, Inventory, Knowledge and Behavioural layers. And finally, I propose overall simulation flow for 2 DAs for pricing options: a) direct auctions; b) online DAs. Although I do not run simulations with a combinatorial exchange in this research, it is still included in the mechanisms list. Figure 3.1 describes these three important aspects of my simulation model.

In underlying market, I simulate two stochastic processes that are relevant for pricing options. First is the asset pricing models such as Brownian model and Jump-Diffusion model. These models simulate continuous and discontinuous paths in asset prices. I use the NASDAQ-100 index for calibrating the parameters of stochastic processes. The second aspect of the simulation of underlying market is the interest rates. Although they are assumed to be constant in

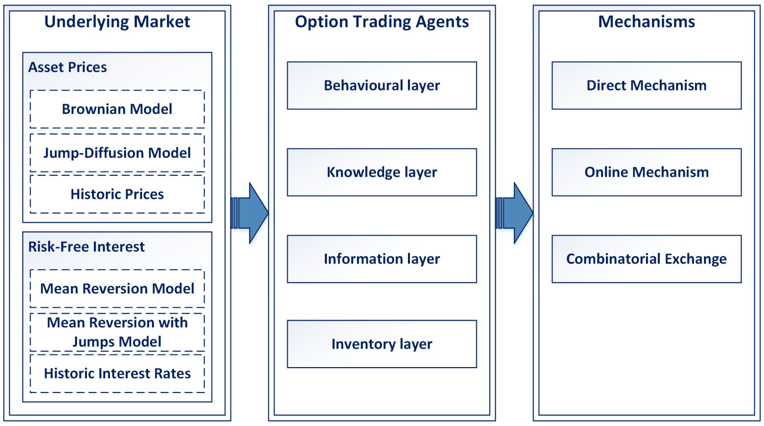


Figure 3.1: Simulation Overview

Black-Scholes framework, the practice shows that most of the time they fluctuate around certain mean throughout the year. This is best modelled using Mean Reverting processes, for that I use standard Vasicek process with an autoregressive parameter. I also try Vasicek model with jumps to mimic the scenario when the interest rate sharply decreases or increases. Historic indices for interest rate are taken from 2-year T-Bill rates.

As for trading agents, I describe the four important layers of its high-level architecture. I determine for each layer what functional roles it possesses, what data it stores, inputs and outputs and how the whole architecture fits together with different categories of traders I developed. I extended Vytelingum's IKB model for CDAs described in his PhD thesis [160] with additional Inventory layer. In this layer, I propose that some of the traders may use their inventory information in order to generate option prices. I proposed an option pricing technique called LMSR and a proxy trading algorithm (i.e. trading algorithm which takes its private value from other source) which directly use inventory information to set bid-ask spread. Besides the inventory layer, the other layers such as information, knowledge and behavioural layers have been applied to option trading scenario. For example, the information layer is devoted to gathering required data such as asset prices, interest rates, volumes, accepted orders, etc from the underlying market and the mechanisms. Knowledge layer is specialised in computing important statistical parameters of the information received such as mean, standard deviation, implied volatility, jump arrival rates and their means and volatility, etc. In behavioural layer agents use techniques specific to their trading behaviour to generate bids and asks.

In the third stage, I run different types of DAs for pricing options. The direct auction holds DSIC property making all the traders to submit truthful orders and everything gets cleared in one round. Although this simplifies the current CDA type markets where the options are traded continuously throughout the trading day, in my approach traders disclose their truthful valuations to the mechanism without engaging into trading algorithms. This, of course, allows us to mix different option pricing methodologies and obtain a competitive aggregate outcome. In second simulation approach, I run CDA along with proxy trading algorithms and compare the prices to theoretical prices.

This chapter is organised as follows. Section 3.2 provides an insight on the simulation of underlying market data and the models that are used. Section 3.3 proposes a new framework for option trading agents and explains the role of each layer in generating option prices. Finally Section 3.4 discusses the mechanisms used in my simulation models and provides scenarios in which they can be applied. In Section 3.5, I compare in detail the similarities and differences of my simulation model to other models proposed as a framework of an option market.

3.2 Underlying Market

The key component of any derivative pricing is the definition of its underlying market, because its price directly depends on the performance of its underlying asset or the factors related to its underlying market such as interest rate. Therefore, one should be able to simulate the dynamics of an underlying market in such a way that it mimics historical data. In this section, I first present my historical data for NASDAQ-100 Index, and provide short analysis of its parameters. Then I try to fit this data into different asset pricing models. As a result, I can find the corresponding simulation parameters for given models. Then I use these parameters to simulate asset prices for my further experiments. In some case, I might alter the parameters of the simulation to stress test certain experimental scenario.

While simulating the asset prices, I take into account the distribution of the returns. Using continuous Brownian motion for asset pricing would imply an assumption that the underlying asset market is highly liquid and efficient, and the asset price trajectory is smooth. The main reason for this is that the asset prices are assumed to be the cumulative sum of infinitesimally small orders submitted to the market. The impact of each order to the asset price is deemed to be negligible. This results in a continuous asset price model with normally distributed returns. Conversely, in a discontinuous model, I pose an assumption that there could be significant event in the market that may sharply alter the asset's value, thus making the asset price trajectory bear random jumps over some period. This is a plausible assumption because the asset price may change significantly due to an important news relevant to the asset. This can also be seen in typical price movements in the real markets. We should also note that discontinuous prices result in fat-tailed distribution of returns which differs from what Black and Scholes initially assumed.

Unlike asset prices, the interest rates usually fluctuate around their long-term mean. No matter where the interest rate is now, it should return back to its mean after certain time. Mean-reverting processes are commonly used models to simulate the interest rate fluctuations. The mean reverting processes can also be continuous or discontinuous depending whether they have sharp jumps within given period or not.

3.2.1 Historic Asset Prices

Historical data for NASDAQ-100 for one calendar year has been obtained from Yahoo! Finance [1]. The data is freely available on Web, and recorded as daily aggregate of opening, high, low and closing prices of 100 stocks listed in NASDAQ. I use adjusted closing prices as the main indicator of asset price for the given day. The reason for using NASDAQ index as an underlying asset price is that it represents most of the technological companies with volatile stocks which gives an additional level of stress testing capability in my experimental models. Dividends are paid for the stocks listed in this index, and one can calculate the aggregate of the total dividends paid to subtract them from asset price's annualised return. However, as it was mentioned in Chapter 1, I shall assume that there are no dividends paid on index based options like NASDAQ in the context of this research. Not including dividends in my model relieves the burden of estimating the uncertain dividends to be yielded per each stock, managing their payments and immediate adjustment of option prices in my simulation model. However once the dividends are known, or assumed constant, it is trivial to adjust option prices accordingly by reducing risk-free rate by the corresponding dividend rate. Table 3.1 displays a snapshot of raw data

[1] http://finance.yahoo.com/q?s=%5ENDX&ql=0

obtained for calendar year of 2014. The Appendix A lists all of the records used for NASDAQ index.

Date	Open	High	Low	Close	Volume	Adj Close
02/01/2014	3575.60	3577.03	3553.65	3563.57	1738820000	3563.57
03/01/2014	3564.94	3567.51	3537.61	3538.73	1667480000	3538.73
06/01/2014	3539.02	3542.52	3512.45	3526.96	2292840000	3526.96
07/01/2014	3539.29	3562.99	3535.50	3557.85	2278220000	3557.85
08/01/2014	3558.30	3575.15	3551.12	3567.54	2345220000	3567.54
09/01/2014	3576.33	3579.40	3541.81	3552.58	2214770000	3552.58
10/01/2014	3565.68	3568.47	3536.45	3565.08	2143070000	3565.08
13/01/2014	3559.39	3572.40	3499.37	3512.80	2322240000	3512.80
14/01/2014	3526.20	3581.60	3525.47	3580.65	2034180000	3580.65

Table 3.1: Snapshot of NASDAQ-100 Indices

I further analyse this data in order to compute the key parameters of it for our further asset price simulations. We have the actual asset price series $S_0, S_1, S_2, \ldots, S_t$ up to some time t, so we can compute the series of asset price returns $\frac{S_1-S_0}{S_0}, \frac{S_2-S_1}{S_1}, \ldots, \frac{S_t-S_{t-1}}{S_{t-1}}$. Let us denote asset price returns with $r_t = \frac{S_t-S_{t-1}}{S_{t-1}}$. Then we can find the sample mean of the series:

$$\hat{\mu} = \frac{1}{N}\sum_{t=1}^{N} r_t \tag{3.1}$$

And the sample variance is given by following equation:

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \hat{\mu})^2 \tag{3.2}$$

This would give us an average of $\hat{\mu} = 0.0006$ daily return and with standard deviation of $\hat{\sigma} = 0.0089$ on NASDAQ-100 over the year 2014. The histogram in Figure 3.2 also shows that the daily mean return is nearly zero. We can also see that the returns are skewed to the right indicating that NASDAQ-100 has grown over the year which is obvious from its price at the beginning of the year \$3563.57 and at the end of the year \$4236.28. Hence the annual growth of the index is 0.1888.

Although the histogram resembles the normal distribution, it is not obvious that it has fatter tails. Figure 3.3 illustrates the plot of the NASDAQ-100 index, its daily returns, Quantile-Quantile (QQ) and the autocorrelation with

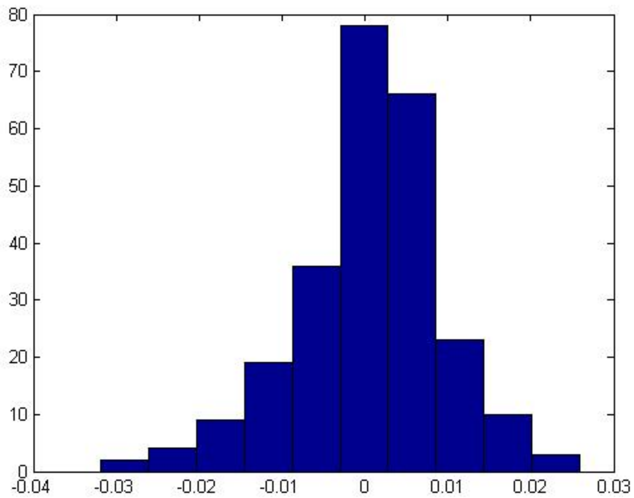


Figure 3.2: NASDAQ-100 daily returns over 2014

different time steps. From the first plot, we can see that the prices had several sharp jumps over the year. We can also see that the returns exhibit certain level volatility making the prices fluctuate. The most important plots, QQ and autocorrelation plots, show how the returns are distributed toward normal distribution and if there is an autocorrelation with past events. By looking at QQ plot we can see that the returns does not line up with normal distribution making the extreme quantiles diverge largely. This would mean that the distribution of the returns has fatter tails compared to normal distribution. Also the autocorrelation plot shows that there is an insignificant autocorrelation with past lags. This would suggest that the NASDAQ-100 returns are somewhat independent from past events. We have discussed two models for simulating asset prices: GBM and jump-diffusion process, so we will try to calibrate these models to NASDAQ-100 index trajectory. At the first glance, it seems that the best model which could simulate NASDAQ-100 index according to the given data is jump-diffusion process because QQ plot shows fat tails. However, I also calibrate the parameters of GBM to stress the difference between two models.

While analysing the prices, I have to admit that this particular price trajectory for NASDAQ-100 in 2014 is just a single instance of some stochastic model. It is impossible to accurately estimate the parameters of its stochastic model from observing the only instance of its price trajectory. However we can count on the estimators such as Maximum Likelihood Estimator (MLE) to guess the correct values of the parameters of the model. The main principle behind MLE

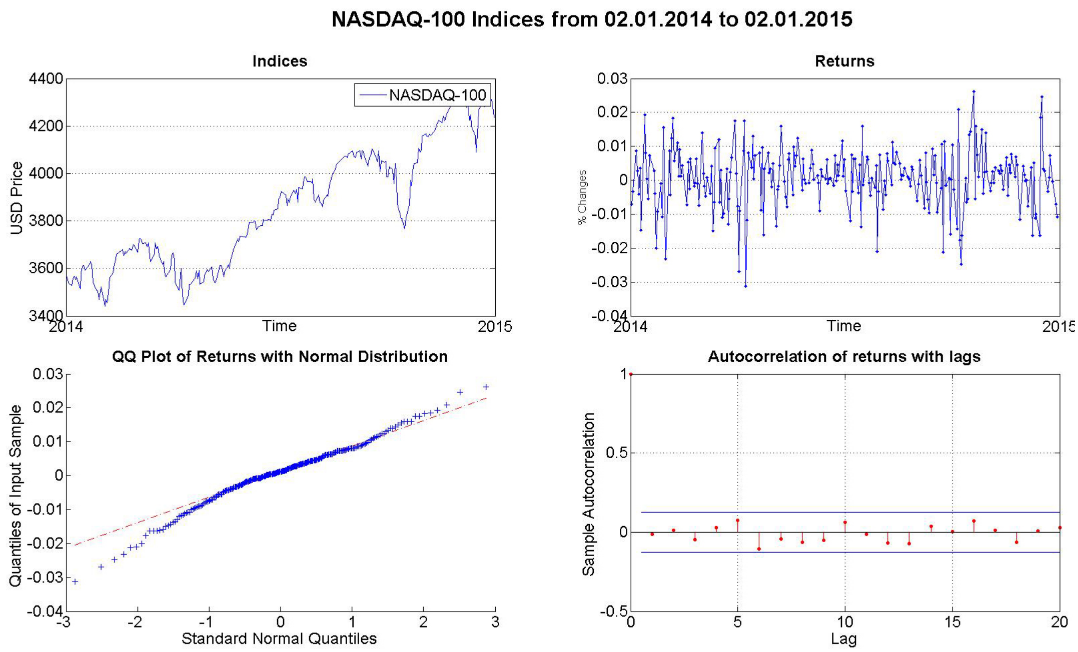


Figure 3.3: NASDAQ-100 Indices from 02.01.2014 to 02.01.2015

is finding the most likely set of parameters Θ of the model that would maximise the likelihood function for given set of data instances $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ of random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$. So we can write the likelihood function as a PDF or a Probability Mass Function (PMF) with free parameters $\boldsymbol{\theta}$ and with pre-configured event $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$. This can be written as following function:

$$\mathcal{L}(\boldsymbol{\theta}) = f_{X_1,X_2,\ldots,X_n;\Theta}(x_1, x_2, \ldots, x_n; \boldsymbol{\theta}) \tag{3.3}$$

If we are looking at $\mathbf{X}$ as the series of asset prices which follow a random walk, then the probability of asset price changing to particular value depends on where it is now. This is also referred as the Markov property of the process. For the likelihood function it can be written as the transition likelihoods between two adjacent steps:

$$\mathcal{L}(\boldsymbol{\theta}) = f_{X_1|X_2;\Theta}(x_1|x_2; \boldsymbol{\theta}) \ldots f_{X_n|X_{n-1};\Theta}(x_n|x_{n-1}; \boldsymbol{\theta}) \tag{3.4}$$

In case of GBM, where the returns are i.i.d., likelihood function can even be simplified to the product of individual PDFs.

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{i=1}^{n} f_{X_i;\Theta}(x_i;\boldsymbol{\theta}) \tag{3.5}$$

However the product of probabilities can be very small number which can cause numerical issues. Therefore log-likelihood can be used as an objective function to maximise.

$$\log \mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{n} \log f_{X_i;\Theta}(x_i;\boldsymbol{\theta}) \tag{3.6}$$

We can find the maximal likelihood either analytically, or numerically, but the latter one is easier due to the ready-made software such as `fminsearch` and `fzero` in MATLAB. After solving the maximisation problem for normal likelihood on index returns, we can obtain that the mean is $\hat{\mu} = 0.0007$ and standard deviation $\hat{\sigma} = 0.0088$ which are very close to sample mean and standard deviations calculated earlier. So we can use for simulating the either of proposed asset pricing models.

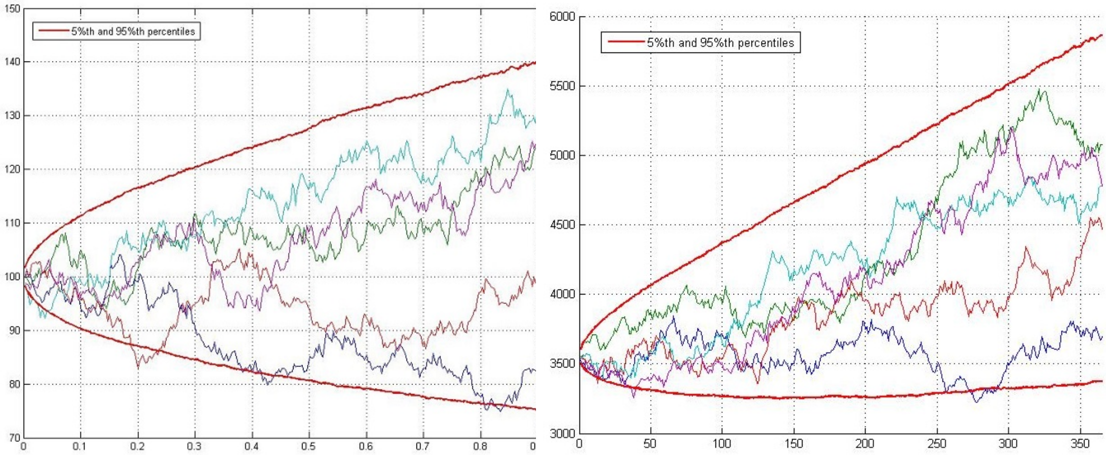
To sum up, in order to simulate asset prices that would mimic the historical trajectory, we should use μ and σ which are estimated to be $\hat{\mu} = 0.0007$ and $\hat{\sigma} = 0.0088$ as daily return and volatility parameter. Further we will see how well these parameters fit with GBM and jump diffusion models.

3.2.2 Geometric Brownian Motion

We have already defined the GBM in Chapter 2 and specified its main components such as drift and diffusion. As it was mentioned earlier, such Stochastic Differential Equation (SDE)s simulate ideal conditions in the asset market where the market is highly efficient and liquid with no frictions and the traders are independently distributed. In such market, prices are supposed to be continuous and non-stationary. GBM model perfectly simulates this kind of behaviour. In below simulation, we have pre-set the parameters of the process as shown in Table 3.2.

Asset price at $t = 0$	$S_0 = 100$
Annual mean	$\mu = 0.05$
Annual standard deviation	$\sigma = 0.1$
Time interval	$\Delta t = 1/365$

Table 3.2: Parameters of the asset price simulation using GBM process



(a) Virtual asset (b) NASDAQ-100 indices in 2014

Figure 3.4: 5 possible instances of price trajectories generated using GBM process

I have simulated 10000 instances of asset prices using GBM process and displayed 5 instances out of them in Figure 3.4a. It can be seen that price trajectories do not contain sudden jumps, and the changes are infinitesimally small which make them continuous. Also we can see that the prices can move in any direction with no tendency to revert to the mean. Hence there is no effect of past decisions, and the prices should exhibit very small autocorrelation. This would make the prices non-stationary too. Figure 3.4a also shows the 5%th and 95%th percentiles of 10000 simulations to illustrate the confidence boundaries of price trajectories at each time step. It can be seen that as the time progresses, the uncertainty about the future asset prices grows with it according to the definition of GBM. The percentiles of the asset prices can be used to draw the probability distribution of an asset price at given time t.

I also simulated the asset prices using the parameters obtained from NASDAQ-100. These parameters has been already estimated in previous section using MLE method for normal i.i.d.log-likelihood (3.6). I provided the settings in Table 3.3. These settings will be used to simulate prices for the experiments with trading agents and mechanisms presented in this research. Figure 3.4b

NASDAQ-100 at $t = 0$	$S_0 = \$3563.57$
Daily return mean	$\mu = 0.0006$
Daily return standard deviation	$\sigma = 0.0089$
Time interval	$\Delta t = 1$

Table 3.3: Calibrated parameters of GBM process on NASDAQ-100 returns

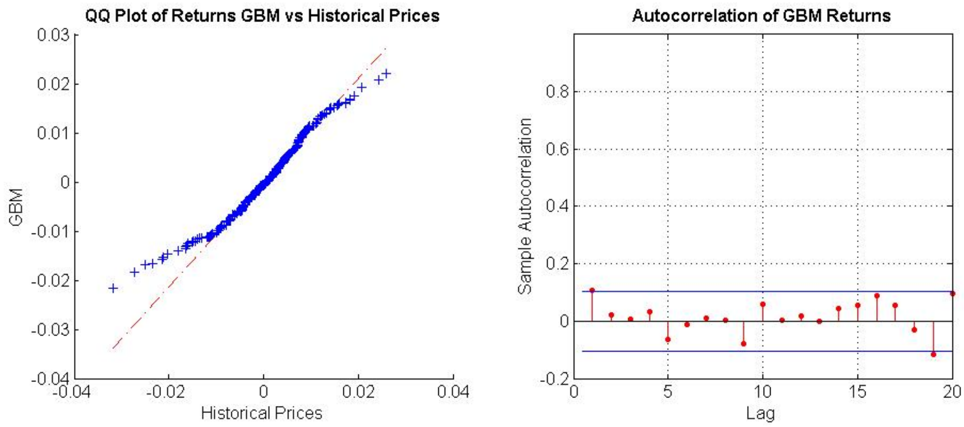


Figure 3.5: GBM: QQ Plot with Historical Returns and Autoregression

illustrates 5 possible trajectories of NASDAQ-100 out of 1000 generated. The percentiles of all trajectories simulated are also presented here. Due to the positive daily return, the percentiles move in upward direction asserting that in most scenarios, NASDAQ-100 index would grow.

To test if GBM model aligns well with actual historical returns, we take any arbitrary instance of GBM simulation and plot QQ along with historical returns. We can see that GBM diverge from the actual returns at tails. Figure 3.5 illustrates it clearly in the first chart. Second chart shows the autocorrelation of GBM returns which are below significance level. This is an expected outcome because in GBM, as there is no dependence on past returns.

Given accurate μ and σ, the GBM simulation of the whole path can be skipped, if all trader needs is the asset price S_T at time T. This shortcut formula (3.7) helps to compute it analytically:

$$S_T = S_0 e^{(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)T + \hat{\sigma} dW_T} \tag{3.7}$$

where $dW_T \sim \mathcal{N}(0, \sqrt{T})$

3.2.3 Jump-Diffusion Model

We have talked about jump-diffusion processes in Chapter 2 previously. Merton has introduced this asset pricing model back in 1976 and has given a closed form solution for pricing a European option whose underlying asset follows jump-diffusion process [114]. The model consists of 3 main uncertainties: GBM's infinitesimal random shocks, Poisson jumps and normally distributed magnitude of the jumps. The SDE of the model can be written as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_t dJ_t \tag{3.8}$$

where W_t is Wiener process, and J_t is a jump process defined as follows:

$$J_T = \sum_{j=1}^{N_T} (Y_j - 1) \tag{3.9}$$

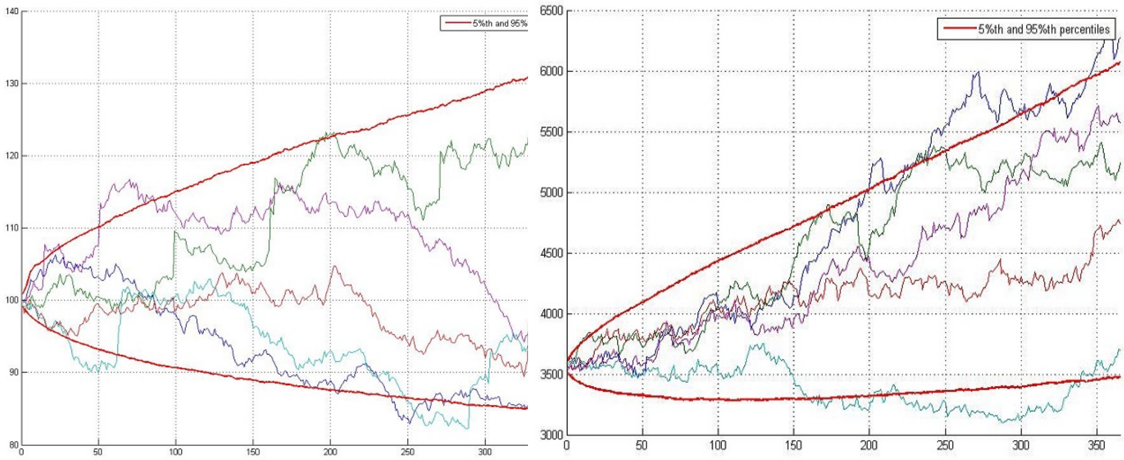
where N_t follows Poisson process with intensity λ indicating the number of arrivals per time t. Hence that $N_T \sim Poisson(\lambda T)$ is distributed according to Poisson distribution. Y_j represents a log-normally distributed random variable $\log Y_j \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$ which indicates the magnitude of j-th jump with respect to the current price. In order to avoid negative values for S_t, I have to convert the series into logarithmic series like I did for GBM. In this way, extending the (3.7) with jumps we can obtain following SDE for log-prices (remember $s_t = \log S_t$):

$$ds_t = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t + \log Y_{N_t} dN_t \tag{3.10}$$

As I did for GBM, the random end values of asset price at time T including jumps can be computed analytically. If we solve (3.7) and convert it back to asset prices, we obtain following formula for asset price S_T at time T. Agents can run this formula without doing the simulation in order to estimate the possible values of S_T.

$$S_T = S_0 e^{((\mu - \frac{1}{2}\sigma^2)T + \sigma W_T) \prod_{j=1}^{N_T} Y_j} \tag{3.11}$$

I have simulated the jump-diffusion process to illustrate if it exhibits sharp changes in price due to random Poisson arrivals. This is an example simulation which does not calibrate to any historic data, and thus the parameters used in this simulation are arbitrary. Table 3.4 provides the values of the parameters that I used. Besides standard GBM parameters μ and σ, I have also set the jump arrival rate or jump intensity equal to 0.01 which would mean that the jumps must occur every 100 days on average. Also the log magnitude of a single jump is drawn from $\mathcal{N}(0.05, 0.002)$. It can be observed from Figure 3.6a that there are couple of upward jumps happening on every instance due to the positive mean of the jump magnitude. Also we can see that the jumps are uniform all the time which means that only one jump is happening at given



(a) Virtual asset (b) NASDAQ-100 indices in 2014

Figure 3.6: 5 possible instances of price trajectories generated using jump-diffusion process

time. If there were several jumps arriving at the same time, the magnitude of the jumps would be twice or N_t times greater. The percentiles from 10000 simulations show the estimated boundaries of the price trajectories. As in GBM, the boundaries get wider as the time progresses.

Asset price at $t = 0$	$S_0 = 100$
Daily mean	$\mu = 0.05/365$
Daily standard deviation	$\sigma = 0.1/365$
Daily jump intensity	$\lambda = 0.01$
Jump mean	$\mu_Y = 0.05$
Jump standard deviation	$\sigma_Y = 0.002$
Time interval	$\Delta t = 1$

Table 3.4: Parameters of the asset price simulation using jump-diffusion process

To estimate the parameters of jump-diffusion model $\boldsymbol{\theta} = (\mu, \sigma, \lambda, \mu_Y, \sigma_Y)$ to NASDAQ-100 prices, I again use MLE method where the parameters that maximise the log-likelihood are found. Taking into account that the Poisson arrivals and the asset price changes are independent events, we can define the likelihood function for given return x_i as the sum of all possible Poisson cases for given return x_i:

$$f_{X_i;\Theta}(x_i; \boldsymbol{\theta}) = \sum_{j=0}^{+\infty} P(n_t = j) f_{\mathcal{N}}(x_i; (\mu - \sigma^2/2)\Delta t + j\mu_Y, \sigma^2\Delta t + j\sigma_Y^2) \quad (3.12)$$

If Δt is very small period, then Poisson random variable is $N_t = \{0, 1\}$ meaning that there is either one jump or there is no jump at all within Δt period. This should simplify (3.12) to following equation:

$$
\begin{aligned}
f_{X_i;\Theta}(x_i; \boldsymbol{\theta}) =& \lambda \Delta f_{\mathcal{N}}(x_i; (\mu - \sigma^2/2)\Delta t + \mu_Y, \sigma^2 \Delta t + \sigma_Y^2) \\
&+ (1 - \lambda\Delta) f_{\mathcal{N}}(x_i; (\mu - \sigma^2/2)\Delta t, \sigma^2 \Delta t)
\end{aligned} \tag{3.13}
$$

Then I can use (3.6) MLE objective function to find maximising parameters of $\boldsymbol{\theta}$. I used the same optimisation subroutine provided by MATLAB to accomplish this task. We found following calibrated parameters for jump-diffusion process that would match NASDAQ-100 price trajectory. Table 3.5 presents these parameters: In Tables 3.5 and 3.3, the calibrated parameters show that

NASDAQ-100 at $t = 0$	$S_0 = \$3563.57$
Daily return mean	$\mu = 0.00068$
Daily return standard deviation	$\sigma = 0.0046$
Daily jump intensity	$\lambda = 0.8$
Jump mean	$\mu_Y = -0.0021$
Jump standard deviation	$\sigma_Y = 0.0083$
Time interval	$\Delta t = 1$

Table 3.5: Calibrated parameters of jump-diffusion process on NASDAQ-100 returns

the mean returns μ of both GBM and jump-diffusion models are almost the same, however the standard deviation $\sigma = 0.0046$ for jump-diffusion process is twice smaller than GBM. This shifts the responsibility for sudden price changes to jumps introduced in the model, hence the standard deviation of GBM part of the model is evaluated less that in pure GBM. We can also see that the intensity of jumps are considerably frequent making an expected jump once in every 1.25 days. We can also observe from the negative value of mean μ_Y of jump magnitudes the index was expected to drop by -0.21% on every jump with standard deviation of 0.83%.

I have simulated the 10000 possible paths of NASDAQ-100 index for 2014 using the calibrated parameters, and presented 5 instances of them in Figure 3.6b. Comparing the Figures 3.4b and 3.6b, we can see that the percentiles for both models cover nearly the same range, thus making the confidence interval for using either model to be the same. My simulation model uses the unseen asset price trajectory for agents to compute option prices, although the parameters of the simulation such as mean of returns, standard deviation of returns, daily

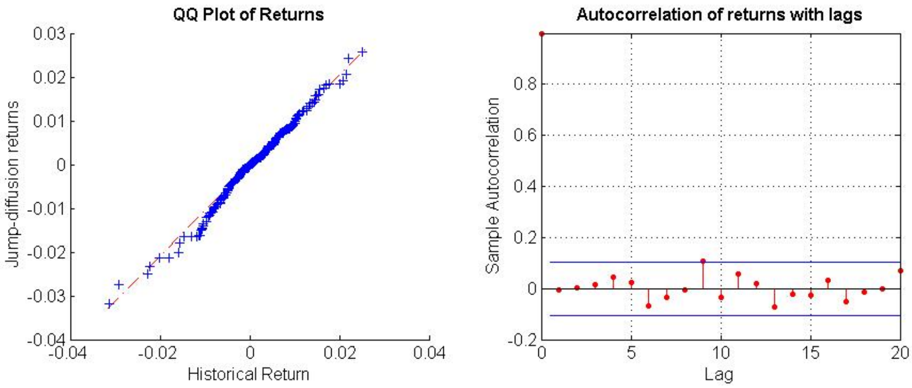


Figure 3.7: Jump-Diffusion: QQ Plot and Autocorrelation

jump intensity, jump mean, jump standard deviation are assumed to be known to agents as they can use the above described techniques to calibrate chosen asset pricing model to historic data. For example, Monte Carlo option trading agents simulate certain number of asset price paths with given parameters to estimate the risk-neutral value of the option.

As I did in GBM model, I also test how simulated returns fit with the actual historical returns in QQ plot. Figure 3.7 shows that historical returns and simulated returns align along single line. This suggests that historical returns have the same distribution as jump-diffusion model. We noted that jump-diffusion models are used to simulate fat-tailed distributions. Also we can see that there is no significant autocorrelation in returns confirming the index is somewhat independent from past events. I have mentioned earlier in the section that NASDAQ-100 prices could be simulated best using jump-diffusion model, and this confirms this statement.

3.2.4 Mean-Reverting Process

Mean-reverting process, or sometimes referred as stationary process, is good model for simulating time-series data such as interest rates [88, 92], exchange rates [14], electricity rates [107, 55] which are supposed to remain stable over long period of time. For example, US T-bills are traded in stock market with nominal value of \$100 at the time of expiration. Its present value which is normally less than \$100 is determined through the financial market where traders bid and ask for this security. If its present value is \$90, then this would mean that at the time of T-Bill's expiration, the investor makes $(100-90)/90 \approx$ 11% return. Because US economy is huge and comparatively stable, there is

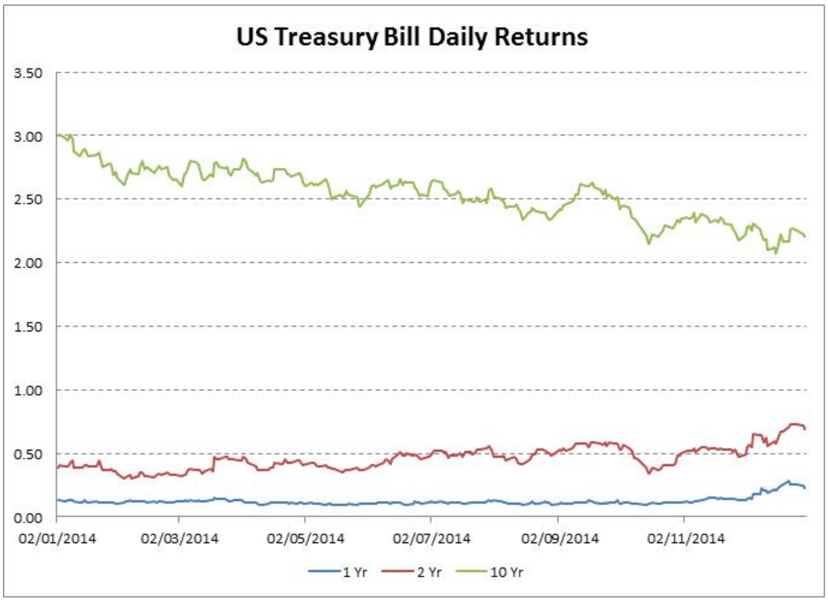


Figure 3.8: Daily returns of US Treasury Bills with different expiration terms

very little risk in US economy going default and not being able to pay back promised \$100 per T-Bill at its expiration. Therefore this 11% return can be viewed as risk-free investment rate. This overall established trust in US economy can hardly be influenced in short period of time, and so the traders in the market submit more or less the same quotes for T-Bills with very little volatility in day to day trading. There could be temporary rises or falls of risk-free rates for short period, but because the US economy is not going to change dramatically overnight, the rates return back to their respective mean by exhibiting mean-reverting behaviour. This can also be seen in below chart 3.8 showing the daily returns of US T-Bills in 2014. We have included the full data for this chart in the Appendix A for future references.

Because in Black-Scholes model the risk-free rate is accepted as constant, in most of our experiments I assume that this assumption is true. However, simulations can equally assume cases where the interest rate itself is a stochastic process. Interest rates do change because in practice, it is computed as the return from T-Bill which is openly traded in the market, and hence subject to continuous changes in value. In order to simulate the dynamics of risk-free rates, I use the exponential Vasicek model [158] discussed in Chapter 2. The reason for choosing exponential Vasicek model is that it simulates only positive values for risk-free rates in the same way as GBM does for asset prices. In order to transform standard Vasicek model to exponential form we have to

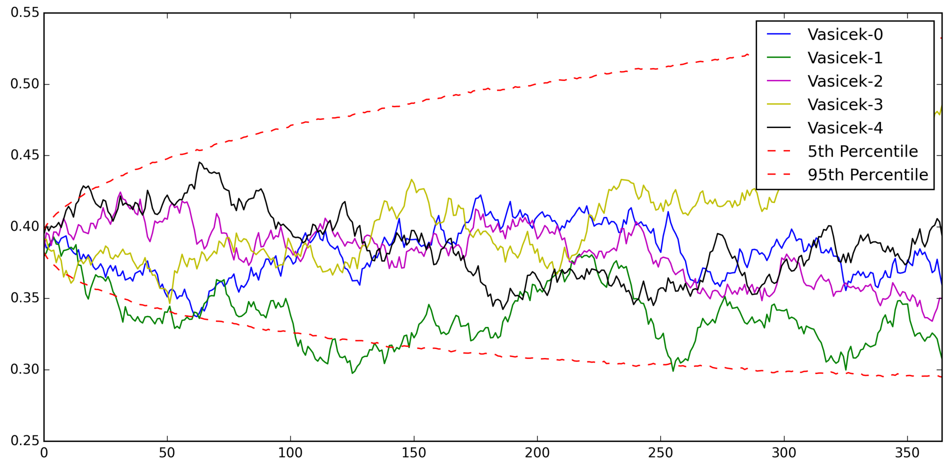


Figure 3.9: Simulation of risk-free rates using Vasicek model

define the log-returns s_t as standard Vasicek process:

$$ds_t = \theta(\mu - s_t)dt + \sigma dW_t \tag{3.14}$$

where parameters θ represents speed of reversion, μ long-term mean and σ volatility.

In this way, we can have the risk-free rates $S_t = e^{s_t}$, and by applying Ito's lemma [91] (i.e. second-order Taylor expansion with a stochastic term), we obtain following formulation of exponential Vasicek process:

$$dS_t = \theta S_t(\mu + \frac{\sigma^2}{2\theta} - \log S_t)dt + \sigma S_t dW_t \tag{3.15}$$

The main statistical parameters such as mean and standard deviation of US T-bills can be obtained from given historical data above. Hence for 2-year T-bill, the mean is $\mu = 0.46$ and the standard deviation is $\sigma = 0.09$. I use these parameters along with setting an arbitrary speed of reversion, such as $\theta = 0.1$ to simulate Vasicek model. Figure 3.9 illustrates simulation of Vasicek process with above parameters. It can be seen that Vasicek price even if it fluctuates, it does not leave the mean risk-free rate. Red lines depict 5% and 95% percentiles of 10000 instances of Vasicek simulation. It can be seen from these instances that all these simulated paths are not diverging from the mean, in 90% confidence interval fluctuate between approximately 0.1 and 0.8.

I present the autocorrelation and partial autocorrelation graphs in Figure 3.10 to highlight the effect of past prices to the current price. In autocorrelation

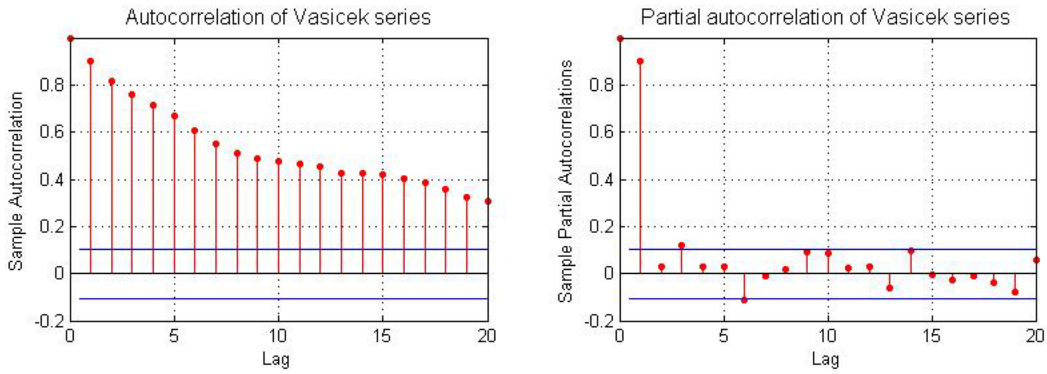


Figure 3.10: Autocorrelation and Partial Autocorrelation of Vasicek prices

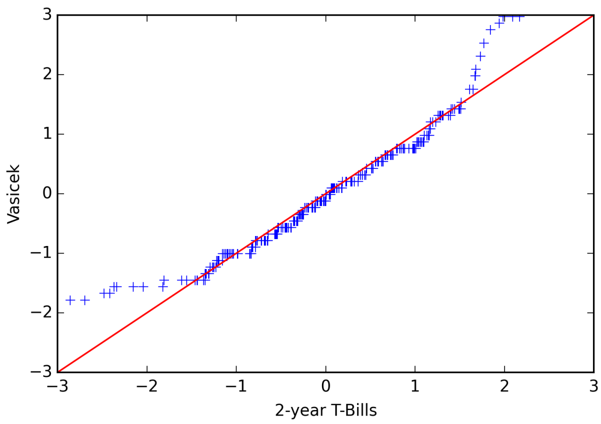


Figure 3.11: Vasicek model vs 2-year T-bills

graph, we can see that the diminishing influence of past prices on current price due to the recursion in Vasicek's model. Partial autocorrelation shows which lags directly affect the next price. Because the mean-reversion model is the instance of AR(1) (Autoregressive) model, the partial autocorrelation shows significant effect of previous value on current value.

We can also look at the QQ-plot in Figure 3.11 between an arbitrary instance of Vasicek model simulation, and the historical returns of 2-year T-Bills. This clearly shows that both series are distributed similarly, although Vasicek series are seen more volatile and covering wider range of possibilities.

3.2.5 Mean-Reverting Jump Process

In this model, I introduce random jumps to mean-reverting model. This incorporates a situation when there is an important news which could cause dramatic changes in US T-Bills market, and hence sharply changes risk-free

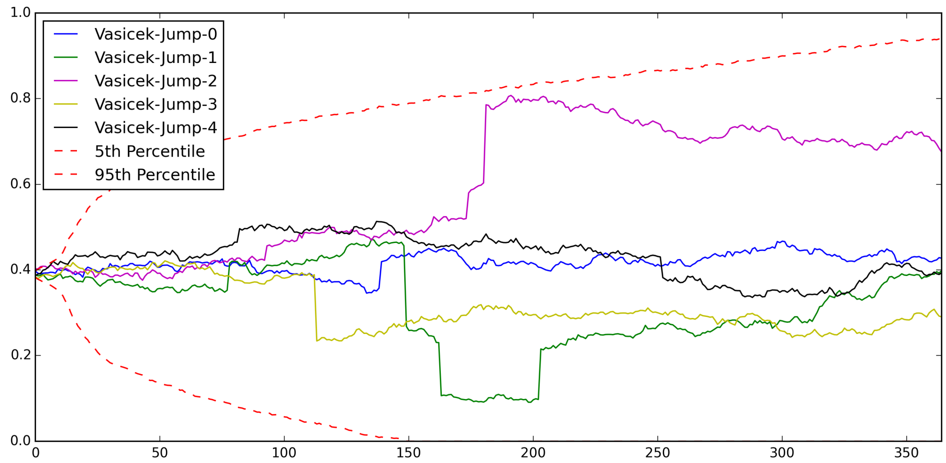


Figure 3.12: Simulation risk-free rates using Vasicek-Jump model

rate for a limited period of time until the process returns back to its mean. We have observed how jumps can be added into standard GBM in earlier sections. Now I shall add the jumps to Vasicek model. It can be as straightforward as adding extra jump term dJ_t to Vasicek model of log-returns (3.14)

$$ds_t = \theta(\mu - s_t)dt + \sigma dW_t + dJ_t \tag{3.16}$$

where jump process dJ_t is given by (3.9).

Figure 3.12 illustrates 5 instances of Vasicek-Jump model simulation. I have used the same parameters as in previous Vasicek model, setting $\mu = 0.46$ and $\sigma = 0.09$. I set the jump arrival rate $\lambda = 4$, but it can be set to any arbitrary number depending the experimental scenario. Jump mean $\mu_\lambda = 0.2$ and $\sigma_\lambda = 0.1$ are also arbitrarily set. We can also look at the percentiles of 10000 instances determining the 90% confidence interval. Although we cannot observe the full convergence of paths to the mean, it is still noticeable that the trajectories lean towards the mean even after sharp jumps.

We can also check how an instance of a Vasicek-Jump model simulation complies with historic T-bill returns in QQ-plot shown in Figure 3.13. There is a significant divergence at higher quantile because this particular instance has sharp jump in its trajectory. This shows that Vasicek-Jump does not perfectly fit the T-Bill returns. I do not include the experiments with the use Vasicek-Jump process simulating the interest rates in the scope of this work.

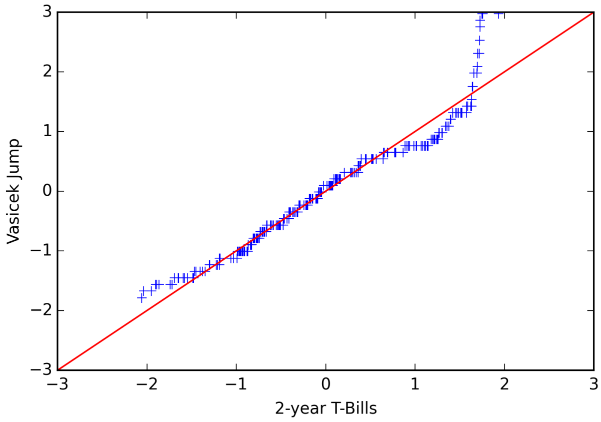


Figure 3.13: Vasicek-Jump model vs 2-year T-bills

3.3 Option Trading Agents Framework

This section provides a general framework for trading agents that I designed and implemented for different environments. I slightly extended Vytelingum's IKB model for CDAs described in his PhD thesis [160] and incorporated the requirements of this research into it. Vytelingum's framework has minimalistic separation of concepts and it is easily adaptable for variety of applications. It perfectly fits into scenarios I am going to simulate in this thesis, because it makes agents independent from the mechanism they are interacting with. This, of course, helps us to replace and to reuse the modules developed for each aspect of trader's behaviour. According to the framework, agent requires information about itself and the environment first, and it has a limited capability to extract the knowledge from given raw data. Once it extracts the knowledge, its behaviour decides how to trade using this knowledge.

I define the structure of generic trading agent using three layers of IKB model plus one more layer for the Inventory the agent holds. Agent's inventory, or also referred as portfolio, in the context of this thesis may also affect his market behaviour. I refer to this new framework with a new acronym IIKB framework. Its main aspect is illustrated in Figure 3.14. This is an abstract inclusive model for any type of trader in this research. This means that not every component described below may necessarily exist in concrete implementation of the trader. For example, Zero Intelligence (ZI)traders do not have components responsible for processing the live information from market, or maintaining an inventory. Below sections provide detailed explanation of the role of each layer in the design of option trading agents.

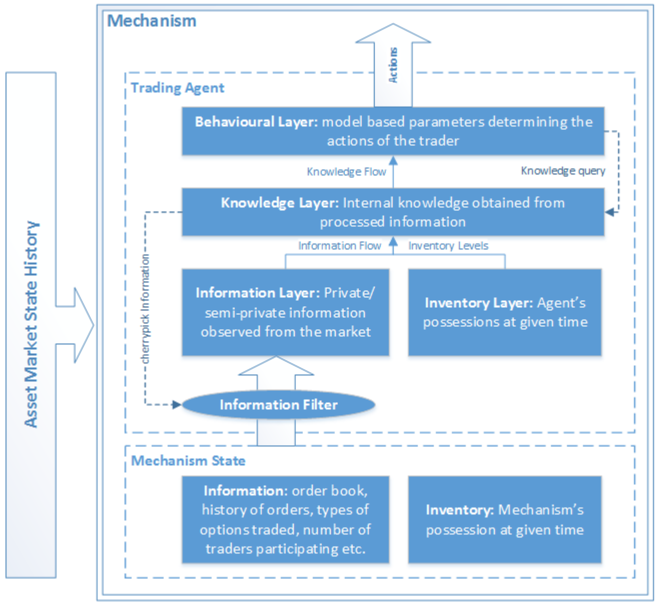


Figure 3.14: Structure Of the IIKB Model

3.3.1 Information Layer

In this layer, agent collects raw data about the mechanism and its current state. Although it sounds confusing that the data is referred as an information in the original framework[160], I shall stick with the same naming convention. So in order to avoid confusion, further across this thesis, I shall use information layer as the layer responsible for collecting and storing raw data. The data collected is mostly the order book, the history of option clearing prices, option types traded, the depth of the market, and other constants specific to particular mechanism or underlying market. Depending on a type, agents may drop information about the certain aspects of the mechanism state such as the number of participants, because their behavioural model does not require this information in order to produce option prices. The information is collected based on the needs of the agent. For example, agent can be myopic and does not need the full historic trend in an online market. This type of agent has a limited capability of learning or encompassing the full information about current state of the orders.

We can classify the information into following categories according to Mas-Collel [112]:

- *Complete/Incomplete:* An agent is said to possess complete information if he knows the complete structure of the market which involves the actions of other participants, their corresponding utilities and expectations, the rules of the mechanism and other data which could completely describe any phenomenon that happens in the market. Otherwise, agent is said to possess incomplete information.

- *Perfect/Imperfect:* An agent is said to possess perfect information if he is certain of all the information he knows including his own type, market's current state and its corresponding history and future. Otherwise, he is said to have an imperfect information.

Mostly the information the agents will be dealing with is incomplete and imperfect because of computational and functional limitations of the agent. Also it is impractical to assume that trader might have access to the utility functions of other agents. In real markets, traders may not guess the action sets of other traders, and could not base their decisions on that. The only available information is publicly posted orders and volumes. Information that agents use throughout the thesis is also imperfect, because the expectation of future asset prices bear uncertainty, and even more, the priors that agents use to model the asset prices bear uncertainty too. Therefore agents cannot fully predict the future of the asset prices, not only because it is a random process, but mostly because they have no control over it. Uncertainty that may arise inside the mechanism can be the behaviour of other market participants, how they are going to react to a certain event in assets market and what their predictions are going to be. This all depends on the population and the diversity of option traders.

Agents deal with both aggregated and generated information while making buying or selling decisions, however they can collectively influence only the aggregated one. Talking about the types of information by its source, we can break it down into two categories:

- *Aggregated Information:* This information is obtained from the mechanism (i.e. option market) as a result of a one-period or continuous interactions of trading agents with the mechanism. Agents can influence the flow of this information through the aggregate effect of their actions.

- *Generated Information:* This is an information that cannot be influenced by option trading agents. It is assumed that this information is generated using stochastic process. Its flow is independent of any transactions happening in the mechanism. We can classify the state of the asset prices $state_S$ in to this category. Agents take this information as it is and calibrate their option pricing models according to its statistical parameters.

3.3.2 Inventory Layer

There are two reasons for maintaining this layer in proposed framework for option traders. Firstly this layer imposes certain rules on holding cash and other liquidities to certain agents such as dealers and portfolio holding agents. Thus it affects their option preferences and price setting behaviour. Secondly, it allows to bookkeep the agent's transactions, and calculate statistics about his overall performance.

Agents hold different accounts to store the information about these inventories. There are three kinds of inventories agents may buy, sell and hold: cash, underlying asset and options.

- *Cash Account*: Cash is assigned as an initial endowment to the inventory-based agent before the simulation, and can be further obtained through trading. $I_i^c(t)$ is used to denote the amount of cash in cash account of agent i at time t. Keeping record of available cash is important for an inventory-based dealer who uses it as a constraint in determining his bid-ask spread. Also delta hedging strategy involves maintenance of cash account which bears risk-free interest payments.

- *Underlying Asset Account*: The underlying asset is bought or sold at spot market price to the mechanism. Agent i's asset account at time t is denoted as $I_i^S(t)$. This is particularly important if agent is using delta hedging strategy, because he needs to maintain the delta portion of an asset to balance his replicating portfolio between risky underlying asset and risk-free cash.

- *Option Accounts:* Options are the main commodities that are traded inside simulated mechanisms, and there can be multiple types of them traded in the same market. The accounts for holding options are denoted as $I_i^j(t)$ which represents agent's inventory at time t on jth option traded in the mechanism. This account is crucial for LMSR option pricing trader, because it uses the existing portfolio of options to value the price of the option he is going to buy or sell.

I assume that cash is measured in the units of some virtual currency, and because I use NASDAQ-100 index which is measured in USDs, I use this currency to denote cash. Agents use cash to buy or sell assets or options to the mechanism. Options traded in the mechanism are standard European call or put options throughout the thesis unless it is clarified otherwise. It has been discussed in Chapter 2 that European option can be fully defined the number of parameters. Below I list the parameters that are used in my model:

j A positive integer defining the option type traded in the market where even j refers to call option, and odd refers to put option. We can find the intrinsic value of the option using this unified formula: $\max((-1)^j(S_t - K_j), 0)$

r Risk-free interest rate

T Option maturity date (i.e. the time when option gets exercised if its value is above zero).

K Strike price

S_0 Spot asset price

σ Asset price volatility (i.e. the standard deviation of an asset price)

In my simulation model, all options have the same maturity date T to compare them with theoretical price. Also options are created on one type of underlying asset denoted by S_t, and all agents have full information about everything related to the content of the option contract. Thus variables such as S_0, K, T and j are regarded as common knowledge. Moreover agents use the same risk-free interest rate r while pricing option. In some cases, the risk-free rate is not a constant term, and subject to change over time, but these changes are also publicly known to traders. One parameter the agents may vary privately is the implied volatility parameter σ. Because the option pricing methods are different, the implied volatility of the agent may vary too. Implied volatility is commonly considered the way how agents perceive risk. In some scenarios, I deliberately impose perturbed volatility surface to agents pricing options using the marginal risk-neutral distribution obtained from volatility surface. This supports the fact that there are different beliefs about the future of the asset price in the market.

So we can vary options traded in the auction by changing two parameters: option type and strike price K_j. In this way, we can cover whole spectrum of moneyness for options. In real markets, options with the same maturity date, but different strike prices are displayed as option chains like in Table 3.6:

Strikes	**Last Put**	**Put Bid**	**Put Ask**	**Last Call**	**Call Bid**	**Call Ask**
4300	59.21	53.40	56.50	155.08	155.90	161.30
4325	61.15	60.20	63.20	149.40	137.90	144.00
4335	43.47	63.00	66.30	168.77	130.90	136.80
4340	63.00	64.70	67.50	X	X	X
4345	62.23	66.00	69.20	X	X	X
4350	70.12	67.70	70.80	118.78	120.70	125.90
4370	72.30	74.50	77.90	X	X	X
4375	78.72	76.40	79.60	106.80	104.40	109.20
4385	62.82	79.70	83.20	X	X	X
Current Underlying Price: $4399.23						
4400	91.42	85.50	89.20	85.00	89.10	93.90
4420	95.01	93.00	97.90	X	X	X
4425	99.30	93.90	100.20	76.80	74.90	79.00
4440	X	X	X	94.10	67.00	71.10
4445	90.00	102.10	110.40	93.80	64.40	68.00
4450	112.00	105.90	112.40	59.60	62.00	65.50
4460	92.00	110.10	117.60	57.06	57.20	60.70
4475	94.41	118.10	126.20	51.00	50.40	53.80
4480	X	X	X	72.50	48.20	51.30
4485	X	X	X	70.50	46.00	49.80
4500	106.28	131.30	141.10	42.00	40.00	43.50

Table 3.6: NASDAQ-100 (NDX) Option chain for options expiring on 17.04.2015. X denotes options with no open interest.[2]

We can see in Table 3.6 the display of an *option chain* which represents different quotes for options with different strike prices and time to maturity. But in my case, I fix the time-to-maturity part of the option chain. So option chain is the series of open bids and asks for put and call options with the same maturity date and underlying asset, but with varying strike price. Table 3.6, for example, captures the range from \$4300 to \$4500 for NASDAQ index where its spot price is \$4399.23.

All three types of inventory accounts $I_i^c(t)$ cash account, $I_i^S(t)$ underlying asset account and $I_i^j(t)$ option accounts belong to $state_i(t)$ of agent i. We can denote $I_i(t) = I_i^c(t) + I_i^a(t) + \sum_{j=1}^{k} I_i^j(t)$ as agent's total inventory at time t. So $I_i(0)$ is the agent i's initial endowment at time 0. We can write the distribution of agent i's total endowment into inventories using ratio $\pi_i^j(t) = \frac{I_i^j(t)}{I_i(t)}, \forall j \in 1, 2, \ldots, k+2$ where $j = 1$ and $j = 2$ stand for cash and asset accounts. This defines an agent i's *portfolio*, or distribution of agent's total wealth, for different items in the market at given time. It is written as a vector-valued function of t,

$\boldsymbol{\pi}_i(t) = (\pi_i^1(t), \pi_i^2(t), \ldots, \pi_i^j(t), \ldots, \pi_i^{k+2}(t))$ such that $\sum_{j=1}^{k+2} \pi_i^j(t) = 1, \forall t$. Note that $\pi_i^j(t) : \mathbb{R}^+ \to \mathbb{R}$, so it can return both positive and negative real values as long as the feasibility constraint is met. The positive amount represents agent's long position, and the negative is the short position. The total wealth can be written as monetary equivalent of inventory multiplied by corresponding portfolio weights.

We have discussed about well-known option portfolios in Chapter 2. So option portfolio is a combination of short and long positions in options with different moneyness and types. Agents use option portfolios to limit the risk of infinite loss, and for particular agents such as LMSR or Monte-Carlo they play an important role in pricing other options. Traders holding certain option portfolios price options differently based on the final payoff they can get from simulating different asset prices. This is considered as the options inventory of the agent, and stored in option accounts correspondingly.

3.3.3 Knowledge Layer

Knowledge layer is an intermediate layer which bridges information with the behaviour of the trader. Its main function is to infer the key statistical parameters need by the behavioural layer. For example, if the trader is using Black-Scholes valuation, then it needs to find out the implied volatility of the asset prices from previously known information.

In knowledge layer, agent performs three types of operations with available data in information layer. First, it undertakes a statistical analysis of raw data and computes key parameters such as sample mean and variance of the obtained data. These parameters are used by the probability distributions which model random events in the market. For example, by knowing the sample mean and variance of the asset prices, agent can plug these values into normal or log-normal Brownian processes and project the possible trajectories of asset prices at option's maturity date. This can be used to price options using Monte Carlo method.

Second, knowledge layer performs the calibration of unknown parameters of the model to the market data. Sometimes the parameters of the model can be implicit and cannot be calculated by simply aggregating the collected data. For example, agent's using Black-Scholes formula for pricing options have an unknown parameter, volatility σ. During the calibration process, the best fitting volatility is found through a root finding subroutine. However, conversely to Black-Scholes assumption, volatility is not a constant figure, and it changes for each option with different strike and maturity date. Therefore agent's job is not only finding the single value of σ for given strike and maturity date, but formulating an approximation function or simplified mathematical model for finding the value of σ for any given strike or maturity date.

Finally, third important aspect of the knowledge layer is the inference of global information from incoming new market data. Agent's might need to know the global information about the market which might not be very obvious due to the complexity of the interactions between agents. For example, agent's pricing rule involves the probability of having a deal with an informed trader who he thinks is going to outperform him and leave him with loss. In option market, informed trader can be the one who exactly knows what would be the asset price at option's maturity date. Although this assumption is very impractical, there will be a certain portion of traders who beat the dealer and correctly guess the asset price at maturity date. In this way, agent can simply label those traders as informed traders and update his probability parameter for the likelihood encountering such trader next time. This normally done using Bayesian inference.

To add up, the knowledge layer is responsible for aggregating the data, fitting the models to this data and extracting key information from this data. It can also query the necessary information from information layer, and set up a filter to ignore unnecessary information. Knowledge layer can be static too. For example, it may include constant parameters of the probability distributions of ZI traders. They do not change over time, and therefore knowledge layer does not query any information from underlying market at all. This would also make the information layer empty for the ZI trader.

3.3.4 Behavioural Layer

The behavioural layer of the option trading agent is the most important aspect of the whole framework because it is responsible for interacting with the mechanism. It also determines the option prices, other preferences such as quantities and portfolios, and proxy trading algorithms. Agents use different models to price options starting from zero-intelligence pricing and classic Black-Scholes pricing to new LMSR and exponential pricing rules. Besides option pricing, behavioural layer is also responsible for determining the quantity to be traded. Quantities can be randomly picked, or use some linear relationship with the price. In online auctions, behavioural layer is also responsible for running proxy trading algorithms that bargain the option price. Proxy trading algorithm creates a bid-ask spread around computed option prices the breadth of which changes according to the conditions in the market. Agents use proxy trading algorithms only in CDA environment, because it allows continuous trading.

We can classify agents based on their option pricing behaviour, listed below:

- *Zero Intelligence Traders*: Such agent disregards information about mechanism state, current state of the asset market, actions of other agents, history of the mechanism states up to time t, history of the asset market, his own inventory $I_i(t)$ and any other external factors. They obtain option valuation from the random path of asset price simulation. The knowledge layer contains the preconfigured parameters of the PDFs that are used by traders.

- *Risk-Neutral Traders*: Such agents base their decisions only on the events happening in assets market. They evaluate the risk associated with buying or selling certain option from risk-neutral perspective which could be one of risk-neutral option pricing methods such as Black-Scholes pricing, Monte-Carlo pricing or Volatility pricing.

- *Risk-Averse Traders*: Like risk-neutral traders, their decisions are based only on the events happening in assets market. However, they evaluate the risk associated with buying or selling certain option from risk-averse perspective which could be an indifference pricing method using exponential utility function.

- *Portfolio Holders*: Such traders hold an option portfolio, and price options based their current portfolio. The pricing method the agent uses can be LMSR which takes the logarithmic difference of agent's expected payoff from his current portfolio and the portfolio he will have after buying or selling specific option. This approach is described in Section 4.3.

Agents decide buying or selling option by submitting a positive or negative quantity. This involves another sublayer inside behavioural layer which is responsible for the implementation of an algorithm that picks required quantity for the agent to bid or ask. This can be a sophisticated algorithm which takes into account private characteristics of the agent such as liabilities in his current portfolio, budget limit, or some timelined strategy of entry and exit from the market. However in my simulation model, I consider following methods to choose the quantities required for the trading agent:

- *Random integer*: This method uniformly draws a random number from a range $[-q, q]$. However the sign of this quantity subject to change if a corresponding proxy trading algorithm is applied in CDA environment.
- *Linear function*: This method uses linear function with a positive slope to determine the supply, and the negative slope to determine the demand based on the ratio of agent's evaluated option price and risk-neutral price.
- *Option portfolio*: This method determines the quantity of the option needed from the content of an option portfolio assigned to the agent.

Agents use proxy trading algorithms in CDA to adapt their bid-ask spread around evaluated option price. There are 2 types of trading algorithms involved in my simulation. First is the dealer algorithms which post both bid and ask on the option to maintain open market. They change their bid-ask spread based on the events happening in CDA. Second is the trading algorithms which post either bid or ask depending on the previous events in the market. Below is the list of each proxy trading algorithm used in the simulation:

- *Garman's algorithm*: This algorithm models a dealer that is endowed with initial inventory in options and cash. It updates the bid and ask prices for the option based on the rate of order arrivals. The rate of order arrivals subject to the law of demand and supply, which means that the higher is the bid, the lower is the rate of arrival of asks for this bid. The opposite applies to setting asks. Garman's algorithm finds such bid and ask prices which would minimise the probability of trader running short in his inventories.
- *Copeland-Galai's algorithm*: This algorithm models a dealer that believes that there are informed traders in the market who can correctly guess the final prices of the asset. Based on that belief he maintains a bid-ask spread such that it minimises the loss from informed traders, and maximises the gain from noisy traders.
- *ZIP algorithm*: This algorithm models a trader that learns from the previous orders based on their success. This algorithm increases or de-

creases his bid quote taking into account the learning coefficient versus momentum parameters. The same applies to submitting asks too.

- *GD algorithm*: This algorithm models a trader that uses a Bayesian inference in determining the most successful ask or bid from the history of orders accepted and submitted.

I will describe their implementations of above described layers in Chapter 4. In below section, I will show how agents are modelled using object-oriented paradigm.

3.3.5 Conceptual Design

I present the UML class diagram of two types of traders, DA trader and CDA trader. The traders are designed according to IIKB architecture where the behavioural aspects of the trader are modelled using Strategy pattern [159]. Each behavioural sublayer implements a functionality which then passed on to the next layer. First, let us look at Figure 3.15 to illustrate the static relationship of implementations of sublayers in behavioural layer to the DA trader. DA trader compounds 2 sublayers: option pricing and quantity models. We can see that the option pricing itself is connected to the realization of the underlying market. This includes asset pricing and risk-free rate models which are used by the option pricing model to compute the option price. Asset prices and risk-free interest rates are returned in the form of a `Series` object. `Series` object is a table with time and some value columns. It can also include multiple columns for the values of interest. For example, in case of asset prices series, this involves a table with time and the asset price for given time. Based on the simulation of the underlying market, option pricing model computes the option price and returns this value to the trader. The trader then inputs the value to the quantity model which determines whether to sell or buy the option, and in which quantities. These two pieces of information are assembled into an order which is submitted to a direct DA.

It can be seen that LMSR option pricing model involves the inventory to price options. Some other option pricing methods also include private parameters such as risk-averseness, liquidity, etc that should be initialized before use. I will talk about these parameters in detail in Chapter 4 while discussing the implementation of these methods.

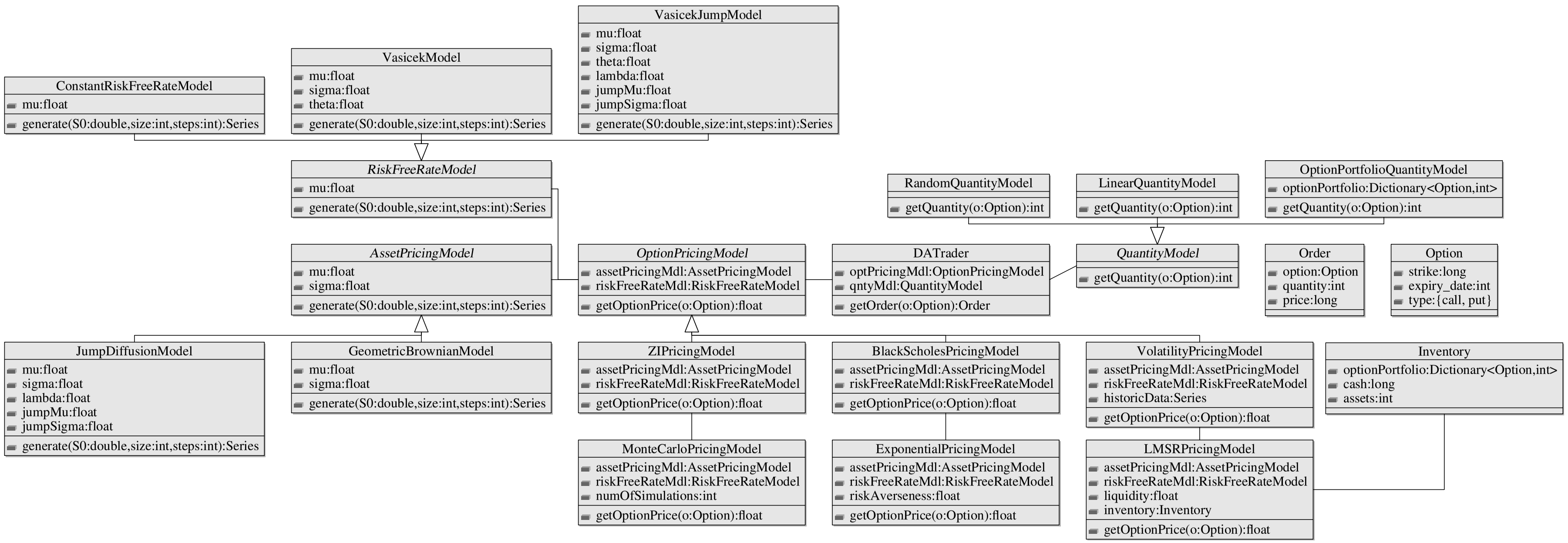


Figure 3.15: UML class diagram of direct DA trader

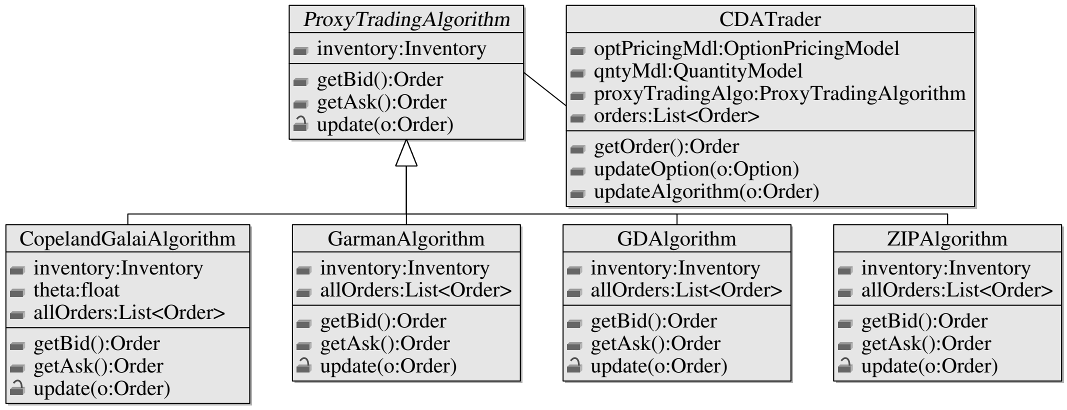


Figure 3.16: UML class diagram of CDA trader

The CDA trader is similar to DA trader, but it also implements the additional proxy trading algorithm behavioural sublayer. Figure 3.16 shows UML class diagram of the CDA trader where all the related classes shared with DA trader are omitted, except the new sublayer for the implementations of proxy trading algorithms. We can see that each implementation of the proxy trading algorithm can have different private parameters to generate bid and ask prices around evaluated option price. CDA trader has `updateOption` method which is called every trading day to use the assigned option pricing model that returns the evaluated option price. Then CDA trader calls `updateAlgorithm` method to update his current bid and ask with the new option price and with the latest information from the CDA. I will explain the details of the implementation of these algorithms in Chapter 4.

3.4 Double Auctions

In this section, I consider simulation models for 2 types of DAs. Namely, I explain the simulation of direct DA and online DA. I do not consider the simulation of combinatorial exchange in this research, however I do provide my remarks on its design in Chapter 7. Each simulation structurally differs from the other one, and involves different types of trading agents and market clearing mechanisms. For example, in direct DA, every new trading day is considered as the new instance of simulation, so everything gets reinitialised. In online DA, however, agents are initialised once, then they continuously make decisions on what to order based on day-to-day changes in the market.

3.4.1 Direct Double Auction Simulation

I construct direct DA for multi-unit bids and asks as part of simulation model. Simulating direct DA involves one round of trades per day, and this process repeats until the option expires. Every time the traders are repopulated in the auction with different configurations and then they submit their truthful orders to the auction. Direct DA uses surplus maximisation in order to allocate options to traders, and it computes DSIC prices to clear the market.

The structure of orders submitted may vary on the information included inside them. It may not necessarily require the traders to submit their price quotes and quantities for options, but instead require the submission of other important parameters that are key in pricing options. For example, the structure of the order may also include the asset price forecasts S_T at time T if the assumption is that all traders are ZI traders, and they compute the value of option based on their private forecast. If traders use Black-Scholes formula, then the parameter which is key for pricing the option is the implied volatility. Due to *revelation principle*, if agents can reveal their true option valuations to the mechanism, they can also reveal the methods they applied to obtain such valuation. In other words, once agents tell the mechanism what type of option pricing method they used, and then reveal their estimated parameters to the mechanism, the mechanism can compute the option prices on their behalf. I will show how this can be done for ZI traders, as their option pricing method is deterministic given the forecast of future asset price. Mechanism can collect these forecasts, find clearing forecasts and compute option prices based on them, and charge traders for this value. It is described in detail in Section 5.3.3. But before that, let me explain how the general simulation flow for direct DA is constructed.

The general flow of the direct DA simulation is described in following Figure 3.17. It starts with opening the trading day, and then initialises option chain to be traded. It also populates the market with option traders and updates the asset price for the given day. Once traders see the asset price and compute their own private valuations of the option, mechanism collects the orders from traders indicating auction specific data in it. Generally I simulate only call option, as the price of the put option can be directly computed from given call option using put-call parity relationship. The order also includes quantity and quoted price. Negative quantity stands for ask, and positive quantity stands for bid. But as it was mentioned earlier, depending on the revelation level,

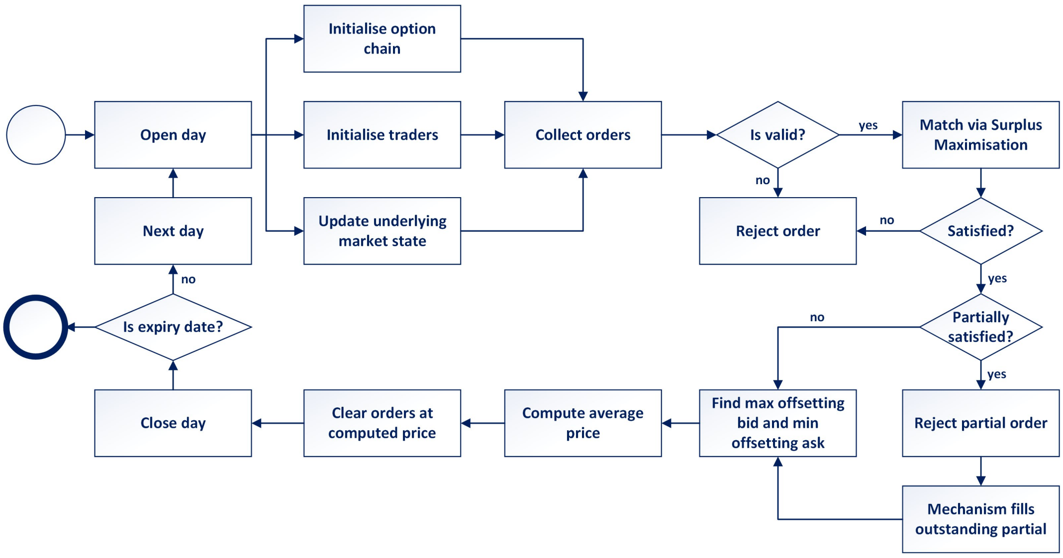


Figure 3.17: Direct auction simulation flowchart

the orders can also contain additional information such as asset price forecasts or volatility parameter. Collected orders are then checked for consistency, so that quoted prices do not exceed the upper and lower bounds of the option price. Inconsistent orders are rejected, and the remaining ones are used to find surplus maximising allocation. After solving the surplus maximisation problem, there will be at most one order with a partially satisfied quantity. If there is such order, it is rejected, and the new outstanding quantities are filled by the mechanism. This is possible because options are not commodities, and can be created at an arbitrary amounts by the mechanism. As it was mentioned before mechanism is not budget-balanced, and this can result in additional liabilities for the mechanism. I use rejected maximum bid and minimum ask to compute the clearing price of the option. Once the option price is determined, the mechanism closes the trading day and switches to the next day unless it is the expiration day of the option.

3.4.2 Online Double Auction Simulation

In online DAs, or in my case CDAs, there is no DSIC mechanism because of the uncertainty in future information or the subsequent actions of other traders. The markets are vast and agents do not have the computational capacity to reflect the complexity of each interaction and its corresponding outcome in one timeline. There is lack of information about the market participants or the events that may happen in future. There are two major uncertainties with

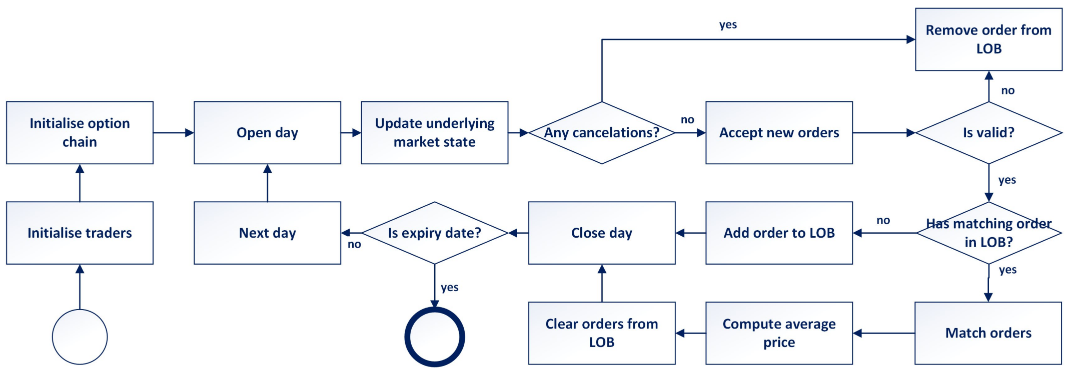


Figure 3.18: Online auction simulation flowchart

online DA: a) if there is any order expected to arrive to the market; b) what information this order is going to bring to the market. Due to these uncertainties, agents cannot have any dominant strategy which would guarantee the maximised payoff. This has been also mentioned by Gode and Sunder [72] through considering CDA as a complex game. However there are online mechanisms that offer BIC prices for traders making them on average better off if they trade truthfully [131, 61, 132]. However I shall limit my research to the simulation of only canonical CDAs.

The simulation flow of online DA significantly differs from the simulation of a direct DA. In this simulation, mechanism first instantiates trading agents and option chain. Adaptive traders use proxy trading strategies to determine what bid or ask they should post based on their private belief of the option value. Because of this continuous nature of the traders in online auction, they are not reinitialised every trading day. Once the trading day is open, the asset price for this day is updated and the previously submitted orders are cancelled. This is because with new information of the asset price, the option value is determined to change, and the previous orders are considered obsolete. Then the traders are randomly requested to submit their orders. Likewise to direct DA, orders are checked for consistency, and rejected if their quotes are found violating the predefined option bounds. New orders are matched with the existing orders in the LOB, and if there is a match, these orders are cleared at the middle price. If the match is not found for the newly submitted order, it is entered into LOB. Once all traders submit their orders and get cleared if they have matching ones, the trading day closes. If the option is not expiring, mechanism proceeds to the next trading day. This simulation flow is illustrated below in Figure 3.18.

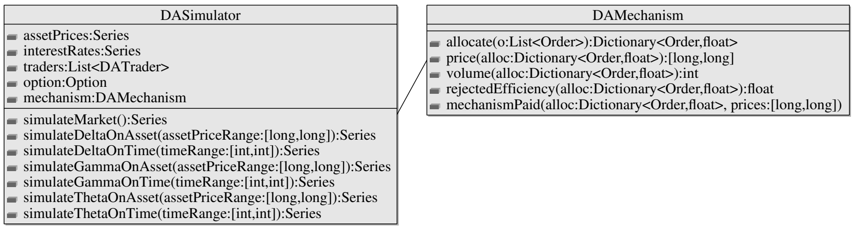


Figure 3.19: UML class diagram of DA mechanism

3.4.3 Conceptual Design

In this section, I describe the UML class diagrams for both mechanisms. I have described the overall simulation flow of both mechanisms in Figures 3.17 and 3.18. These simulations are implemented as `simulateMarket()` function in both mechanisms given below in Figures 3.19 and 3.19. Let us observe the other methods and attributes involved in both mechanisms.

In DA, `DAMechanism` has `allocate()` and `price()` methods which implement the allocation rule and pricing rule of the direct DA proposed in Chapter 5. Given the list of valid orders from trading agents, the mechanism finds the optimal allocation for these orders. Then it returns a dictionary for each order indicating if it has been chosen, and if yes, which portion of its requested quantity is satisfied. It will be shown in Chapter 5 that there is at most one partially satisfied order. For computed allocation, mechanism returns clearing bid and ask prices that are DSIC. Also mechanism gives how much of the trades are covered by the mechanism itself given the allocation and prices inputted into `mechanismPaid()` method. This is because the partially satisfied order is rejected, and exposed matched orders are covered by the mechanism. Mechanism returns this information in its `rejectedEfficiency()` method. Also it can compute the volume of trades in terms of options traded in method `volume()`.

For each mechanism, there is a mechanism, there is mechanism simulator which takes option to be traded, list of agents, and the mechanism itself. Besides implementing `simulateMarket()` function, it also implements the simulation of Greeks covered in Section2.1.7. This requires the valid range of a control parameter to be set, so it can be linearly increased to obtain the sensitivity of the option price to this change. In CDA mechanism, the transactions happen continuously until all of the agents submit their orders. Therefore CDA mechanism should maintain an order book where it stores the outstanding bids and asks. This mechanism has a method `acceptOrder()` which accepts new

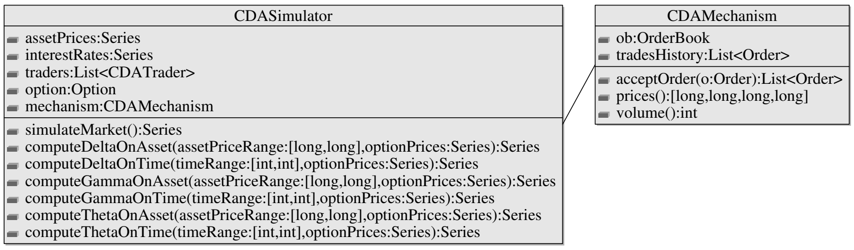


Figure 3.20: UML class diagram of CDA mechanism

order, and tries to match to the existing ones. If the matching order is found, it returns the list of matched orders. If there is no order matched, it adds the order into the order book. Mechanism also keeps the log of the trades happened so far in `tradesHistory` property. `prices()` returns the opening, closing, highest and lowest prices for the options traded so far. `CDASimulator` implements the simulation of the market according to the flow given in Figure 3.18. However unlike DA, the Greeks are not simulated, because CDA traders use proxy trading algorithms that learn from the orders coming to the market, so the linear growth of an underlying asset may result traders aligning their bids and asks for growth, hence bloating the option prices that are used to measure the sensitivity. Therefore, instead I use closing prices of the options to solve the analytical formulas for Greeks. Therefore the simulator's Greeks methods also require the series of option prices.

3.5 Relevance to Other Models

There have been other models proposed involving multi-agent systems simulating the option market. As regards for the market mechanisms used in my simulation model, namely both direct and online DAs, I gave their comparison with the existing implementations in literature in Section 2.2.2. In this section, I will compare my simulation model for option pricing as a whole with two other relevant researches (i.e. Streltchenko [95, 156, 155] and Espinosa [56, 57]) that propose similar mechanisms facilitating the trade of options. I have chosen these two research because they are comparatively recent, present non-trivial approaches and bear some similarities with my simulation model. Of course, this is not the exhaustive list of researches that propose multi-agent simulation of derivatives market or financial markets as a whole. There are many other researches accomplished in this field and I reviewed some of them in Section 2.2. Some of the most relevant works include Ecca's model of an artificial stock option market [51], Saatcioglu's design of a financial portal [143],

Streltchenko *et al.* proposed a multi-agent simulation model with brokers as price-setters and investors as price-takers that collectively rebalance their initially endowed portfolio with financial derivatives to minimise the risk they are exposed to. The second relevant research in this field is accomplished by Espinosa who developed a market mechanism that facilitated the trade of assets and options among multiple agents, but not serve as a platform for price formation. Unlike Streltchenko's and my research, the main focus of Espinosa's research is not directed at determining the price of an option contract, as he uses the Black-Scholes formula for this purpose. The main objective of the research was to build a market mechanism for agents who could trade options for effectively managing the risk in obtaining necessary resources. My simulation model bears both similarities and differences with these two researches. I will describe these aspects in detail for all three components (i.e. underlying market, agents and option market) of my simulation model.

3.5.1 Underlying Market

As it was mentioned previously, in my simulation model the underlying market is a stochastic model of an asset price and risk-free rates that are calibrated to historic data. These prices are not the result of simulated transactions between agents in a market mechanism. Similarly, Streltchenko describes the underlying market as a random walk process which runs on its own without agents being involved in the price formation. In fact, in Streltchenko's model there is a specific agent `StockBroker` who provides the listings of the day's price quotes for different assets to trading agents, so they can buy or sell any of listed assets to `StockBroker` in unlimited quantities. Streltchenko assumes that the price the assets are bought and sold is the same, and there is no other transaction cost involved between traders and `StockBroker`. Similarly, traders can borrow money or short sell assets at the same risk-free rate. In my model, these assumptions also hold, but the agents do not trade any of underlying asset or risk-free asset with a special broker or mechanism. However, it is assumed that they could do that, so they use this information in evaluating the risk associated with buying or selling option. Trading agent's are thought have unlimited amount of cash and stock, except if it is an inventory-based trader. In Streltchenko's model, investors are the traders with budget-constrains and limited portfolio, while brokers have access to unlimited funds and stock.

Although Streltchenko's model allows multiple assets to be posted into the market, the experimental setup uses only one type of underlying asset along with a risk-free investment. This is also true in my simulation model, where multiplicity of different underlying assets is permissible, but this would also involve running multiple option markets for each option on different underlying. Streltchenko did not run multi-period simulation of an asset price (only 2-, 3-step discrete random walk), although her model assumes that underlying market can be discretely simulated for any given time horizon. In my model, I use continuous and discontinuous stochastic models which are described analytically in terms of few parameters that could be controlled to simulate different scenarios in the underlying asset market. Continuous and discontinuous mean-reverting processes are used to simulate the risk-free interest rates. These stochastic models can be simulated numerically as well as they can be calibrated to mimic the possible price trajectory of particular asset or risk-free interest rate.

In Espinosa's model, the underlying market is described as Palmer's simple economic exchange [128] with one type of good which does not bear any dividends. The traded good is indivisible and can be freely traded in unlimited quantities. This is also true in my simulation model where the underlying asset is considered indivisible, non-dividend yielding and can be traded in unlimited quantities. However, in Espinosa's model, the source of the underlying asset prices is considered exogenous to the model, and should be provided as a historic data to run the simulation. In other words, the asset prices change according to a pre-configured scenario, not as a result of traders' transactions. Espinosa's model describes the discount rate (i.e. reciprocal of risk-free interest rate) as a constant drawn from $[0, 1]$. In my model, this is described as either constant obtained from historic mean of T-Bills, or a mean-reverting stochastic process. Table 3.7 summarises the important characteristics of underlying market in Streltchenko's, Espinosa's and my models.

3.5.2 Trading Agents

In my simulation model, I conceptually broke down agents into 4 layers (i.e. Information, Inventory, Knowledge and Behavioural layers) to distinguish the tasks or data each needs to work with. As it was mentioned before, similar

Properties	**Proposed Model**	**Streltchenko's Model** [95]	**Espinosa's Model** [56]
Asset types	One	Many	One
Risk-free investments	One	One	One
Asset pricing model	GBM, Jump-Diffusion	Discrete random walk	Exogenous
Risk-free rate model	Constant, Vasicek, Vasicek-Jump	Constant	Constant
Asset price source	Mechanism	StockBroker	Historic data
Risk-free rate source	Mechanism	StockBroker	Historic data
Who trades assets	None	StockBroker	Asset traders, Option traders
Who lends cash or asset	None	StockBroker	None
Total assets	Infinite	Infinite	Finite
Total cash	Infinite	Infinite	Finite
Underlying market frictions	No	No	No
Asset divisibility	Indivisible	Indivisible	Indivisible
Asset yields dividends	No	Possible	No
Agents can affect asset price	No	No	No
Agents can affect risk-free rate	No	No	No

Table 3.7: Characteristics of Underlying Market in Streltchenko's, Espinosa's and Proposed Models

architecture has been used to design agents in Vytelingum's model [161]. However, in this comparison, I review how agents were designed for an option market proposed by Streltchenko and Espinosa. In Streltchenko, there are 3 types of agents: `StockBroker`, `DerivativeBroker` and `Investor`. `StockBroker` is responsible for trading assets to other agents in unlimited quantities. He is also the sole cash and stock lender. There is only one instance of such agent in the simulation. In my model, the trading agents do not involve in trades with assets or cash, although the mechanism provides information about them. So the mechanism plays the role of `StockBroker` when it comes to reporting the new information about asset prices or risk-free interest rates, but not used for trading them.

In Streltchenko's model, `DerivativeBroker` is an agent with an infinite amount of cash and derivatives, and an agent type who can post limit orders. Hence multiple `DerivativeBrokers` can set the price in the market. They either represent `Investor` or determine their own bid-ask spread for options according to the arbitrage-free evaluation of risk. This is described as a LP which minimizes the difference between the selected ask price and the future payments associated with his current liability. Similarly, the bid price for the option has to maximise the expected cash flow as opposed to the price paid. In other words, `DerivativeBroker` has to select optimal set of bids or asks which would minimize his expected loss and maximize his expected gain. In my simulation mode, every trading agent can buy or sell option contract in unlimited amounts if the quantity is not capped by the mechanism. It is also assumed that they have access to unlimited funds, except inventory-based trader. Also, in my simulation model, the information-based proxy trading algorithm uses similar approach as described in Streltchenko's `DerivativeBroker`. It also minimises the potential loss from informed traders, and maximises the potential gain from noisy traders. However it does this, by determining bid-ask spread around pre-evaluated option price.

Streltchenko also models another set of agents - `Investors` as traders with initial endowment and liabilities. They represent companies such as airline, oil refineries, etc who enter market with a portfolio of assets and liabilities, and hedge their exposed risks in this market. `Investors` have limited budget and they submit their preferences through `DerivativeBrokers`. In my simulation model, traders can directly submit their orders to the market. Also portfolio holding aspect of the traders are best presented in LMSR method for pricing options and inventory-based proxy trading algorithm. In first case, traders are assumed to own an option portfolio, and they have to price the option from the perspective of how this new option contributes to their current payoff structure. In the second case, the inventory-based proxy trading algorithm is given a portfolio of options and cash, along with pre-computed risk-neutral price of the option. Hence he has to decide what bid-ask spread he needs to hold around the given option price, in order not to run short any of his portfolio inventories. Therefore, agent can shift his bid and ask prices up and down to stimulate supply and demand in the market. Also `Investor` has his own discrete projections on the distribution of asset prices in future, also referred as scenario tree. `Investor` finds the minimized expected loss based on this pre-set distribution. Similarly in my model, risk-neutral agents use Monte-Carlo simulation to determine such distribution of future asset prices, and based on this information compute the expected payoff of the option.

In Esponosa's research 6 types of agents are used, where 2 of them were asset trading agents, and the remaining 4 are specialized in trading both asset and options. Agents also maintain inventories on cash, asset and options. Agents can buy or sell assets, and write or hold multiple types of options with different strikes and expiration dates. In my simulation model, agents can have multiple options in their inventory, but they submit their orders to a mechanism which is specialized to only single type of option, unless it is a combinatorial exchange (it is not included in my simulation model, but the framework is described in Chapter 7). One key distinction between the agents in my simulation model and Espinosa's model, that in my model, agents' have heterogeneous beliefs about the option price, while in Esponosa's case, all option trading agents use Black-Scholes pricing model. Thus in Espinosa's model, agents do not contribute to the formation of asset or option prices, but only react to the external changes and employ some trading strategies to manage their risk associated with market fluctuations.

I briefly describe how option trading agents are implemented in Esponosa's model, and draw similarities with the agents proposed in my model. Espinosa's agents use 2 forecasting models: Simple Moving Average (SMA) and α-forecasting to predict the future asset prices upon which the agents made decisions what to buy or sell. Asset trading agents employed 2 strategies: one is the random buy or sell with quantity equal to one; the other is sell if asset price is anticipated to fall, and buy if asset price is to rise. In designing option trading agents, Espinosa uses two types of perceived risk: ρ_L measures the probability of making loss for given action, and ρ_G measures the probability of making gain for given action. Agents use one of 4 trading strategies to pick which action to execute. `OTRnd` strategy randomly selects the next action of the agent from list of K actions such as buy/sell asset, hold/write any type of option listed in the market or pass. In my model, traders only have 2 actions: hold or write single type of option, and their actions are determined by the quantity model they use. Random quantity model uniformly draws an integer from some $[-Q, Q]$ range where Q is the maximum quantity allowed. In linear quantity model, agent buys if the estimated price of the option is lower than the risk-neutral price, and vice versa. In Espinosa, `OTMinR` strategy selects an action which minimises ρ_L, while `OTMaxW` strategy selects an action which maximises ρ_G. Espinosa also proposed `OTMix` strategy where agent uses `OTMinR` if the asset price is predicted to fall, and `OTMaxW` if the asset price is predicted to rise. In my model, inventory- or information-base dealers submit option prices that minimise the loss, and maximise the gain depending on their own perception of the market.

I summarised the similarities and the differences of trading agents in my model with Streltchenko's and Espinosa's models in Table 3.8.

3.5.3 Option Market

In this section, I describe the similarities and the differences of the actual mechanisms used in my simulation model from the ones defined in Streltchenko's and Espinosa's researches. The mechanisms are used as a platform for agents to trade options, so each mechanism implements certain protocol for agents which define what actions should be solicited and in which order. Mechanism also determines the type of the orders it accepts and the rules for matching and clearing them. In Streltchenko, the mechanism assumes the interconnected network of agents who can interact freely to exchange mutual offers, and accept/reject them if they comply or not comply with their internal expectations. In this network, all agents have connection with `StockBroker`, so they can trade assets with him. Also all `DerivativeBrokers` in the market are connected with each other, while the `Investors` are only connected to subset of `DerivativeBrokers`, and are not connected to other `Investors`. All the transactions between agents are visible to concerned parties only. According

[1] See Table 4.1 for reference.

[2] Symbolic, assets account needed for delta-hedging strategy used inside exponential option pricing method.

Properties	**Proposed Model**	**Streltchenko's Model** [95]	**Espinosa's Model** [56]
Option pricing methods	`ZI, BS, MON, VOL, LMSR, EXP`[1]	Binomial	Black-Scholes
Choosing option quantity	`RND, LIN`[1]	Fixed Units	Single Unit
Trading algorithms/ strategies	`GAR, COP, ZIP, GD`[1]	Loss minimisation, portfolio rebalancing	`OTRnd, OTMinR, OTMaxW, OTMix`
Agent's actions	hold/write option	buy/sell asset, hold/write option	buy/sell asset, hold/write option, pass
Agent's knowledge	Knowledge + Information layers	historic asset prices, risk-free interest rates	historic asset prices, risk-free interest rate, standard deviation and mean of asset price
Agent's inventory	cash, assets[2], options	cash, assets, options	cash, assets, options
Agents are option price-setters	Yes	Yes	No
Agents trade assets	No	Yes	Yes
Agents trade multiple options	No	Yes	Yes
Budget limit	Only `GAR` trader	Only `Investor`	All

Table 3.8: Characteristics of Trading Agents in Streltchenko's, Espinosa's and Proposed Models

to Streltchenko, this architecture separates transactional issues from the ones involved in decision making. The simulation runs in discrete steps, so in each step the agents interact with each other and exchange orders.

Streltchenko's model is fundamentally different from my model in terms of the mechanisms used, because I use two market mechanisms direct DAs and CDAs as a single platform for agents to post and clear their orders, where in Steltchenko the flow of orders are not centralised to one place. Although her model is multi-period, the transactions of other agents are still invisible to agents at any state of the simulation. In my CDA model, the past transactions are visible to all traders, and they directly affect the future orders of the agents through a proxy trading algorithm they use. However, in direct DA, agents submit their corresponding valuations of options in sealed bids and asks, and obtain a competitive option price in a single round. Unlike Streltchenko's model, direct DA produces two prices: one for all buyers, and the other is for sellers. In Streltchenko, the prices can be unique per each transaction. Moreover, my mechanism is incentive compatible, efficient and individual rational, while the efficiency of Streltchenko's model strongly depends on the structure of the network between agents. The flow of the simulation in Steltchenko's model is similar to mine. First agents are instantiated with a portfolio and some budget-restrictions if applicable, then they submit orders based on their internal scenario tree on future asset prices. At the end of the simulation, the derivatives are cleared and the obtained wealth is analysed.

Espinosa's model of a marketplace is also different from the mechanisms used in my model. First of all, Espinosa does not require the agents to submit prices, as the prices are assumed to be known both for assets and the option. Therefore it does not matter which order is matched to which as long as the quantities offered and sought are balanced. In other words, Espinosa randomly matches the orders from different agents until there is no either outstanding ask (in overdemand) or outstanding bid (in oversupply) left in the market. Espinosa's model creates option templates at each step of the simulation with different type (i.e. call or put), strike, expiration date, but on the same underlying asset, so the traders can write or hold options complying with these templates. In my model, each mechanism deals with single type of option (i.e. call option), with same strike and expiry date. The simulation flow is similar to my model. First, the mechanism publishes the asset prices and all outstanding options are cleared. Then market publishes the new set of option templates, and agents start posting their orders. At the end of the trading day, all orders are clears as described above, and mechanism goes to the next trading day. When simulation ends, the wealth of agents and the overall performance of the market are analysed. Because mechanism does not produce aggregated option prices, neither their sensitivity to certain factors, nor their comparison to risk-neutral valuation is discussed. This part is analysed in more detail in my model.

I summarised the important characteristics of market mechanisms used in both researches along with mine in Table 3.9

Properties	**Proposed Model**	**Streltchenko's Model** [95]	**Espinosa's Model** [56]
Mechanisms	Direct DA, CDA	Bilateral trading	Random matching
Order types	Limit order	Market and Limit orders	Market order
Order size per item	Multi-unit	Multi-unit	Single-/Multi-unit
Atomicity of orders	Yes	Yes	No
Traded items	One call option type per mechanism	Assets, multiple option types	Assets, multiple option types
Simulation time	Option's lifespan (365 steps)	2-3 steps	10^9 steps
Option exercise time	Once, at the end	Each step	Each step
Number of agents	100	4 sampled to represent the population	10^9
Key data collected	Option prices, Greeks, volume, allocation, budget deficit, agent's inventory	Option prices	Volumes, agent's profit

Table 3.9: Characteristics of Option Market Mechanisms in Streltchenko's, Espinosa's and Proposed Models

3.6 Summary and Contribution

In this chapter, I covered 3 main aspects of my simulation model starting from the simulation of an underlying market to the simulation of direct and online

DAs. In the underlying market, I determined the corresponding parameters of the historic NASDAQ records for GBM and jump-diffusion models and simulated several instances of them. I also simulated the risk-free interest rates as mean-reverting processes using Vasicek and Vasicek-Jump models. I found the corresponding parameters for US 2-year T-Bill returns, which is normally used as a risk-free investment in practice. I also considered the high-level architecture of the trading agents extending IKB framework with an additional inventory layer and justified its need in option trading. I separated the related concepts in option trading into 4 layers. I also proposed the conceptual design of the option traders in UML class diagram following the proposed IIKB framework. I also presented two simulation models for direct and online DAs and described their simulation flows. Also I provided the conceptual design of the mechanisms in UML class diagram and described their main attributes and methods. Finally, I have compared my simulation model with Streltchenko's and Esponosa's researches that propose option trading simulation platform. I highlighted the similarities and the differences of my simulation model with existing models in terms of underlying market, traders and mechanisms implemented.

Chapter 4

Option Trading Agents

4.1 Introduction

In this chapter, we discuss about the design of trading agents, namely, the components involved in forming their trading behaviour. I explain how agents acquire necessary knowledge from the available information in the market. Then we look into three main modules of behavioural layer of the option trading agents. Namely, option pricing module, quantity module and proxy trading algorithms module. I fit these modules into previously defined IIKB framework, and highlight their interactions between different layers of IIKB.

Indeed, trading agents are the main participants of the market who directly or indirectly influence the prices of options, and therefore it is important to model every aspect of their trading behaviour. One of the most important aspects of designing a trading agent is the rationality of agents in making buying/selling decisions. Designing completely rational and homogeneous agents would result in, so called, *'no-trade equilibrium'* outcome described by Milgrom and Stokey[116]. This theorem states that there must not be any trade in the market if all of the following assumptions are true:

1. *efficient market hypothesis* - markets are always in equilibrium state.
2. *rationality assumption* - there are no noise or non-rational traders
3. *full information assumption* - all traders have full access to all market data.

Milgrom and Stokey also state that even if there is a private information about the future of an asset price and the informed trader wants to realise profit from that, it would be a rational decision for an uninformed trader not to participate in this deal, because ultimately he is going to lose his money because there is a rational reason behind the informed trader's decision. Looking at this from rational trader's perspective, the informed trader's order must bring new information to the market and may cause corresponding changes in price quotes. This change in price is not caused by particular trade, but from the assumption of the rationality of other traders. Rational uninformed trader can observe the market and assume that the newly arrived order is the result of a fundamental analysis and an informed decision, so he can simply adjust his quotes to the latest order arrived and still maintain 'no-trade equilibrium' because the newly arrived order already bears the all available information about given stock.

However in this research, I drop some of the assumptions stated above in order to maintain the heterogeneity of prices and hence facilitate the trade. For example, I assume that market was not in an equilibrium state in the first place, and it must converge to an equilibrium price given that agents have full access to all market information, and they act in their best interest. This assumption is critical in the simulation of direct DAs, because otherwise the realised outcome must not result in trade. In online DAs, I consider agents have bounded rationality, so they make their decisions based on limited information. For example, they do not take into consideration the actions of other traders or any other exogenous factors. Also, there is an uncertainty about the future events and agents need to act based on the currently available information which could be the risk-neutral behaviour of the agent.

Further in this chapter, I discuss the details of the implementation of trading agents. Figure 4.1 illustrates the relevant components involved in each layer of IIKB framework. Behavioural layer involves functional modules such as option pricing, choosing quantities and proxy trading algorithms. These modules consist of various implementation of respective functions. In knowledge layer, I present statistical and other relevant parameters obtained from analysing the information available. Information layer contains data coming from underlying market and the simulated mechanism. Finally, in inventory layer, traders keep their cash, underlying asset and options accounts. I highlight how these inner components are interrelated with each other while discussing their particular implementations.

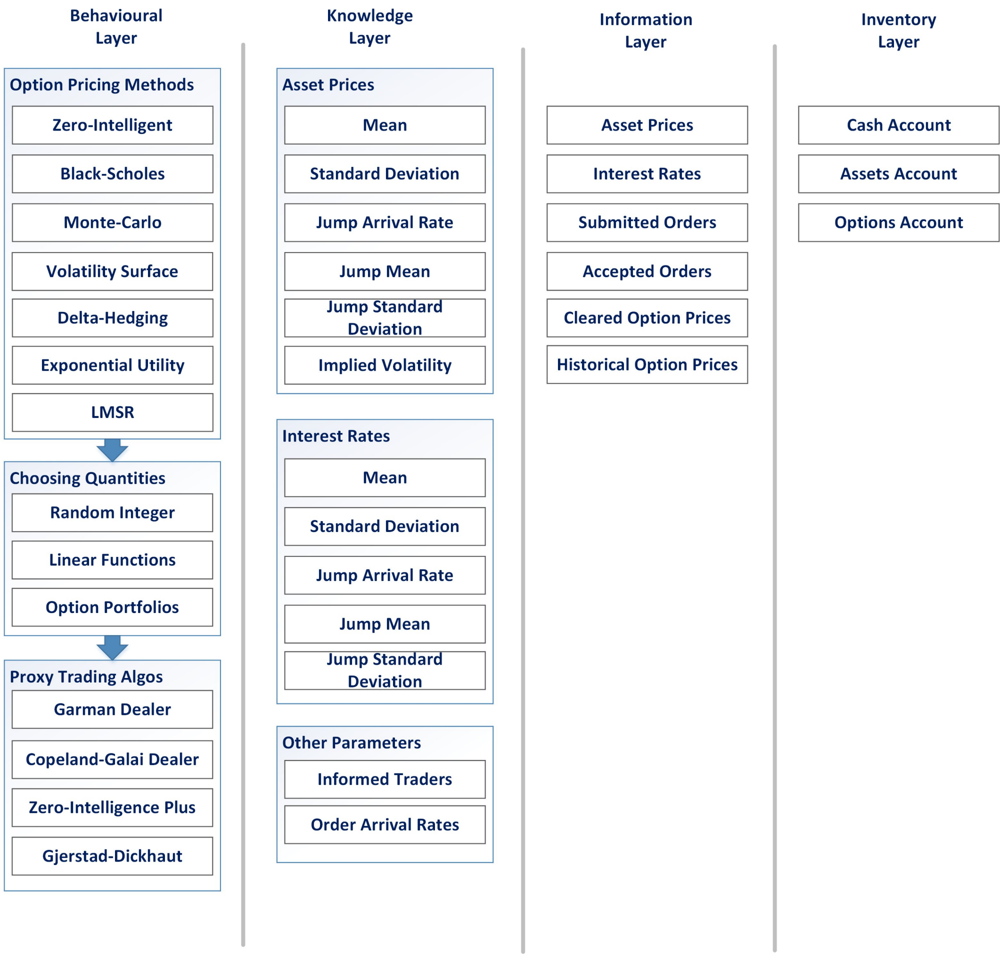


Figure 4.1: Overview of trading agent components

For easy referencing of traders used in my mechanisms, I devised a naming nomenclature for the traders based on their respective trading behaviour. I use three aspects of the behavioural layer to name them: a) option pricing method; b) choosing the quantity; and c) proxy trading algorithm to name the agent. For example, if the agent uses ZI option pricing and ZIP proxy trading algorithm to submit bids and asks to the market, and picks random integer as its desired quantity, I name this agent as `ZI-RND-ZIP`. Table 4.1 summarises the keywords that are used to define the names of the traders.

Option Pricing	**Choosing Quantity**	**Proxy Trading Algorithm**
ZI - Zero-Intelligence BS - Black-Scholes MC - Monte Carlo VOL - Volatility Surface EXP - Exponential Utility LMSR - Log Market Scoring Rule	RND-Random Integer LIN- Linear Quantity PORT - Option Portfolio	ZIP - Zero-Intel. Plus GD - Gjerstad-Dickhaut GAR - Garman's Model COP - Copeland-Galai Model

Table 4.1: Nomenclature for naming agents

I organised this chapter in following way. Section 4.2 describes how agents can learn implicit information about the market using calibration and Bayesian learning methods to obtain necessary components of knowledge layer. I dropped the description of finding the basic statistical parameters such as mean and standard deviation. Section 4.3 illustrates different methods such as zero-intelligence, risk-neutral and risk-averse to price options intrinsically. In Section 4.4, we discuss how agents choose the quantity to trade. Section 4.5 then provides four algorithms for adaptive traders that bid and ask around computed option prices. Section 4.6 summarises the key contributions of the chapter.

4.2 Learning About Markets

In this section, I review how agents can learn about the parameters of their models from the market data. They use optimisation routines and Bayesian inference techniques to gain useful knowledge about the markets. All of these tools are used in agent's knowledge layer in proposed IIKB model. Agent

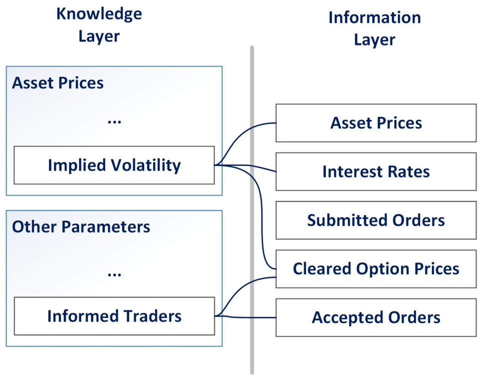


Figure 4.2: Learning About Volatility and Informed Traders

can harness the knowledge to generate bids and asks on various options. For example, agent can find unknown volatility parameter of Black-Scholes by solving the equation for given market price of the option. For complex models which require more information about the global parameters of the market such as the population of informed traders in the market, agents can use Bayesian learning technique.

Figure 4.2 illustrates how different components of the trading agent are related to gathering knowledge about implied volatility and the number of informed traders. These components are taken from the overall architecture presented in Figure 4.1. We can see that in order to compute the implied volatility, we need 3 pieces of information: asset prices, option prices and interest rates. In order to learn about the informed traders in the market, we need to know about accepted orders and the option prices. I shall explain each case in detail in below sections.

4.2.1 Volatility Surface

Pricing options is based on an assumption that the evolution of the underlying asset follows some stochastic process and the value of the option directly depends on how this stochastic process behaves as the time progresses. Hence, we need to be able to simulate such stochastic process in order to observe its behaviour and draw corresponding conclusions on the option price. The simulation routine might require two sets of parameters as input, which are a) contractual/market parameters and b) model parameters. Contractual/market parameters represent the certain aspects of the option such as strike price, maturity date or the market such as risk-free interest rate, the asset price volatility etc. Model parameters are subjective to agent's inner behavioural model that is used for pricing options.

I show how agents can find out one of the implicit parameters of the asset market - implied volatility. It can be found by modelling the asset price as a stochastic process, and then calibrating the value of the volatility parameter to fit the market data. Calibration is the process of evaluating a parameters set such that model prices comply with the market prices with minimum discrepancy. In its essence, it is an optimisation problem with an objective to minimise the distance between factual market price with the model price. The model used by an agent typically has a set of parameters $\boldsymbol{\theta}$ which can be tuned to find an optimum fit to the current market state.

If the number of parameters is greater than the number of data points that model has to calibrate to, this is called an *over-parametrised* calibration model. This can be a case when there are few illiquid orders or no order at all in the market for the given option, and agent has more number of parameters to adjust to his model. This would result in an inconclusive outcome which cannot have an optimal solution because of an unbounded parameters. In this situation, agent either needs to somehow fill in the missing data points and construct an appropriate optimisation problem or make arbitrary assumptions on certain parameters of the model. On the contrary side of the problem is the *under-parametrised* calibration models. This would involve much factual information available for a relatively simple model with few parameters. The typical example for this case can be Black-Scholes formula (2.28) where there is only one free parameter, implied volatility σ that is unknown and should be adjusted to the market. Volatility cannot be determined as a single numeric value, because solving Black-Scholes for σ on options with different strikes and maturity dates would result in different volatilities. Implied volatility is obtained through numerically solving Black-Scholes formula for unknown σ. So in other words, implied volatilities of options with similar parameters not matching one single value suggest the fact that the Black-Scholes model with only one free parameter is clearly under-parametrised.

In the rest of this section, I explain how I can use different techniques to obtain more information about the volatility surface of the market which is a 3D graph indicating the respective implied volatility per strike and maturity date. Volatility surface is a critical instrument for an agent who is willing to price his options in a risk-neutral way. Therefore it constitutes the major

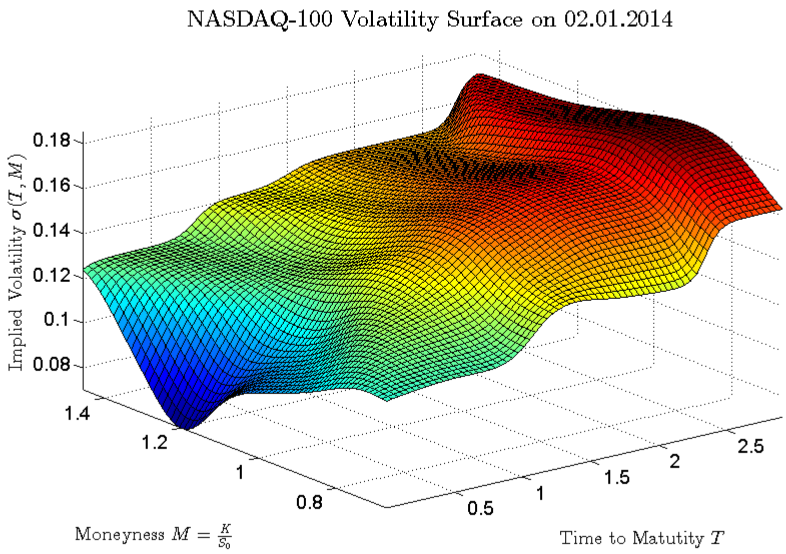


Figure 4.3: NASDAQ-100 Volatility Surface on January 2, 2014

part of agent's knowledge base, because agents will base their private option valuations on their estimated volatility surfaces.

Implied volatility is normally described as a function of strike K and time t, or most of the time with more relevant arguments such as moneyness $M = K/S_0$ and time to maturity normalised to years T. We can obtain the implied volatility for given quotes from NASDAQ-100 on January 2, 2014 by solving Black-Scholes for unknown σ and plot a surface graph to see how it changes for each parameter. Figure 4.3 shows the plot with a risk-free interest rate set to $r = 18\%$ annually and the opening NASDAQ-100 index $S_0 = \$3575.60$ on January 2, 2014.

From Figure 4.3 shows that the implied volatility is not a constant. So I have to model a volatility function $\sigma(K/S_0, T)$ which can be further used to evaluate the price of an option of any moneyness. We can find implied volatility function $\sigma(K/S_0, T)$ by numerically solving following equation (4.1) using Newton's method, or MATLAB function `fzero`:

$$C_{mkt}(S_0, K, T) = C_{BS}(S_0, K, T, r, \sigma(K/S_0, T)) \tag{4.1}$$

where $C_{mkt}(S_0, K, T)$ is the option's factual price, $C_{BS}(S_0, K, T, r, \sigma(K/S_0, T))$ is Black-Scholes price of the option.

Agent has to find all the values of $\sigma(K\S_0, T)$ which correspond to the market data. However the strike prices and maturity dates are not continuously defined in the market, and there are significant intervals between different strikes and maturity dates of options listed in option chain. Normally, the option chain defines constant intervals ΔK between every strike price and ΔT between every maturity date. These chain intervals are used to define options with different strike and maturity date, and the market participants can submit their orders for these contracts. However sometimes there is no any order for a particular type of option in the option chain due to the lack of demand or supply of it. This creates an additional information gap in the grid of strikes and maturity dates.

In order to resolve the issue of missing option prices, I have to interpolate the intermediate missing option prices based on the existing ones. Generally there are two approaches for handling this problem: one is the use of approximation functions between 2 data points to interpolate the value in between and to make a continuous smooth transition between discrete data points; or model volatility surface as a stochastic function with several unknown coefficients $\boldsymbol{\theta}$ and find them using an optimisation routine. I describe both of these methods which would help us to draw a continuous smooth volatility surface.

In first approach, I use kernel smoothing method to approximate the missing value in between two data points. The kernel is defined as a function $K(x)$ which must hold 2 conditions: a) it must integrate to 1 making it an acceptable PDF; b) its mean must be equal to the sample's mean. There are multiple kernel functions which are commonly used in many probability distributions, and the most familiar one is the Gaussian kernel function. Gaussian kernel is defined below:

$$D(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{4.2}$$

In order to smoothly transition over missing data points, the Euclidean distance of given sample data point X_i from generated data points X_0 with an interval of $h(X_0)$ are passed to a kernel function (4.2) to obtain a weighting of smoothed function $Y(X_0)$ for given X_i. I describe the method used for constructing the volatility surface proposed by Gatheral [64]. Let us define the kernel function as follows:

$$K(X_i, X_0) = D(\frac{\|X_i - X_0\|}{h(X_0)}) \tag{4.3}$$

where

$h(X_0)$ is an interval between generated data points.

X_0 generated data points

X_i given sample data point

Using Nadaraya-Watson [120, 138] kernel-weighted average function, we can estimate the approximate values of $\hat{Y}(X_0)$ for generated data set X_0. It is defined below:

$$\hat{Y}(X_0) = \frac{\sum_{i=1}^{N} K(X_i, X_0)Y(X_i)}{\sum_{i=1}^{N} K(X_i, X_0)} \tag{4.4}$$

In implied volatility case, we deal with 2 parameters moneyness M and maturity date T, and because the data points are independently generated for each parameter, we can simply multiply the kernels of two parameters and make joint kernel function for both parameters. Then by using (4.4), we can obtain following estimation of volatility surface for generated data points of M_0 and T_0.

$$\hat{\sigma}(M_0, T_0) = \frac{\sum_{i=1}^{N} K(M_i, M_0)K(T_i, T_0)\sigma(M_i, T_i)}{\sum_{i=1}^{N} K(M_i, M_0)K(T_i, T_0)} \tag{4.5}$$

where

M_0 generated data points of option moneyness

M_i moneyness given by an option chain

T_0 generated data points of maturity date

T_i maturity date given by an option chain

$\sigma(M_i, T_i)$ implied volatility obtained from factual option prices for given moneyness M_i and maturity date T_i.

In Figure 4.3, I have modelled the volatility surface using approximation formula (4.5). Although the non-parametric approximation methods like I used may not always produce smooth curves that are differentiable, it is very flexible to reflect every aspect of the empirical data. It is not restricted to particular shape and does not require the agent to undertake an estimation of certain parameters. The first- and second-order derivatives of the volatility surface with respect to any of the arguments K or T can be computed numerically using any of well-known finite difference methods. The derivatives of the volatility surface are key to the evaluation of option's risk-neutral price, because it produces an option Greek vega parameter which can be used to model the option's price in respect to changes in volatility.

In order to be able to obtain the derivatives of the volatility surface analytically, I have to fit the volatility surface into some well-defined mathematical function with unknown parameters $\boldsymbol{\theta}$. I can use polynomial or exponential functions with unknown coefficients to represent the overall volatility skew and fit the coefficients of the model to empirical data. Hirsa *et al.* [83] suggested following simple exponential model for determining the volatility surface:

$$\sigma(K,T) = \frac{1}{2}(c_1 e^{K/c_2} + c_3 e^(- K/c_2)) e^{\beta T} \tag{4.6}$$

where c_1, c_2, c_3 and β are the unknown coefficients (let them denote $\boldsymbol{\theta}$) that must be fit to the market data.

We will use following optimisation routine for calibrating the unknown parameters of the mathematical model of volatility surface defined in (4.6). The objective function is the squared difference between model $\sigma(K,T;\boldsymbol{\theta})$ parametrised by coefficients $\boldsymbol{\theta} = \{c_1, c_2, c_3, \beta\}$ and implied volatility obtained from the market data $\hat{\sigma}(K,T)$. In a continuous calibration cases, it is important to note that agents do not expect calibration parameters change dramatically in short period of time. Therefore agent also imposes a penalty for diverging from previous parameters $\boldsymbol{\theta}_{prev}$. It can be formulated as $\lambda\|\boldsymbol{\theta} - \boldsymbol{\theta}_{prev}\|^2$ - the squared norm of the difference between two parameter sets multiplied by λ penalising factor. Hence the objective function can also include a penalising term for divergence from previous coefficients.

$$\min_{\boldsymbol{\theta}} (\sigma(K,T;\boldsymbol{\theta}) - \hat{\sigma}(K,T))^2 + \lambda\|\boldsymbol{\theta} - \boldsymbol{\theta}_{prev}\|^2 \tag{4.7}$$

Above I have shown how risk-neutral agent can construct a knowledge base in the example of volatility surface from using available market data for options. I described two different methods that agent use to calibrate parameters of his model to factual data. Every agent has a model with set of unknowns $\boldsymbol{\theta}$, and agent's task before producing option valuations is to match his unknown parameters correctly to the market data.

4.2.2 Informed Traders

Some agents like information-based agents use Bayesian inference to estimate the parameters of the probability of certain events in the market. This can be the probability of the population of certain type of traders in the market, the probability if the asset is undervalued or overvalued, the distribution of option valuations, the distribution of arrival rates of certain orders etc. As in model calibration, I denote this set of parameters as $\boldsymbol{\theta}$. Unlike model calibration, in Bayesian inference, parameters are described as distributions. Various estimators can be used to obtain a concrete value for these parameters. Also in Bayesian inference models parameters obtained as a result of agent's internal beliefs and observation of incoming data. The distribution which summarises agent's beliefs and observations on parameter $\boldsymbol{\theta}$ is referred as *posterior distribution*. In order to obtain a posterior PDF for possible values of $\boldsymbol{\theta}$, agent needs to establish initial prior PDF of $\boldsymbol{\theta}$ and a corresponding likelihood function for data X parametrised by $\boldsymbol{\theta}$. The product of prior and likelihood is then divided by the marginal PDF of the data X which can be found by the law of total probabilities. This involves the integration of the conditional probabilities over all values of $\boldsymbol{\theta}$. Bayes rule is defined as follows in its general formulation:

$$f_{\boldsymbol{\Theta}|X}(\boldsymbol{\theta}|x) = \frac{f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})f_{X|\boldsymbol{\Theta}}(x|\boldsymbol{\theta})}{f_X(x)} \tag{4.8}$$

where

$f_{\boldsymbol{\Theta}|X}(\boldsymbol{\theta}|x)$ is the agent's posterior probability distribution of the parameters of his model for given market data x.

$f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$ is the agent's prior probability distribution of the parameters of his model.

$f_{X|\boldsymbol{\Theta}}(x|\boldsymbol{\theta})$ is the agent's likelihood function for a parameter $\boldsymbol{\theta}$.

$f_X(x)$ is the marginal probability distribution of agent's estimates on X.

To simplify things even further, I assume that the parameter set $\boldsymbol{\theta}$ consists of a single variable, and it can be some unknown parameter of Bernoulli, Binomial, Beta, Normal or Poisson distributions which represents the PMF or PDF of interested random variable. Depending on particular case, agent can use either of these distributions to model the random events happening in the market. For example, if agent is interested in the population of informed traders in the market, he can try to find out a Bernoulli parameter $\theta \in [0, 1]$ which represents the probability of encountering an informed trader in the market. He can learn about θ through observing the arrival of orders and classifying them to informed/uninformed orders. The agent can find out if the order came from an informed trader only after observing the future. In case of options, if agent bids

\$5 for the call option with strike $K = \$105$ and spot price $S_0 = \$100$, then the trader is implicitly expecting the asset price end up at least at \$110 at time T if the risk-free rate r is zero. If the price, indeed, ends up at \$110 or above at time T, then the agent can assume that the bid he filled arrived from an informed trader. Of course, this is a very naive way of detecting informed traders, and there could be a bias from liquidity traders who accidentally guessed correct price for the option, but in order to simplify this process, I use this method to classify orders into informed/uninformed orders. In this way, agent can count the number of informed orders arrived so far and denote it with random variable X. If agent assumes that the arrival of informed/uniformed orders are i.i.d.and the whole order arrival process simulates the random sample of the trader population, this can be modelled as a Binomial likelihood which is written below:

$$p_{X|\Theta}(x|\theta) = \binom{N}{x}\theta^x(1-\theta)^{N-x} \tag{4.9}$$

where

N number of orders agent has accepted.

x number of informed orders agent has detected.

θ probability of encountering an informed order.

Binomial likelihood is derived from Binomial distribution which has a discrete PMF for each case of X out of N orders.

The prior PDF of θ can be modelled as $Beta(a, b)$ distribution which is a very flexible PDF. It can take the shape of the most of the unimodal PDFs depending on its positive real parameters a and b. Figure 4.4 shows how it looks like for different parameters of a and b. Hence I can use it to cover variety of initial beliefs about the unknown parameter, in my particular case, θ which represents the proportion of informed traders in the market.

Formally, Beta distribution is defined as follows:

$$f_\Theta(\theta) = \frac{1}{B(a,b)}\theta^{a-1}(1-\theta)^{b-1} \tag{4.10}$$

where

$B(a, b)$ is a beta function which is a normalising factor of the distribution.

The main reason for choosing Beta distribution for representing the prior is that it is conjugate to Binomial likelihood. This means that the posterior

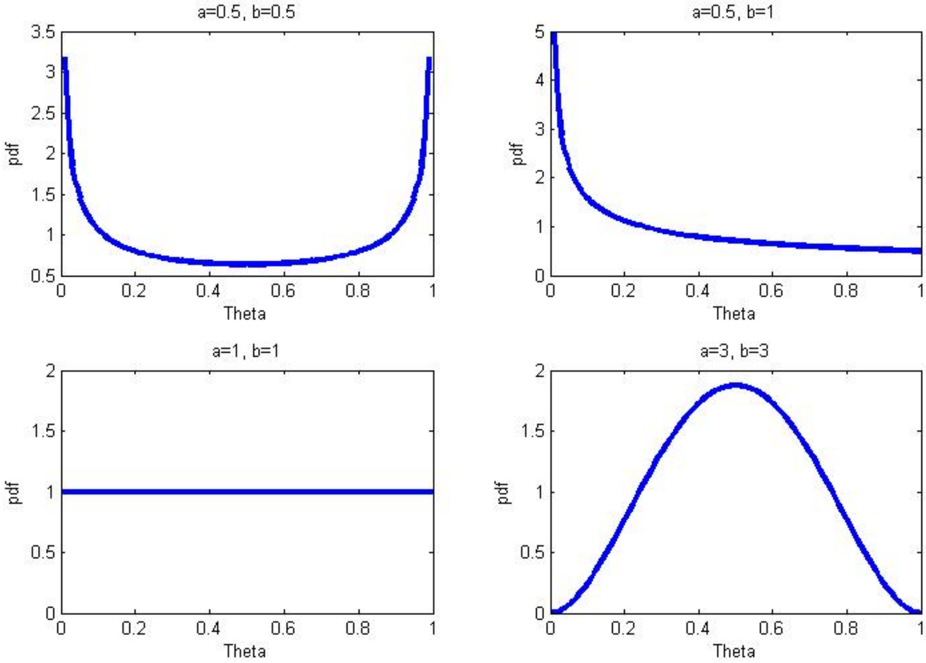


Figure 4.4: Beta Distribution with different values for a and b

distribution is also going to be a Beta distribution, and its parameters are clearly defined from the parameters of the prior distribution and likelihood. Although conjugate prior restricts us to use only Beta distribution for the prior, it provides a closed form solution to a posterior distribution function and does not require the agent going through the Bayesian inference process defined in (4.8). The solution is analytically accurate and requires no effort from the agent to compute the result. I took the posterior distribution formula for given Binomial likelihood $p_{X|\Theta}(x|\theta)$ function (4.9) and Beta prior $p_{\Theta}(\theta)$ function (4.10) from Gelman's textbook[65]:

$$p_{\Theta|X}(\theta|x) = \frac{1}{B(a+x, N+b-x)}\theta^{a+x-1}(1-\theta)^{N+b-x-1} \tag{4.11}$$

This makes new Bayesian updated θ's distribution also a Beta distribution with parameters $Beta(a+x, N+b-x)$.

Once agent has the posterior distribution of θ for given x, agent can find out a *posterior-predictive distribution* of X to compute the expected value of the random variable X itself. The general formula for posterior-predictive distribution is defined below:

$$p_{X'|X}(x'|x) = \int p_{X'|\Theta}(x'|\theta)p_{\Theta|X}(\theta|x)d\theta \tag{4.12}$$

where

$p_{X'|\Theta}(x'|\theta)$ likelihood of having data x' for given parameter θ

$p_{\Theta|X}(\theta|x)$ posterior distribution obtained from the given data X.

By using the equation given in (4.12), I can derive posterior-predictive distribution for Binomial likelihood and Beta prior cases which model the population of informed traders in the market. The integration of the product of Binomial likelihood (4.9) and Beta posterior distribution (4.11) over θ would result in following new PMF named a Beta-Binomial Distribution:

$$p_{X'|X}(x'|x) = \binom{N'}{x'} \frac{B(x+a, N-x+b)}{B(a,b)} \tag{4.13}$$

where

N' the predictive size of the sample

x' the predictive number of informed traders

N the obtained size of the sample

x the obtained number of the informed traders

a, b the parameters of the initial Beta prior.

Agent then can use the posterior-predictive distribution to find the expected number of the informed traders in the market based on the information he gathered so far. The knowledge about the number of informed traders in the market can be useful in estimating the probability of encountering an informed trader $\hat{\theta} = \frac{\mathbb{E}[X'|X=x]}{N'}$ for an arbitrary order. This is a key parameter for information-based traders using Copeland and Galai's model [33], Glosten and Milgrom's model [70, 71] and Easley and O'Hara's model [49, 48].

Above I showed that if agent chooses a prior distribution for parameter θ which is conjugate to the likelihood function, then the posterior distribution is also defined same as the prior distribution but with different parameters. Hence the trader can simply obtain the new posterior distribution by plugging in the data he observed. He can also use this knowledge to estimate the parameter θ from the posterior-predictive distribution.

However, not all the time, the agent can find a conjugate prior distribution for his likelihood function. This would mean that the agent has to manually compute the posterior distribution using the Bayesian inference rule (4.8).

With posterior distribution on his hand, agent can use various estimators such as Maximum A Posteriori (MAP) or conditional expectation to determine the estimated value of $\hat{\theta}$. MAP is good if agent wants to use the most likely value of θ given the market data x, while conditional expectation would give an estimate which minimises the squared error between estimated $\hat{\theta}$ and the real θ.

Below formulas show how to compute $\hat{\theta}$ using MAP and conditional expectation. MAP is defined as follows:

$$\hat{\theta} = \arg\max_{\theta} f_{\Theta|X}(\theta|x) \tag{4.14}$$

And conditional expectation is defined as follows:

$$\hat{\theta} = \mathbb{E}[\theta|X = x] = \int \theta f_{\Theta|X}(\theta|x)d\theta \tag{4.15}$$

After obtaining the estimated $\hat{\theta}$, agent can plug this knowledge into his corresponding behavioural model which dictates how bids and asks are submitted in the market. In my case, agent having the correct probability of encountering an informed trader and assuming that the informed trader can outrun him because of the exclusive knowledge, he can set up a corresponding bid/ask spread to compensate this risk. I show how it is used in Copeland and Galai's model [33] later in this chapter.

In this section, I have described how agent can use Bayesian inference technique to estimate the probability of certain events or conditions in the market using the empirical data. In particular, I showed how agent can identify the proportion of informed traders in the market by using Binomial likelihood and conjugate Beta prior distribution as an initial guess, and obtaining a Beta posterior adjusted to incoming data. I also showed how agent can draw the PDF of encountering X' informed traders out of N' traders given that X out of N has been already recorded. Of course, agent can use Bayesian inference for other purposes, with other likelihood functions and conjugate priors such as Normal or Poisson distributions depending on the nature of the events, but these events are outside the scope of my thesis. There are well defined formulas for calculating the posterior of such distributions in Bayesian statistics textbooks such as Gelman [65], etc.

4.3 Option Pricing

In this section, I specify how agents evaluate option prices based on their behavioural models. Option pricing is the integral part of behavioural layer of proposed IIKB model and it is responsible for computing the intrinsic value of the option to an agent. Agent's can use different models to price options starting from zero-intelligence pricing to risk-neutral pricing models. I have already reviewed most of the arbitrage-free option pricing methods such as Black-Scholes and Monte Carlo methods in Chapter 2, but in this section I am also going to introduce several other pricing techniques. Agents have their own subjective asset valuation schemes that they use to trade assets in the market. This type of derivative pricing is also referred as *indifferent pricing* [24] which must satisfy agent's utility function in such a way that agent is indifferent to sell or buy the asset at given price or remain idle. Some of the recent works in pricing options using indifference schemes are researched by Othman *et al.* [42], deMarzo *et al.* [40], Musiela *et al.* [118], etc. In this research, I use 2 very simple indifference pricing models: one is standard risk-averse exponential model and the other is LMSR model inspired by *prediction markets* [26] and developed by Othman [126]. Among other option pricing methods, I employ Monte Carlo pricing methods, especially for determining the price of option portfolios using a volatility surface.

The main purpose of using different option pricing techniques is to maintain the liquidity in the market. Agents with different valuations would ensure that there are overlapping interests in the market which facilitate the trading process. For example, ZI traders produce most of the unstable prices due to their randomised nature. But they are mostly used as a forecasting traders who pick an option portfolio based on their forecast.

Figure 4.5 shows the relationship between various option pricing methods and the rest of the layers in IIKB framework. For example, delta-hedge pricing method involves the use of asset and cash accounts in inventory layer. While LMSR method uses the existing portfolio of options to evaluate the price of an option. Instead of linking each component inside the module with another component in different layer, I generalised the relationship as a bold line between 2 modules. So the bold lines indicate relationship between groups of components. This would mean that every option pricing method uses knowledge about asset prices and interest rates. Also I have shown the relevance of

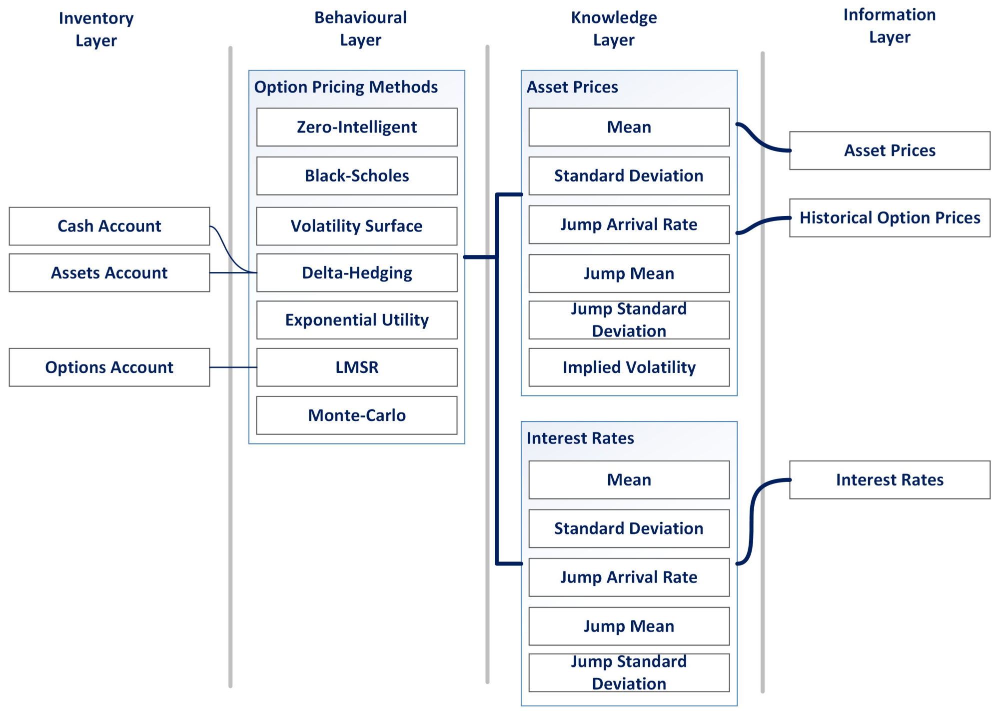


Figure 4.5: Components involved in option pricing

information in order to obtain certain components in the knowledge layer.

Below I describe the implementation of each option pricing method and provide my simulation results. I discuss the important aspects of each pricing model and determine the reason behind using them in my proposed mechanisms.

4.3.1 Zero-Intelligence Pricing

As I mentioned earlier, ZI agents are mostly needed to mimic the forecasting traders and also provide additional liquidity to the market. ZI agents had been studied first in a CDA environment by Gode and Sunder [74, 72] to measure how humans and ZI reach equilibrium state in the market. It turned out that ZI traders with constrained budget may closely replicate rational human traders in the market. So the results demonstrated that the dominant factor in obtaining equilibrium prices in the market was not associated with the rational strategy of its participants, but with the order matching mechanism employed by the market-maker. Thus ZI agents quickly became common benchmark model for testing multiple mechanisms in economics, finance and even physical sciences due to their simplicity and ease of its implementation [98, 1]. ZI traders are mainly designed to determine the role of the market mechanism in forming the prices ignoring the strategic behaviour of the traders. This would allow market designers to separate which aspects of the resulted price originated from the mechanism itself, and not from the strategic behaviour of the traders. Because this research also proposes a DA design for pricing option portfolios, ZI traders appear to be a natural choice to measure the effect of proposed mechanisms in determining option prices. The other variations of noisy traders have been implemented and experimented by Botierslevi *et al.* [20] and Friedman [60]. Othman applied noisy traders for pricing options [42, 126].

In this section, I simulate ZI agents which are adapted to pick option prices at random constrained to the bounds defined by the option's intrinsic value. I have listed these boundaries for call c and put p below:

$$e^{-rT}\max(\hat{S_T}-K,0) \leq c \leq e^{-rT}\hat{S_T} \tag{4.16}$$

$$e^{-rT}\max(K-\hat{S_T},0) \leq p \leq e^{-rT}K \tag{4.17}$$

Thus ZI traders are constrained by above boundaries to prevent generating absurd option prices that does not conform with their definition. Also the boundary values are discounted with risk-free interest rate r to conform with the time value of money. The only unknown $\hat{S_T}$ is obtained through randomly choosing a potential expiration value of the asset price following any of the above discussed asset pricing processes. Formula given in (3.7) shows us a clear formulation on how one can generate random expiration prices of S_T for GBM pricing process. For generating jump-diffusion prices at time T, agent has to use (3.11). I use risk-neutral drift $\mu = r$, and the estimated volatility of asset prices in simulated experiments. Note that the ZI agents do not use implied volatility which is obtained by solving Black-Scholes formula. Instead they use the factual volatility of the asset prices. Once ZI trader computes his private valuation for the call, he can use put-call parity (2.6) to compute the value of the put with the same strike and expiration date.

Below is the Table 4.2 showing option prices generated by ZI trader using GBM and jump-diffusion pricing models. Options expire in one year, and the annualised risk-free return is set to $r = 0.05$, and the annual asset price volatility is set to $\sigma = 0.02$. Although the 95% confidence interval for the asset price at T is $101.12 \leq S_T \leq 109.24$ and the drawn agent prediction is $\hat{S}_T = \$105.11$, the strikes cover even the extremal cases starting from 50 to

150. This could be the case for jump-diffusion prices where the prices may vary significantly due to the number of jumps in the past. For example, the current draw of jump-diffusion price at time T is $\hat{S}'_T = \$113.85$ which easily crosses the confidence boundaries of GBM process with parameters $\lambda = 0.1$, $\mu_Y = 0.05$ and $\sigma_Y = 0.02$. This would simulate asset price having an expected 0.05 more jump in return every 10 days on average.

Strikes	**GBM Calls**	**GBM Puts**	**Jump-Diff Calls**	**Jump-Diff Puts**
50.00	53.50	0.00	60.73	0.00
60.00	43.99	0.00	51.22	0.00
70.00	34.47	0.00	41.71	0.00
80.00	24.96	0.00	32.20	0.00
90.00	15.45	0.00	22.68	0.00
100.00	5.94	0.00	13.17	0.00
110.00	0.00	3.57	3.66	0.00
120.00	0.00	13.09	0.00	5.85
130.00	0.00	22.60	0.00	15.37
140.00	0.00	32.11	0.00	24.88
150.00	0.00	41.62	0.00	34.39

Table 4.2: The option prices generated by ZI traders using GBM and Jump-Diffusion pricing models

In Table 4.3 shows the potential option valuations of ZI traders on NASDAQ-100 prices using both GBM and jump-diffusion processes. I used 2-year nominal U.S. Treasury Bill annual rates [1] as risk-free interest rate which on 02.01.2014 was equal to 0.12% to discount the intrinsic option values. I have used already calibrated daily volatility $\sigma = 0.0088$ of the asset and annualised it to $\sigma = 0.1681$. The time parameters was set to $T = 16/365$. For jump process, I re-calibrated NASDAQ parameters with new risk-free interest rate, and obtained $\lambda = 0.8$, $\mu_Y = -0.004$ and $\sigma_Y = 0.0083$ and used them to simulate random jumps in asset pricing process. We can see from the Table 4.3 that this particular ZI trader underpriced the option compared to market price, while ZI trader simulating jump-diffusion asset price happened to be within bid-ask spread of the market. As cautionary note that these are just one instances of ZI trader simulation, and there is no reason to assume that GBM ZI trader will always price below the market price. In another instance it could be equally higher too. Another important aspect of market prices are that puts are priced positively where their intrinsic values for ZI traders are zero meaning that the

[1] http://www.treasury.gov/resource-center/data-chart-center/interest-rates

price GBM and jump-diffusion prices predicted are less than strike for calls, and opposite for puts.

4.3.2 Risk-Neutral Pricing

This type of agents use standard option pricing techniques described in classic finance literature. The main idea behind pricing derivative products is the arbitrage free pricing assumption. In this pricing methodology, the expected future cash flows must be compensated by the price paid today, so that there is no opportunity to make risk-free profit. I have discussed about arbitrage free pricing in Chapter 2. I also covered most of the canonical option pricing methods such as Black-Scholes, Binomial model and Monte-Carlo simulations in that chapter. Here I simulate the option prices using each of these techniques. Particularly I showed how one can use Black-Scholes framework to obtain a volatility surface of an underlying earlier in this chapter. Agent use this surface to compute the risk-free valuation of any option, including the option portfolios. Volatility pricing method is used to obtain risk-neutral PDF directly from the market itself. I use Monte-Carlo simulations to approximate the risk-neutral price of different kinds of options.

Black-Scholes Pricing

The Black-Scholes prices are used as main benchmark pricing to compare the behaviour of other trading agents in proposed mechanisms. The Black-Scholes prices can be directly computed from the closed formula given in (2.28) once all of its parameters are given. As it was mentioned earlier, volatility parameter is unknown and it changes for every different setting. Agents can simply use the asset price volatility calibrated from the available sample, or model a volatility surface from historical option prices and use that implied volatility to price other options. I assume that the agent knows the volatility of the asset price, so he can directly plug it into Black-Scholes formula and compute risk-neutral option prices. For special NASDAQ-100 case I have already found the calibrated volatility parameters, so I use these parameter settings while pricing NASDAQ-100 options.

Table 4.4 shows the prices of Black-Scholes (BS) trader for different strikes of call and put of the same maturity date. The maturity date of the option is set

Strikes	GBM Calls	GBM Puts	J-D Calls	J-D Puts	Mkt Call Bid	Mkt Call Ask	Mkt Put Bid	Mkt Put Ask
3500.00	63.57	0.00	83.88	0.00	81.10	86.40	16.20	18.10
3505.00	58.57	0.00	78.88	0.00	77.20	82.60	17.20	19.00
3510.00	53.57	0.00	73.88	0.00	73.30	78.70	18.30	20.20
3515.00	48.57	0.00	68.88	0.00	69.40	74.30	19.50	21.50
3520.00	43.57	0.00	63.88	0.00	65.80	70.90	20.70	22.60
3525.00	38.57	0.00	58.88	0.00	62.10	66.70	22.10	24.00
3530.00	33.57	0.00	53.88	0.00	58.60	63.10	23.40	25.50
3535.00	28.57	0.00	48.88	0.00	55.40	59.80	25.00	27.10
3540.00	23.57	0.00	43.88	0.00	51.80	56.20	26.60	28.80
3545.00	18.57	0.00	38.88	0.00	48.70	52.20	28.30	30.50
3550.00	13.57	0.00	33.88	0.00	45.60	49.20	29.90	32.40
3555.00	8.57	0.00	28.88	0.00	42.60	45.90	31.70	34.50
3560.00	3.57	0.00	23.88	0.00	39.70	42.70	33.50	36.60
Underlying Price on 02.01.2014: $3563.57								
3565.00	0.00	1.43	18.88	0.00	36.80	39.90	35.90	39.10
3570.00	0.00	6.43	13.88	0.00	34.10	36.70	38.20	41.00
3575.00	0.00	11.43	8.88	0.00	31.50	34.40	40.60	43.40
3580.00	0.00	16.43	3.88	0.00	29.00	31.70	42.30	46.00
3585.00	0.00	21.43	0.00	1.12	26.70	29.10	45.50	48.60
3590.00	0.00	26.43	0.00	6.12	24.50	27.40	47.80	51.80
3595.00	0.00	31.43	0.00	11.12	22.30	24.70	49.80	54.30
3600.00	0.00	36.43	0.00	16.12	20.40	23.00	53.90	57.40

Table 4.3: Comparison of ZI trader prices with NASDAQ-100 historical option prices on 02.01.2014 for options expiring on 18.01.2014

to 1 year, and risk-free rate is $r = 0.05$ and volatility 0.02. The virtual asset price at time $t = 0$ is equal to $S_0 = \$100$.

Strikes	**BS Calls**	**BS Puts**
50.00	52.44	0.00
60.00	42.93	0.00
70.00	33.41	0.00
80.00	23.90	0.00
90.00	14.39	0.00
100.00	4.88	0.00
110.00	0.01	4.64
120.00	0.00	14.15
130.00	0.00	23.66
140.00	0.00	33.17
150.00	0.00	42.68

Table 4.4: The option prices generated by BS traders using closed formula

In Table 4.5, we can see similar comparison of BS trader prices versus the actual market prices. We can see that BS prices are slightly overprices towards market prices and the reason for that could be anything starting from the transaction costs that add-up to the risk-neutral value of the option down to the complex strategies employed by market participant along with market's matching mechanism at that time. Even if it is assumed that all traders use Black-Scholes pricing, the choice parameters for the formula can be different which as a result will cause variance in price. In our case, we used U.S. Treasury Bill rate for risk-free rate and the the GBM calibrated volatility as we did previously for ZI traders. However the volatility parameter is thought to be different for each particular case below, because of volatility skew in the market. But BS trader will hold on to an assumption made by Black-Scholes on the invariance of volatility of the asset price.

Volatility Pricing

In volatility pricing method, agents harness the information available from the option market itself to price other types of options such as option portfolios, etc. They use the volatility surface obtained earlier in this section to find out about the risk-neutral probability distribution. In Section 4.2.1, I have described two major ways of obtaining volatility surface from given data, and further in this section I show how agent can use it as main knowledge base

Strikes	**BS Calls**	**BS Puts**	**Mkt C Bid**	**Mkt C Ask**	**Mkt P Bid**	**Mkt P Ask**
3500.00	87.86	24.09	81.10	86.40	16.20	18.10
3505.00	84.44	25.67	77.20	82.60	17.20	19.00
3510.00	81.10	27.33	73.30	78.70	18.30	20.20
3515.00	77.84	29.07	69.40	74.30	19.50	21.50
3520.00	74.65	30.87	65.80	70.90	20.70	22.60
3525.00	71.53	32.76	62.10	66.70	22.10	24.00
3530.00	68.49	34.72	58.60	63.10	23.40	25.50
3535.00	65.53	36.76	55.40	59.80	25.00	27.10
3540.00	62.65	38.88	51.80	56.20	26.60	28.80
3545.00	59.85	41.08	48.70	52.20	28.30	30.50
3550.00	57.12	43.35	45.60	49.20	29.90	32.40
3555.00	54.48	45.71	42.60	45.90	31.70	34.50
3560.00	51.92	48.14	39.70	42.70	33.50	36.60
Underlying Price on 02.01.2014: $3563.57						
3565.00	49.43	50.66	36.80	39.90	35.90	39.10
3570.00	47.02	53.25	34.10	36.70	38.20	41.00
3575.00	44.70	55.92	31.50	34.40	40.60	43.40
3580.00	42.45	58.68	29.00	31.70	42.30	46.00
3585.00	40.28	61.51	26.70	29.10	45.50	48.60
3590.00	38.19	64.42	24.50	27.40	47.80	51.80
3595.00	36.18	67.40	22.30	24.70	49.80	54.30
3600.00	34.24	70.46	20.40	23.00	53.90	57.40

Table 4.5: Comparison of BS traders prices with NASDAQ-100 historical option prices on 02.01.2014 for options expiring on 18.01.2014

in pricing options. The reason why I stressed the importance of obtaining volatility surface is that it can be used to evaluate the risk-neutral value of options with different payoff structures. Dupire has developed a methodology of using volatility surface for finding the risk-neutral value of any option, be it an exotic one or a bundle of options generating certain income structure [45, 46]. Dupire suggested using the second derivative of call option $C_{mkt}(S_0, K, T)$ with respect to strike K as the risk-neutral probability density $\varphi(K; T)$ for evaluating the expected undiscounted payoff of an option at time T. In a continuous scenario it is formulated as follows:

$$\varphi(K; S_0, T) = \frac{\partial C^2}{\partial^2 K} \tag{4.18}$$

Then agent can use this PDF to find the risk-neutral value of any option with various payoff structures. For that he needs to find the expectation of the payoff function $f(S_T; S_0, K)$ for given maturity price S_T which is written below for any option:

$$C(S_0, K, T) = \int_0^\infty f(S_T; S_0, K)\varphi(S_T; T)dS_T \tag{4.19}$$

For example, for European call option, we already know its payoff function from (2.2) which is $f(S_T; S_0, K) = \max(S_T - K, 0)$, so we can use formula (4.19) to find the risk-neutral value of it. Because for the asset prices below K, the option's value is set to zero we can drop that range from the integration space and start the integration directly from K. This will also take away maximum function around $S_T - K$ and simplify the integration process. Below is the concrete formula for computing the price of a European call option using the volatility surface [64]:

$$C(S_0, K, T) = \int_K^\infty (S_T - K)\varphi(S_T; T)dS_T \tag{4.20}$$

Derman and Kani suggested a numerical way of finding risk-neutral PDF $\varphi(S_T; T)$ using the present value of a butterfly call spread (2.35) in the market. The butterfly call spread must be constructed by selling one OTM call with strike $K + \Delta K$, one ITM call $K - \Delta K$ and buying 2 ATM calls at strike K. Hence we can write its value as follows:

$$B_{mkt} = C_{mkt}(S_0, K - \Delta K, T) - 2C_{mkt}(S_0, K, T) + C_{mkt}(S_0, K + \Delta K, T) \tag{4.21}$$

If we assume that all traders are risk-neutral, then the price of butterfly call spread must be equal to the discounted risk-neutral expectation of its payoff at T. This can be written as:

$$
\begin{aligned}
B_0 = \mathbb{E}_0^Q[e^{-rT}B_T] &= e^{-rT}P(K-\Delta K \leq S_T \leq K+\Delta K)\,\mathbb{E}[B_T] \\
&= e^{-rT}\varphi(K;T)(2\Delta K)(\frac{\Delta K}{2}) = e^{-rT}\varphi(K;T)(\Delta K)^2
\end{aligned} \tag{4.22}
$$

Using Equations (4.21) and (4.22), we can obtain the numerical second-order differentiation of call price C with respect to strike price K [44, 43]:

$$
\varphi(K;T) = e^{rT}\frac{C_{mkt}(S_0,K-\Delta K,T) - 2C_{mkt}(S_0,K,T) + C_{mkt}(S_0,K+\Delta K,T)}{(\Delta K)^2} \tag{4.23}
$$

The density function should be multiplied to eliminate the risk-neutral discount factor that market used to price European call options. In this way, we can numerically determine PDF from obtained volatility surface and use this knowledge to compute the risk-neutral value of options. I have to note that the expiration time for all options must be the same to use this PDF. Moreover option must be exercisable only on maturity date, and its value must depend on only the spot asset price at time T. For example, options such as Asian or Barrier options cannot be calculated by using this PDF, because their value depends on whole history of the stock price upto time T, and not only on the asset price at time T.

I can summarise the steps of computing the risk-neutral value of the option using the volatility surface as follows:

1. Construct a volatility surface from available option prices.
2. Use (4.23) to compute risk-neutral PDF $\varphi(S_T;T)$ for small changes in strike K. The unavailable price of the call option can be interpolated by inputting the corresponding approximate of a volatility parameter taken from the volatility surface into Black-Scholes formula.
3. Find the risk-neutral expectation of the option payoff $f(S_T;S_0,K)$ parametrised by the maturity price of an underlying asset S_T using obtained PDF $\varphi(S_T;T)$.

I cannot obtain a volatility surface for the virtual asset without simulating the mechanism, because there is no sample simulation of the bids and asks

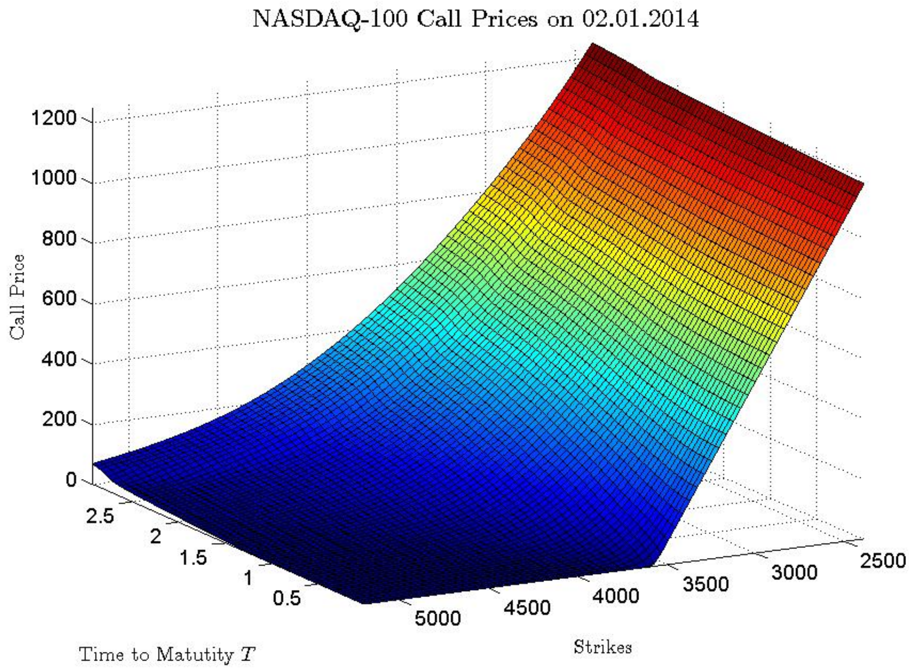


Figure 4.6: NASDAQ-100 call prices for different strikes and maturity dates on 02.01.2015

generated by volatility (VOL) traders at this stage. Volatility traders base their option pricing methodology on the realised prices in the mechanism. I will demonstrate the bids and asks generated by volatility traders while simulating the actual mechanisms, because volatility trader needs feedback from the mechanism in order to adjust its σ to the current market value of the option. However I show how agent can price options using the available data from historic NASDAQ-100 call prices. As we have already constructed the volatility surface, and presented it in Figure 4.3, agent use this data to compute the approximated value of call option for any given strike or expiration time.

Agent can further use the data presented in Figure 4.6 to calculate the risk-neutral probability $\varphi(K;T)$, and use it to compute the expected payoff for options with different payoff structure such as option portfolios. Note that the payoff of the option must only depend on the maturity price of the asset, and nothing else. The option portfolios such as butterfly spread mentioned in Chapter 2 use OTM, ATM and ITM options with the same maturity date. Therefore their payoff is only dependent on the maturity price of an asset. Agent can also compute the price of butterfly spread using the available information from NASDAQ options. Below Figure shows the possible prices for butterfly spread with different OTM and ITM strikes. The ATM strike $3560

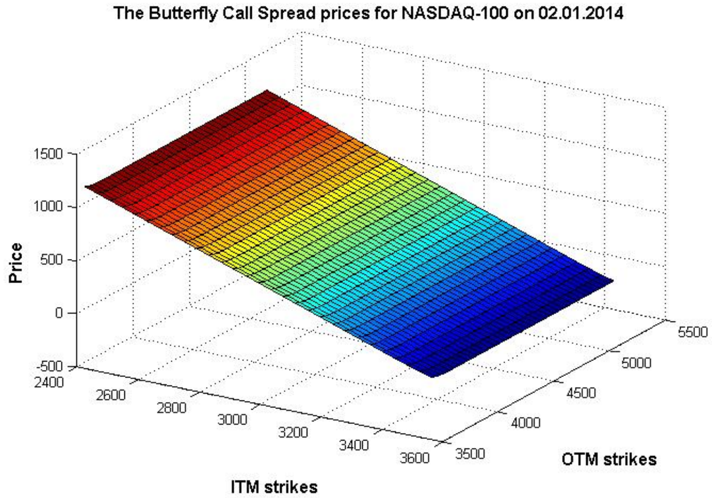


Figure 4.7: NASDAQ-100 butterfly call spread prices for different OTM and ITM strikes on 02.01.2014

is chosen as the nearest strike to the spot price of the asset \$3563.57.

Monte Carlo Pricing

Monte Carlo methods are excellent tools for pricing derivatives with complex payoff structure or asset pricing model for which there is no analytical solution. The core idea behind Monte Carlo pricing is that the option price is equal to its expected value, and through multiple numerical simulations of its underlying, agent can obtain the sample of its possible values. It does not require analytical knowledge of the agent to produce approximately correct risk-neutral prices. All he needs is sufficient number of simulations to achieve desired accuracy in estimation. The sample size is the main parameter in computing Monte-Carlo simulations, as the accuracy of the prices directly depends on how many instances are used to find the mean value of the option. The 95% confidence interval for Monte-Carlo computed option price can be related to the sample size as shown below:

$$\left[\hat{c} - 1.96\hat{\sigma}/\sqrt{N}, \hat{c} + 1.96\hat{\sigma}/\sqrt{N}\right] \tag{4.24}$$

where $\hat{c}$ is the Monte Carlo computed option price, $\hat{\sigma}$ sample standard deviation, N sample size.

I use the sample size of 10000 which significantly tightens the confidence interval and hence improves the accuracy of agent's estimate at least 100 times. I reviewed Monte Carlo simulation in Chapter 2, and here I simulate the prices of options with different underlying asset pricing models. Also I will use Monte Carlo simulations for evaluating the prices of option portfolios commonly used in option market. The general method that Monte-Carlo agent is going to follow can be described in following steps:

1. Simulate N underlying asset prices and extract necessary information which is necessary to compute the value of the option. In our context, the European option's value depends only on the maturity price of the asset S_T, this is the only value we need to simulate. But for other cases such as Asian options, the whole trajectory of the underlying asset is required to evaluate its price.

2. Compute the intrinsic values of the options based on the simulated asset prices, and discount them with risk-free rate.

3. Find the mean value of the discounted options' intrinsic values.

In Table 4.6 displays the results of simulation of a MC (Monte-Carlo) trader using GBM and jump-diffusion prices of a virtual asset. I have set the parameters for the simulation as $\mu = 0.05$,$\sigma = 0.02$, $\lambda = 0.1$, $\mu_Y = 0.01$ and $\sigma_Y = 0.005$. It can be seen that the MC J-D traders prices are slightly higher than MC GBM because the former expects upward jumps in return with average magnitude of 1% every 10 days, hence his final asset prices are higher than those who uses GBM model to simulate prices.

Strikes	**MC GBM Calls**	**MC GBM Puts**	**MC J-D Calls**	**MC J-D Puts**
50.00	52.44	0.00	52.69	0.25
60.00	42.93	0.00	43.18	0.25
70.00	33.42	0.00	33.67	0.25
80.00	23.90	0.00	24.15	0.25
90.00	14.39	0.00	14.65	0.26
100.00	4.88	0.01	5.90	1.02
110.00	0.01	4.64	1.16	5.80
120.00	0.00	14.15	0.11	14.26
130.00	0.00	23.66	0.01	23.67
140.00	0.00	33.17	0.00	33.17
150.00	0.00	42.68	0.00	42.68

Table 4.6: The option prices generated by MC traders using GBM and Jump-Diffusion asset pricing models

MC traders can also price different portfolios of options such as butterfly call spread or bullish spread that we discussed in Chapter 2. Otherwise values of constituent options are priced separately and added to find the corresponding option portfolio price. I have simulated the MC trader pricing butterfly call option using GBM asset price model. Table 4.7 shows the butterfly call prices for various combinations of strikes computed by MC trader and compares them with theoretical Black-Scholes prices. Because both Black-Scholes and MC use GBM as their asset pricing model, the prices are almost the same. The sample size of 10000 instances has been used to compute Monte Carlo prices. Simulation tries different OTM and ITM strikes of call option to construct a

Strikes 1	**Strikes 2**	**Strike 3**	**BS Prices**	**MC Prices**
50.00	100.00	150.00	42.677	42.676
60.00	100.00	140.00	33.164	33.163
70.00	100.00	130.00	23.652	23.651
80.00	100.00	120.00	14.140	14.139
90.00	100.00	110.00	4.636	4.636

Table 4.7: The butterfly call prices generated by MC traders using GBM asset pricing model

butterfly spread. Its price is better visualised using a surface plot given in Figure 4.8. We can see from the plot that the deeper into the moneyness the trader goes with his ITM option choice, the higher is the price he is expected to pay. We can also see that the OTM choice of the butterfly spread does not affect the final price of the bundle significantly. Although I do not present a simulation model (I present only the design of such combinatorial exchange) for pricing option portfolios using a combinatorial exchange, agents can use Monte-Carlo method to price option portfolios in order to operate in a combinatorial exchange environment. Monte-carlo simulations for other option portfolios such as Bullish Spread, Iron Butterfly, etc. are given in Appendix B.

4.3.3 Indifference Pricing

Indifference pricing is a method of pricing derivatives with respect to agent's private utility function. This type of pricing method is relative to the agent's own portfolio or his subjective opinion in contrast to the market's objective arbitrage free pricing methods. The indifference price is the price at which an agent would have the same expected utility by participating in the trade as by not participating in it. Usually the indifference price is a bid-ask spread

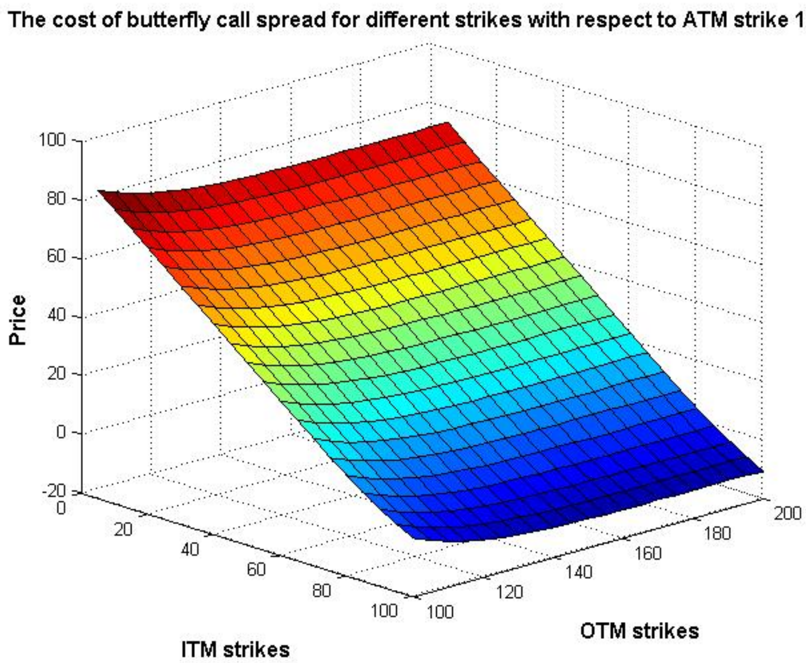


Figure 4.8: MC simulated butterfly call spread prices for different OTM and ITM strikes. ATM strike is 100.

for some agent. One important aspect of indifference pricing is that it allows agents to be subjective and private about derivative pricing and relates the risk associated with buying or selling derivatives to agent's current portfolio or inner beliefs [24]. This can be justified with the fact that every agent has his own methods for managing the risk, and certain policies of the institution that are enforced on him. I will use 2 basic indifference pricing techniques to compute options prices: one is using risk-averse exponential function and the other is the cutting edge LMSR method originated from prediction markets.

Exponential Utility

Most traders are more afraid of loosing money, rather than making profit because of human nature and potential implications of failure. This defines the rationale behind risk-averseness of trading agents. For this reason, the utility functions designed to model risk-averseness are concave functions. One of the commonly used utilities is the exponential utility which is defined below [24, 4]:

$$u(x) = \frac{1 - e^{-ax}}{a} \tag{4.25}$$

where a is the risk-aversion parameter.

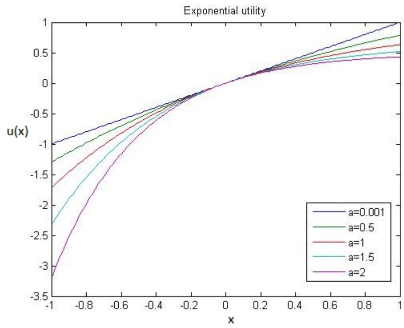


Figure 4.9: Exponential Utility

Figure 4.9 illustrates the concavity of the utility function. For a very small risk-aversion coefficient $a = 0.001$, the agent's utility grows almost linearly making him almost risk-neutral to given payoff. However for larger $a = 2$, it can be seen that agent's utility grows slower than the payoff offered for the option, making him to ask for larger payoffs to compensate the possible losses. This nicely fits to the behaviour of a risk-averse trader who asks higher prices for lower risks.

We can see from (4.25) that the higher is the risk-aversion, the less is the utility of the trader, making him more reluctant to not participate in the trade, even it has a positive expected payoff. Agents have indifferent prices relative to some ways of managing the portfolio of assets. In delta hedging method, agent keeps changing his portfolio by changing the corresponding weights of assets and cash after selling a derivative such as option. This would help to reduce the cost of fulfilling his liability if the sold option is exercised. At the maturity day, agent has a balance consisting of the assets and cash in his inventory. I have described delta-hedging while explaining Black-Scholes formula and replicating portfolios in Chapter 2.

For example let us say that agent simulates multiple paths of GBM for underlying asset and delta hedges the premium offered for the option c. At option's maturity date T agent can check how much his portfolio is worth for each GBM path by computing the series of final payoffs $\mathbf{p}$. These payoffs are the potential gains or losses that agent can have if he follows the delta hedging strategy for received premium c. However, he is reluctant to receive c because his possible payoffs $\mathbf{p}$ may also include losses, and he is afraid of them more than gaining c in case if the option sold is not exercised. Therefore he needs to charge such c' which could make him indifferent on average, so that his utility is zero as if he stayed idle. Below equation illustrates the concept mathematically:

$$\mathbb{E}(u(c' e^{rT} + \mathbf{p})) = 0 \tag{4.26}$$

Such c' can be numerically found using MATLAB's `fzero` routine. To sum up, agent can speculate any strategy, be it delta strategy, gamma strategy, or just simply placing premium received to risk-free account, to analyse his potential gains and losses, and then make a risk-averse bid on what to charge for given option so that on average he as good as if he has never traded. Table 4.8 highlights how option prices change based on traders risk-averseness parameter a. As it can be seen, the prices slightly grow as the risk-averseness of the agent is also growing. Moreover we can see that the risk-averse prices are slightly higher than the risk-neutral Black-Scholes price. I have simulated prices using GBM model using parameters annualised $\mu = 0.05$, $\sigma = 0.2$, $T = 1$ and $S_0 = 100$. I also used higher volatility to push the prices up to stress the difference between traditional risk-neutral valuation and indifference pricing.

Strikes	**BS Price**	**a=0.06**	**a=0.50**	**a=1.00**	**a=4.00**	**a=8.00**	**a=16.00**
50.00	52.44	52.44	52.44	52.44	52.44	52.45	52.55
60.00	42.94	42.94	42.94	42.94	42.94	42.95	43.04
70.00	33.54	33.54	33.55	33.55	33.57	33.67	33.96
80.00	24.59	24.59	24.60	24.61	24.79	25.30	25.67
90.00	16.70	16.70	16.72	16.74	16.89	17.11	17.35
100.00	10.45	10.46	10.49	10.52	10.89	11.56	11.97
110.00	6.04	6.05	6.08	6.12	6.46	6.94	7.28
120.00	3.25	3.25	3.27	3.30	3.73	4.30	4.67
130.00	1.64	1.63	1.64	1.66	1.84	2.23	2.58
140.00	0.78	0.78	0.78	0.79	0.88	1.21	1.57
150.00	0.36	0.35	0.35	0.35	0.37	0.43	0.67

Table 4.8: Option prices for different risk-aversion factor a in comparison with Black-Scholes prices

Logarithmic Market Scoring Rule

This pricing rule emerges from the concept of *prediction markets* which are the mechanisms developed for aggregating information about uncertain events from market experts [135]. The market-maker, or commonly referred as *information aggregator* in the context of prediction markets, devises a contract which pays/receives certain amount of money for each possible future outcome.

For example, the aggregator might be interested in gathering predictions about the weather for tomorrow, and pay the same amount as the factual temperature tomorrow. Similarly, in financial markets, the future price of an asset is uncertain and the European option's value is directly dependent on this uncertain value. Hence the aggregator can use an option contract as his main payoff structure for compensating the predictions on the possible future prices of an asset. The payoffs on each future price can be defined as the intrinsic value of the option, $(S_t - K)^+$ for calls and $(K - S_t)^+$ for puts.

In prediction markets, the informants are allowed to change the aggregator's payoff structure for a corresponding payment. For example, if aggregator is accepting bets for teams A and B on a football match, and his current payoff structure is (300,200) meaning that the aggregator has to pay \$300 in total if team A wins, and \$200 if team B wins. However one would like to bet on team A, and he expects to receive \$50 if his bet is achieved. The aggregator changes his payoff structure to (350,200) by accepting the bet, and he also needs to decide how he can charge the client for accepting his bet. The most common method for evaluating the cost of accepting the bet in prediction markets, LMSR, has been developed by Hanson [76, 77] and it found its corresponding applications in a number of companies such as Yahoo!, Microsoft, Inkling Markets, Consensus Point and Gates Hillman Prediction Market[127]. It is defined as a cost function for the vector of payoffs $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ on the probability space of events $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$:

$$C(\mathbf{x}) = b \log \left(\sum_i \exp(x_i/b) \right) \tag{4.27}$$

where $b > 0$ is a liquidity parameter. The larger values of b produce tighter bid/ask spreads, but may also incur larger worst-case losses capped by $b \log(n)$ [42]. Follmer and Schied regard LMSR as an *entropic risk measure* in their book on stochastic finance [59].

The agent who wishes to change the payoff from $\mathbf{x}$ to $\mathbf{y}$ has to pay the difference between the costs $C(\mathbf{y}) - C(\mathbf{x})$. In our above example, given that $b = 100$, the aggregator accepting bets must charge the client $C((350, 200)) - C((300, 200)) \approx \$39$ for the bet.

The same principle can be used for the option trader who holds a certain portfolio of options that generate certain payoffs for different asset price outcomes, and prices other options from the point of his own payoff structure. The agent can virtually simulate buying or selling particular type of option and compute the changes it makes to its payoff structure. For example, let agent take butterfly call spread with buying ITM call at strike $K_1 = 80$ and OTM call at $K_3 = 120$, and selling 2 ATM calls at $K_2 = 100$. We can compute his discounted payoffs for the range of possible prices where the asset price can end up at time T. Let this payoff structure be $\mathbf{x}$, and it can be depicted as in Table 4.9. The trader feels bullish and wants to buy one more call option at strike $K_4 = 130$. This should change his payoff structure to $\mathbf{y}$ as shown in Table 4.10.

Asset Prices	**Payoffs x**
<70	0.00
75	0.00
80	0.00
85	4.75
90	9.51
95	14.26
100	19.02
105	14.27
110	9.51
115	4.75
120	0.00
>125	0.00

Table 4.9: Potential payoffs of the trader holding butterfly call spread BEFORE buying call at $K_4 = 130$

Asset Prices	**Payoffs y**
<70	0.00
75	0.00
80	0.00
85	4.75
90	9.51
95	14.26
100	19.02
105	14.27
110	9.51
115	4.75
120	0.00
125	0.00
130	0.00
135	4.75
140	9.51
>145	$e^{-rT}(S_t - 130)$

Table 4.10: Potential payoffs of the trader holding butterfly call spread AFTER buying call at $K_4 = 130$

As it can be seen from tables 4.9 and 4.10, buying a certain type of option can significantly change the payoff structure of the trader. The agent's bid for buying an OTM option at $K_4 = 130$ can be computed from the difference of his payoff structures $C(\mathbf{y}) - C(\mathbf{x})$, and in our particular case is \$1.42 given that the liquidity parameter is $b = 2500$. We can also compute the Black-Scholes price of such option given parameters $T = 1$, $r = 0.05$, $S0 = 100$, $\sigma = 0.02$ and it is \$2.52. This would mean that the trader places a bid for given OTM option less than its risk-neutral value. In this scenario, the trader is not supposed to loose. However if trader decides to sell this OTM option, and computes the price in similar manner, he will end up with approximately \$1.42 (which is close to his bid due to tight liquidity parameter b) and may loose money because he is selling below option's risk-neutral value. However unlike other option traders in the market, the LMSR emerges from prediction markets, and it is designed for gathering information and thus he is more interested in liquidity, rather than profit. Therefore we can assume that such traders may exist in the market to improve the liquidity of options. I have simulated the bids and asks that butterfly call spread holder would use to trade various options in the market. Table 4.11 shows LMSR trader's bids and asks for different options on virtual asset. I have to note that LMSR trader is holding butterfly call spread which makes positive payoff only when the asset price remains mostly the same.

We can see in Table 4.11 there is a small spread between bid and ask for different options, and the overall prices significantly vary from Black-Schole's prices. For example, butterfly call spread holder using LMSR would undervalue the options deep in-the-money, and bid much cheaper price for option with strike $K = 50$, it is \$45.04 while the risk-neutral price is \$52.44. However for the options in the proximity of ATM, the option prices are highly overpriced, for example for $K = 100$ option, LMSR trader's bid is \$10.73, while the risk-neutral price is only \$4.88. This is due to the exponential function intersecting with the piece-wise payoff function of Black-Scholes. Figure 4.10 illustrates the bids and asks of LMSR trader holding butterfly call spread. Because $b = 2500$, the bid-ask spread for the options are very tight and barely noticeable. Also the bids and asks for put option do not cross the Black-Scholes price, because I use put-call parity to compute the put price from given call price. So we can see that put prices conform with the boundaries of option price.

I have also generated bids and asks for traders holding other option portfolios such as bullish spread, bearish spread, iron butterfly, etc in Appendix C. I used liquidity parameter $b = 100$ to show how it may affect the bid-ask spread for LMSR trader.

To sum up, the indifference pricing methods like LMSR take into account the agent's current possessions in the inventory layer and make option pricing decisions based on this information as well. This plays well with our proposed IIKB framework where there is a layer responsible for bookkeeping the agent's current inventory levels. Later in the chapter, we will also see how this layer can be used to support proxy trading algorithms widely used in CDAs.

Strikes	**BS Call**	**BS Put**	**LMSR Call Asks**	**LMSR Call Bids**	**LMSR Put Asks**	**LMSR Put Bids**
50.00	52.44	0.00	45.34	45.04	0.00	0.00
55.00	47.68	0.00	40.81	40.52	0.00	0.00
60.00	42.93	0.00	36.53	36.25	0.00	0.00
65.00	38.17	0.00	32.47	32.21	0.00	0.00
70.00	33.41	0.00	28.65	28.42	0.00	0.00
75.00	28.66	0.00	25.07	24.86	0.00	0.00
80.00	23.90	0.00	21.73	21.55	0.00	0.00
85.00	19.15	0.00	18.62	18.46	0.00	0.00
90.00	14.39	0.00	15.75	15.62	1.36	1.23
95.00	9.63	0.00	13.12	13.02	3.49	3.38
100.00	4.88	0.00	10.73	10.65	5.85	5.77
105.00	0.86	0.74	8.58	8.52	8.46	8.40
110.00	0.01	4.64	6.67	6.63	11.31	11.26
115.00	0.00	9.39	5.00	4.97	14.39	14.36
120.00	0.00	14.15	3.57	3.55	17.72	17.70
125.00	0.00	18.90	2.38	2.37	21.28	21.27
130.00	0.00	23.66	1.43	1.42	25.09	25.08
135.00	0.00	28.42	0.71	0.71	29.13	29.13
140.00	0.00	33.17	0.24	0.24	33.41	33.41
145.00	0.00	37.93	0.00	0.00	37.93	37.93
150.00	0.00	42.68	0.00	0.00	42.68	42.68

Table 4.11: The bids and asks of LMSR trader holding a butterfly call spread

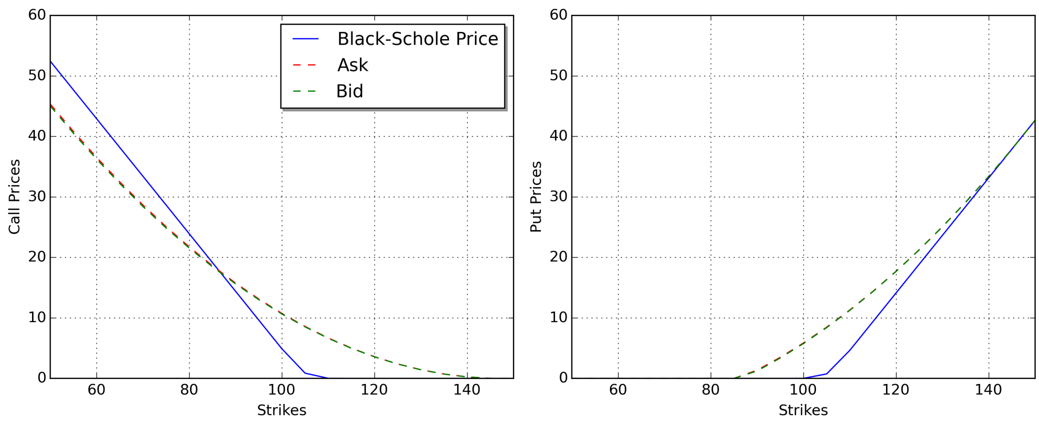


Figure 4.10: Bids and asks of LMSR trader holding butterfly call spread in comparison with Black-Scholes prices.

4.4 Choosing Quantities

In traditional algorithmic trading with assets, the traders have to decide when to enter and when to exit the market. Entry and exit is only applicable in online DAs, because the traders are continuously involved in trading. In direct DA, I ignore this idea, because traders are re-instantiated every trading day. In proposed simulation of online DA, I assume that all traders enter the market every trading day. Traders who do not want to engage in trading submit orders with zero quantity. This would mean that the traders are inactive at that position. However if they wish to buy or sell, they order a quantity other than zero. I assume that the trader opens long position if the option is underpriced according to his private valuation. Similarly trader takes short position if the option is overpriced in his view. It is also worth to mention that the ability of opening and closing market positions are attributed to traders only and not to dealers. I talk about traders and dealers in more detail in the next Section 4.5. Unlike traders, dealers always maintain a bid-ask spread for reasonable amount of options in online DA.

The exit from the market means either closing the market position or exercising the option on its expiry date. Option ending up in-money is always exercised, and its value is equal to the payoff it generated on its expiry date. Before option expiration, trader needs to buy or sell the same number of options to close his short or long position in the market. For example, if trader holds call option, he can write call option with the same strike and expiry date in order to close his market position. Trader closes his current position when he decides to switch his market position from long to short, or vice versa. This decision is again governed by trader private option valuation method and his entry decision on particular side of the market.

In this section, I describe how the quantities for options are chosen by traders. Traders can control their orders to be bids or asks by setting the quantities either to positive integer or negative integer. If trader sets the quantity to 0, then it means that he is not interested in buying or selling this option. In this way, by setting the quantity $q_j \in \mathbb{Z}$ for jth option in the option chain, trader can express his preference whether he wants to buy or sell particular type of option, and if yes, in which quantities. I propose 3 main techniques that determine how traders choose the quantities for the desired options, and explain the reason behind them.

4.4.1 Random Integers

Agents choose quantities using random integers within reasonable bounds to define the demand and supply for particular types of options. The main assumption for having this method to determine the quantities is there is an enormous amount of option contracts varying in their types, underlying asset, strike price and expiration date in the real life markets, which makes particular kind of option highly uncompetitive. In fact, if there are d underlying assets available in the market, and $|K|$ strike prices and $|T|$ expiration dates in option chain, the total number of options in the market would be equal to $2 * \sum_{i=1}^{d} |K|_i \times |T|_i$, let alone options that are more exotic than European ones. Due to this sparse supply and demand on different kinds of option, it is difficult to observe a continuous interest over longer period of time for any particular option. We can also see the absence of interest to certain kind of options in NASDAQ-100 option chain shown in Table 3.6. Therefore I assume that the supply or demand is not driven by the competing forces of the market, but rather emerges arbitrarily from the intrinsic needs of traders.

4.4.2 Linear Functions

In this method, agents take linear approach in determining the quantities to buy or sell from the given option value $\hat{p}$. Although in the context of financial markets, it is less likely to see a linear relationship between option prices and

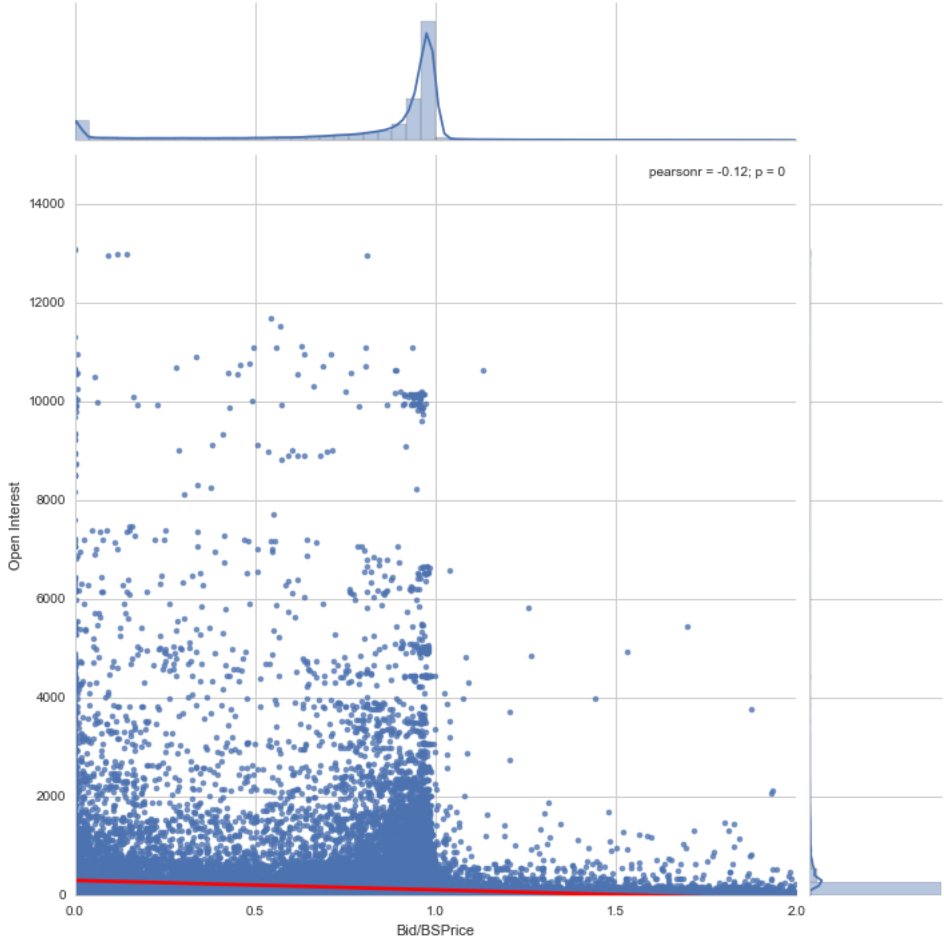


Figure 4.11: Bid Price/Black-Scholes Price vs Open Interest of NDX-100 Options in 2014

the quantities ordered, there is a certain level of rationality in choosing these quantities. We can compare the posted bid prices to Black-Scholes prices and observe how the open interest (i.e. outstanding volume) changes based on that. Figure 4.11 illustrates the scatter plot for the ratio of submitted bid prices and Black-Scholes prices versus the overall open interest in options. If we compare the quantities ordered in both undervalued ($ratio < 1$) and overvalued ($ratio > 1$) option cases, there is clearly more interest in options if the prices are below theoretical value. However we can also see that the quantities if the option is underpriced do not seem to have any linear relationship with the ratio itself. The difference is only between underpriced and overpriced groups. The red line in Figure 4.11 shows the linear regression between overpriced and underpriced bid quantities. We can see that it resembles the monotonically decreasing nature of the demand curve in classic economical perspective. Therefore agents can also use linear function to determine the quantity for the options to be traded.

Let $q_a(\cdot)$ and $q_b(\cdot)$ be supply and demand functions of the agent. These functions take option price p as an input, and return an integer indicating the quantity of the option to be traded. We model supply and demand with following assumptions:

1. Given the option value $\hat{p}$ (i.e. result of option pricing method), agent assumes that this price is the true price of the option, and hence it must be the equilibrium price of the option. This would imply $q_a(\hat{p}) = q_b(\hat{p})$.
2. $q_b(\cdot)$ can be found using linear regression of previously submitted bids and the outstanding volume as we did in Figure 4.11.
3. $p_a = 0 \Rightarrow q_a(p_a) = 0$. Agent has no incentive in supplying options at zero price, hence the supply is zero too.

Given $q_b(\cdot)$ demand function, we can find the analytical formula for $q_a(\cdot)$ supply function. Let $q_b(\hat{p}) = \hat{q}$, then from above assumption 1), we can see that $q_a(\hat{p}) = \hat{q}$. From assumption 3) we know that $q_a(0) = 0$. Having 2 points of the supply line we can derive its linear formula:

$$q_a(p) = \frac{\hat{q}}{\hat{p}} p \tag{4.28}$$

4.4.3 Option Portfolios

Option portfolios and their usage have been extensively covered in Chapter 2. Most of the option portfolios, except calendar option portfolios, are made up using options with different moneyness to cut the risk of infinite loss, and take advantage of agent's future forecast. Option portfolios are traded only in multi-item multi-unit DAs or combinatorial exchanges where traders submit their orders for bundles of options. For that purpose, I use traders specifying their orders in terms of option portfolios. Option portfolio can be represented as a quantity vector $\mathbf{q} \in \mathbb{Z}^m$ where m is the size of the option portfolio. Although the size of the option portfolio can be arbitrarily extended, most of the popular option portfolio sizes range from 2 to 4. They classify options based on two parameters: type (i.e. put/call) and moneyness (i.e. OTM, ATM, ITM). Hence there are 6 types of options used to build an option portfolio. Of course, the moneyness of the option can be further extended depending how close is the strike price to the current spot price. So there could be multiple

OTMs or ITMs available. Traders may include two OTMs with different strikes into option portfolio creating combinations such as Condor. However I assume there are only one OTM and one ITM defined to make an option portfolio. Hence the traders cannot construct Condor option portfolio from the available options.

Table 4.12 lists some of the option portfolios commonly used by traders. The quantity of option type to buy or sell is specified in positive or negative numbers respectively. The table also tells about the forecast direction of each option portfolio, so agents can choose based on their forecast. This is more applicable to ZI option pricing agents, because their option price depends on their forecasted asset price $S_{i,T}$ at time T. The strategies are classified as bullish, bearish and neutral based on the income they generate in case if asset price ends up in expected place. Bullish traders expect asset price to end up higher than its current spot price, while bearish expect the opposite. The neutral traders expect asset price remain in its current spot. For example if agent i's forecast is in between some $S_0 - \epsilon \leq S_{i,T} \leq S_0 + \epsilon$, then agent will choose *neutral strategy.* If $S_{i,T} > S_0 + \epsilon$, then agent will choose *bullish strategy.* And finally if $S_{i,T} < S_0 - \epsilon$, the agent will choose bearish strategy. ϵ denotes the fixed step size in strikes inside the option chain. Also it is worth noting that I regard option as ATM option if its strike price K_j is in ϵ vicinity of current asset price S_0. By definition of ATM option, its strike price must be equal to the current asset price, but because there are only discrete K_js in option chain, ATM option is the option whose strike is approximately the same as the spot price. Strike prices beyond $(S_0 - \epsilon, S_0 + \epsilon)$ are either considered OTM or ITM.

Traders pick random portfolio among other portfolios with the same direction. However some option portfolios can be both bullish and bearish, and Long Straddle option portfolio is one example of this. I assume that both bullish and bearish traders will be equally interested in this option portfolio. It is also possible that option portfolio is more bullish, than bearish, or vice versa. For example, Strip option portfolio generates greater payoff when prices go up. Therefore there is a biased chance for a bearish trader to choose Strip option portfolio among other bearish option portfolios because it is less bullish. Let us name the algorithm for selecting option portfolio as a $Strat(S_0, S_{i,T})$ function which returns $\mathbf{q}_i$ quantities vector for each agent i based on his forecast. *Strat* function is defined in listing below 1.

Name	c_{ATM}	p_{ATM}	c_{OTM}	p_{OTM}	c_{ITM}	p_{ITM}	Direction
Long Call	1	0	0	0	0	0	bullish
Long Put	0	1	0	0	0	0	bearish
Bull Call Spread	0	0	-1	0	1	0	bullish
Bear Call Spread	0	0	1	0	-1	0	bearish
Butterfly Put Spread	0	-2	0	1	0	1	neutral
Long Call Ladder	-1	0	-1	0	1	1	neutral
Short Call Ladder	1	0	1	0	-1	0	bullish > bearish
Long Put Ladder	0	-1	0	-1	0	1	neutral
Short Put Ladder	0	1	0	1	0	-1	bearish > bullish
Iron Butterfly	-1	-1	1	1	0	0	neutral
Long Straddle	1	1	0	0	0	0	bearish and bullish
Short Straddle	-1	-1	0	0	0	0	neutral
Long Strangle	0	0	1	1	0	0	bearish and bullish
Short Strangle	0	0	-1	-1	0	0	neutral
Strip	1	2	0	0	0	0	bullish > bearish
Strap	2	1	0	0	0	0	bearish > bullish

Table 4.12: Some Option Portfolios

Algorithm 1 Option Portfolio Selection

Require: $S_0, S_{i,T}, \epsilon$
  if $S_0 - \epsilon \leq S_{i,T} \leq S_0 + \epsilon$ **then**
    return random neutral option portfolio
  else if $S_{i,T} < S_0 - \epsilon$ **then**
    return random bearish option portfolio
  else if $S_{i,T} > S_0 + \epsilon$ **then**
    return random bullish option portfolio
  end if

4.5 Proxy Trading Algorithms

In this section, I present the ways how agents can employ proxy trading algorithms in online mechanisms. Proxy trading algorithms take evaluated option price $\hat{p}$ along with other specific parameters and generate adaptive bids and asks around $\hat{p}$. I use 2 types of agents: *dealers* and *traders*. Dealers are the agents that are passively involved in trading, as their main concern is market-making and servicing the traders who wish to buy or sell options. Therefore they are always forced to maintain both long and short open positions in online DA. Also they are not actively involved in improving their limit orders to achieve their fast execution, but rather interested in maintaining the liquidity of the market. It is important that they stay in business as long as they can without running into loss or failing to fill the orders. I propose 2 kinds of algorithms designed specifically for dealers: one is an Garman's inventory-based dealer who sets his bid-ask spread based on the probability of failure for given rate of order arrivals, and the other one is Copeland-Galai's information-based dealer who sets his bid-ask spread based on his belief on the proportion of informed traders in the market. We look at these two methods in Sections 4.5.1 and 4.5.1.

Traders, on the other hand, are more actively involved in trading process, because their main objective is to bargain a good price from the online mechanism and make a profit. So they have the freedom of entering and exiting the market when there is a profitable deal. As it was mentioned earlier, traders decide to enter the market when their option price is above/below the traded option price, and they decide to exit if they want to change their market position. Traders constantly update their orders according to the changes in the market to ensure that they adapt to the trend of the market. This would involve can-

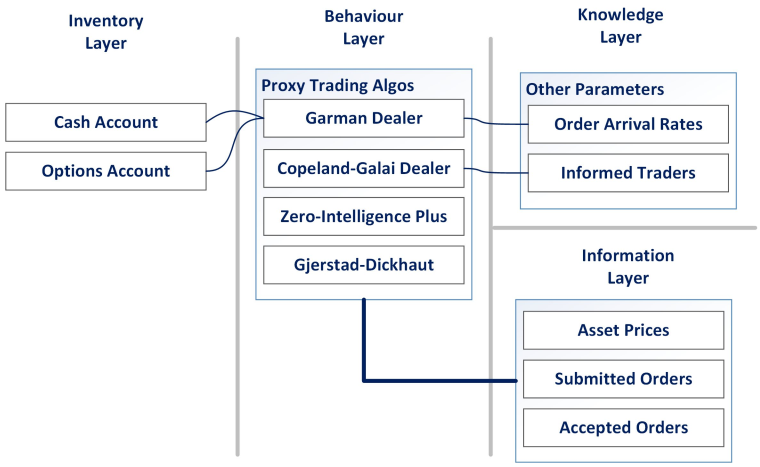


Figure 4.12: Components involved in proxy trading algorithms

celling the previously submitted orders every trading day and submitting the new ones with updated quotes. I provide 2 algorithms widely used in CDAs: one is ZIP which implements a recursive learning function to adjust agent's bids/asks to current market quotes, and the other is GD algorithm which finds an optimal bid-ask spread that maximises the probability of success. These topics are explained in more detail in Sections 4.5.2 and 4.5.2.

Figure 4.12 shows the relationship between IIKB layers that are used in order to run our proposed proxy trading algorithms. Dealer algorithm, Garman's model (i.e. inventory-based model), uses the inventory levels in assets and options, and order arrival rates and option prices to find the optimal bid-ask spread that could minimise its failure. Another dealer algorithm, Copeland and Galai model (i.e. information-based model) requires knowledge about informed traders in order to decide which bid-ask spread to use, so he can secure himself from definite loss of trading with informed traders. The bold line between proxy trading algorithms and pieces of information indicates that every algorithm uses either all or some of the information listed in given group.

Note that agents do not use any proxy algorithm for direct and combinatorial mechanisms because given that they are DSIC, the best way to submit bids and asks are submitting them truthfully to the mechanism. Therefore positive bid-ask spread is unnecessary, and usually the trader's bid and ask are equal to his private valuation.

4.5.1 Dealers

Studying the markets from the perspective of dealers and their private incentives have been pioneered by market microstructure researchers [125]. One of the main goals of this field is to model the price setting rules that evolve in markets based on the information available in the market. The market microstructure theory has an important application in the design and implementation of new trading algorithms and development of new market regulations. I extend some of its core methods to option trading while implementing dealer agents.

I addressed 2 types of dealers: a) inventory-based dealer and b) information-based dealer. Inventory-based models define trading agents with limited endowment of initial inventory and cash which they need to maintain or increase by engaging in trading process with other agents. Dealer's main objectives are not to run out of stock or cash, and at the same time to make a positive return [125, 147]. One of the famous examples of such models is Garman's one-period model [63] for setting bid and ask prices based on the probability of running out of cash or stock. In Garman's model the arrival of orders is modelled as a Poisson process with varying rate $\lambda(p)$ which is dependent on current price. Ask orders rate $\lambda_a(p_a)$ is monotonically decreasing function because the higher prices would ratchet up supply, and thus decrease interarrival time of ask orders. Conversely, bid orders rate $\lambda_b(p_b)$ is monotonically increasing function, as an increasing prices would result in lower demand, and thus longer interarrival time for bids. Garman in his model tries to minimise the probability of running out of cash or inventory, and at the same time maximise the potential profit through finding the optimal bid-ask spread. Another example of inventory-based model is Ho and Stoll's model which extends Stoll's initial idea [154] on optimising exponential utility function to maintain a risk-averse spread over some time horizon. Ho and Stoll extend Garman's model into multi-period level where the dealer's portfolio value can be modelled as a Brownian motion. Dealer's portfolio consists of cash account which subject to risk free interest return, and the inventory the value of which is computed using the sum of random short rate changes in stock. One difference between cash account and inventory position is that dealer uses bid and ask prices to account the cash outflow and inflow, and use book value of the stock for computing the value of total inventory he holds. Thus the dealers optimization problem can be formulated as the maximisation of the conditional expected utility for bid and ask prices. I highlight the details of the implementation of Garman's model for our inventory-based dealer later in this section.

The other category of dealers, information-based dealers, take into account the market data such as the orders flow, the population of other traders and any other information that could affect the behaviour of traders in the market. In contrast to inventory-based dealers, information-based dealers are assumed to have unlimited budget and stock, and thus they neglect anything but information while pricing the assets. In information-based paradigm, the traders that the dealer will be interacting with can be broken into *informed* and *uninformed* traders. Informed traders have some private information about the future of asset prices and they can exploit it by either buying or selling the option. Uninformed traders are simply liquidity traders who have other reasons such as hedging, speculating, etc to participate in the trade. Galai and Copeland demonstrated how the trader can behave while setting the bid/ask spread for anonymous traders. In this approach, trader simply tries to maximise his profit model which involves the loss from dealing with an informed trader who will ultimately outperform the market, and the gain that can be made from dealing with an uninformed trader [33]. In other words, Galai and Copeland suggested that dealer can balance the gains and losses through setting up a corresponding spread. Of course, dealer has no information about with whom he is dealing with, but may have a certain prior on the probability of informed and uninformed traders in the market. Trader uses these probabilities to calculate the corresponding regret-free spreads. On the other hand, another famous information-based model, Glosten-Milgrom model [71], utilises the Bayesian learning technique to adjust the new information to the pricing scheme. I provide the details of the implementation of Copeland-Galai's dealer later in this section.

Inventory-based Dealer

In a continuous market where orders are submitted subsequently, the equilibrium of supply and demand is unstable and some imbalances may emerge due to stochastic nature of order arrivals to the market. This temporal imbalance stresses the uncertainty of maintaining an inventory for the market-making agent, or dealer. Inventory-based dealer is mostly concerned about maintaining a sufficient stock of options and cash, so he does not run out them at some point in time. He can control the inflow of bids and asks by setting bid-ask spread in such a way that it maximises his profit and at the same time reduces the probability of failure. The inventory-based dealer can be modelled with following assumptions:

- Bid and ask orders for each option j arrive independently according to Poisson process with stationary intensity rates $\lambda_a^j(p_a^j)$ and $\lambda_b^j(p_b^j)$ respectively, where $\lambda_a^j(p_a^j)$ is a monotonically decreasing function representing the intensity of demand for option j directed at dealer, and $\lambda_b^j(p_b^j)$ is a monotonically increasing function representing the intensity of supply for option j towards dealer.
- Dealer cannot produce or borrow cash, options or assets. He is initially endowed with $I(0) = I^c(0) + I^a(0) + \sum_{j=2}^{k+2} I^j(0)$ worth inventory, where $I^c(0)$ stands for initial cash, $I^a(0)$ initial assets, $I^j(0)$ option of type j.
- Dealer is said to be broke or failed if he runs out of cash $I^c(t) = 0$ or any of the options he is trading with $I^j(t) = 0$ at some $t \geq 0$.
- Options' intrinsic values, asset price dynamics and all other market information except dealer's inventory levels and order arrival rates are disregarded.

Because the amounts of dealer's current cash and options determine whether dealer continues to play in the market or not, we can draw following formulas for dealer's cash account $I^c(t)$ and jth option account $I^j(t)$.

$$I^c(t) = I^c(0) + \sum_{j=2}^{k+2} \left(\int_0^t p_a^j dN_a^j(t) - \int_0^t p_b^j dN_b^j(t) \right) \tag{4.29}$$

$$I^j(t) = I^j(0) + \int_0^t dN_b^j(t) - \int_0^t dN_a^j(t) \tag{4.30}$$

where $dN_a^j(t) \sim Poisson(\lambda_a^j(p_a^j), dt)$ and $dN_b^j(t) \sim Poisson(\lambda_b^j(p_b^j), dt)$ are Poisson random variables such that $dN_a^j(t), dN_b^j(t) \in \{0, 1\}$ representing whether there is an order occurred for option j within an infinitesimal time interval dt or not.

Let us assume that the inventory dealer sets his bid-ask spread for one type of option. This also makes notation much more readable. Let this option type be of jth type. We drop superscript j from every variable, and work with dealer's bid price p_b and ask price p_a for this option j.

The dealer can control the width of his bid-ask spread to optimise his position.

If he choose very wide bid-ask spread, he risks loosing orders and affect his potential profit. If the bid-ask spread is very narrow, then the dealer can go broke or run out of stock due to imbalance between the arrival rates of asks and bids. In order to solve this issue, let $Q(t,k)$ denote the probability of having k units of cash at $I^c(t)$ at time t. Then Garman derives the agent's probability of running out of cash in an infinite time horizon as described below [63]:

$$\lim_{t\to\infty} Q(t,0) \approx \left(\frac{\lambda_b(p_b)p_b}{\lambda_a(p_a)p_a}\right)^{I^c(0)/\hat{p}} \qquad \text{if } \lambda_a(p_a)p_a > \lambda_b(p_b)p_b \tag{4.31}$$

$$\lim_{t\to\infty} Q(t,0) = 1 \quad \text{otherwise} \tag{4.32}$$

where $\hat{p}$ can be either the realised price of the option in the market, or dealer's private option valuation.

The similar relationship is defined for the options hold by a dealer where $R(t,k)$ defines the probability of holding k options at account $I^j(t)$ at time t for some single option j.

$$\lim_{t\to\infty} R(t,0) \approx \left(\frac{\lambda_a(p_a)}{\lambda_b(p_b)}\right)^{I^j(0)} \qquad \text{if } \lambda_b(p_b) > \lambda_a(p_a) \tag{4.33}$$

$$\lim_{t\to\infty} R(t,0) = 1 \quad \text{otherwise} \tag{4.34}$$

From above equations, we can draw an objective function for the dealer. The dealer needs to minimise the probability of either running out of cash or inventory. I assume that these two events may happen independently, so agent sums these two probabilities to make his objective function subject to corresponding constraints. It can be formulated as given below:

$$\min_{p_a,p_b} \quad \left[\left(\frac{\lambda_b(p_b)p_b}{\lambda_a(p_a)p_a}\right)^{I^c(0)/\hat{p}} + \left(\frac{\lambda_a(p_a)}{\lambda_b(p_b)}\right)^{I^j(0)}\right] \tag{4.35}$$

$$\text{s.t.} \quad \lambda_a(p_a)p_a - \lambda_b(p_b)p_b > 0 \tag{4.36}$$

$$\lambda_b(p_b) - \lambda_a(p_a) \geq 0 \tag{4.37}$$

$$p_a, p_b, \lambda_a(p_a), \lambda_b(p_b) > 0 \tag{4.38}$$

In the first term of the objective function, we can see that dealer is better off bidding less because both $lambda_b(p_b)$ and p_b are monotonically increasing and they are in the numerator of the fraction. From the second term of the objective function, we can learn that the dealer is better of raising his ask

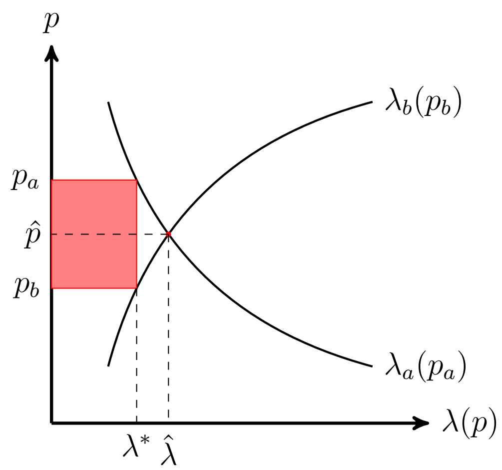


Figure 4.13: Optimal bid-ask spread for Garman's dealer

price, because $\lambda_a(p_a)$ is a monotonically decreasing function. But this must correspond with large enough bid price p_b. The objective function subject to the constrains drawn from (4.31) and (4.33) equations. However we changed the constraint (4.37), because we assume there could be an equilibrium state where there is λ^* such that demand and supply volumes are approximately the same, so their rate of arrivals is equal to single value.

The relationship between ask and bid prices and their corresponding arrival rates are illustrated using a chart similar to supply and demand curves from microeconomics. The dealer is better off maximising the surplus generated from the bid-ask spread he holds. Figure 4.13 shows this surplus highlighted using a shaded rectangle.

In order to simulate proposed algorithm, let us set arrival rates them to some functions, although it is going to be different for each mechanism. Let us assume that the rates of arrival are linear functions ensuring that $\lambda_a(\cdot)$ has negative slope, and $\lambda_b(\cdot)$ has positive slope. When arrival rates are equal, the inventory of the dealer is in its stable state, so the incoming orders balance each other. Dealer assumes that the rates of the arrival balance only if the bid and ask prices are close to $\hat{p}$. Hence the arrival rates should satisfy $\lambda_a(\hat{p}) = \lambda_b(\hat{p})$. I denote this arrival rate as $\hat{\lambda}$. We also know that there won't be any orders coming if dealer's bid is zero. This is because in Section 4.4.2 I have made an assumption that no one is willing to supply option at zero price. Hence $\lambda_b(0) = 0$. Also the last assumption about the arrival rates is that they change at the same rate but in opposite directions. This would mean that the slope of $\lambda_b(\cdot)$ is the negative of the slope of $\lambda_a(\cdot)$.

Using above assumptions, we can construct linear functions to estimate the optimal bid-ask spread that would minimise dealer's failure probability based on our made-up order arrival models. In case if the probabilities are the same for the range of bid-ask spreads, the dealer would choose the narrowest of them, so he can provide enough liquidity to the market. Below steps summarise the approach taken by our inventory-based trader to set a bid-ask spread around given option price taking into account his inventory levels:

1. Given $\hat{p}$ and $\hat{\lambda}$, agent can find analytical form of $\lambda_b(\cdot)$ using 2 data points: $(\hat{p}, \hat{\lambda})$ and $(0, 0)$. Hence, given:

$$\lambda_b(p_b) = \frac{\hat{\lambda}}{\hat{p}} p_b \tag{4.39}$$

2. Given that arrival rates are symmetrical in opposite directions, agent linear transform the $\lambda_b(\cdot)$ to $\lambda_a(\cdot)$:

$$\lambda_a(p_a) = \hat{\lambda} - \frac{\hat{\lambda}}{\hat{p}} p_a \tag{4.40}$$

We can see that above defined arrival functions also intersect at $(\hat{p}, \hat{\lambda})$ giving the inventory-based dealer the most desired position. Using above defined linear functions, agent can solve the optimisation problem given in equation (4.35) and find ask p_a and bid p_b prices that minimise the probability of failure. It can be seen from the objective function of the optimisation problem that the probabilities depend on the inventories $I^c(0)$ cash and $I^j(0)$ option j of the trader. The higher are the resources of the dealer, the less probable is the failure. This means that the dealer's bid-ask spread may change over the course of the trading day in an online mechanism as he acquires or sells options. In case if the minimised probabilities are almost the same with tolerable absolute error, the dealer tends to select the bid and ask prices with the narrowest possible spread. This facilitates the trade between other traders and improves the liquidity in the market.

In Figure 4.14, I have simulated the possible cases for Garman's dealer with different inventory levels. The vertical axis represents bids, while the horizontal one is for ask prices. I assume that the option price given to the dealer is $\hat{p} = 5.0$ with $\hat{\lambda} = 4$ orders arrived at this price. I also used above method to compute corresponding linear functions for $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$, and then plotted the color

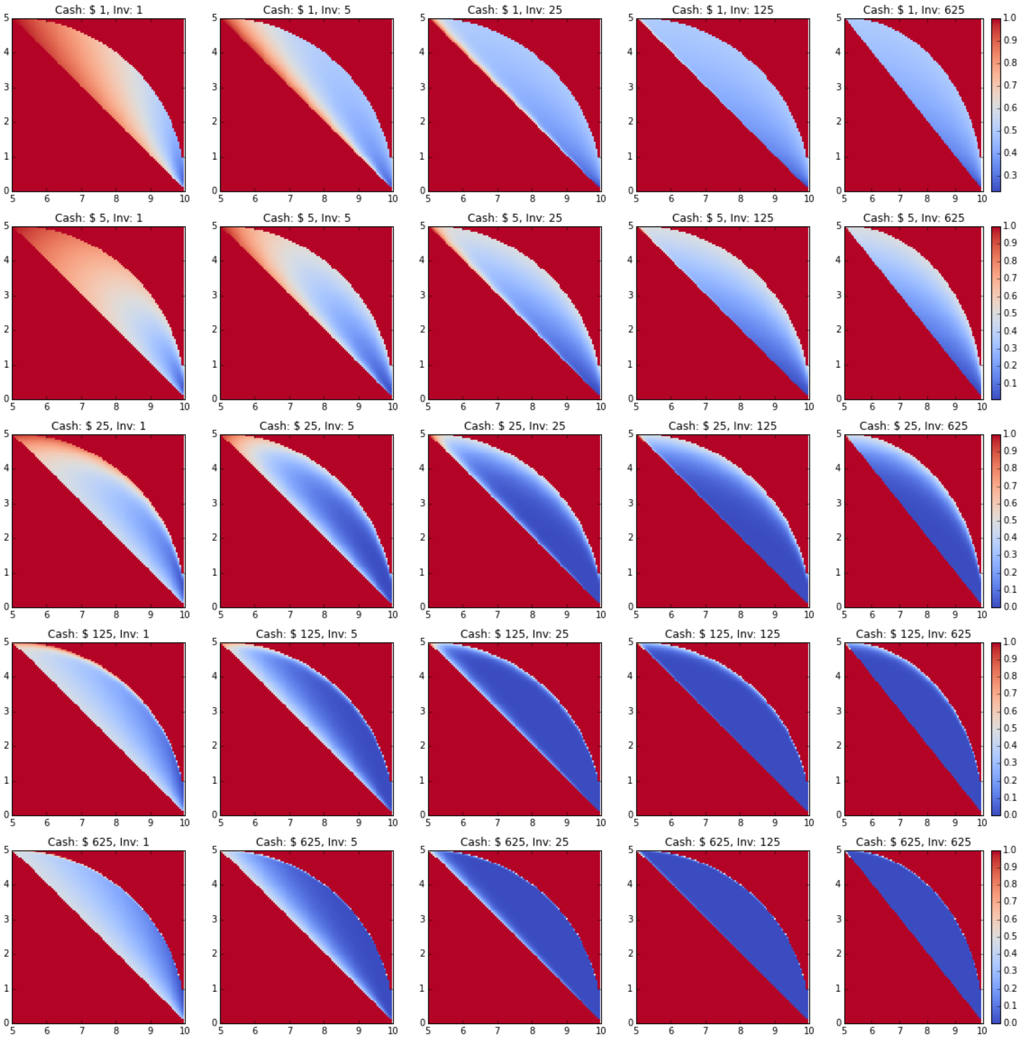


Figure 4.14: Dealer's bid-ask choices based on probability of failure

map indicating the probabilities of failure for the different combinations of bid-ask spread. The red area indicates that the dealer will fail almost surely, because it violates Garman's constraints (4.36) and (4.37). The bluer the area, the less is the probability of failure. For example, when dealer has only \$1.0 and only one 1 option in his inventory, he is almost destined to fail unless he bids near zero price, and asks near \$ 10 price, stretching his bid-ask spread to the maximum level for given linear $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$. However for the dealer having \$625.0 in cash and 625 in units of option a very thin bid-ask spread such as (\$4.95, \$5.05) is rather unlikely to result in failure of the dealer.

To sum up, I have used linear functions for $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$, but in reality these functions can be different, and not necessarily continuous and well-defined. This requires thorough calibration of arrival rates into more complex models which may include stochastic elements as well depending the auction setting. I will not go beyond the complexity of linear functions to model the arrival rates in online DAs, and determine the regression line between order arrivals and submitted bid-ask spreads from simulated mechanism. Also I have presented a dealer who trades only one type of option, but this is a unrealistic example. Dealers in the market can trade with every kind of option listed in the option chain and has to use income generated from all of these options as his disposable cash. In given example, the cash can only be generated through the trade of only one option. Moreover in real market situation, dealers can generate cash from other activities such as borrowing or short selling assets. Also they can produce options at their own risk and price them corresponding to their own option valuation method. However in our presented Garman's model these capabilities of the dealer are restricted. I use this proxy trading algorithm for simulating dealers upon option pricing methods discussed earlier, and use `GAR-` prefix to denote this method.

Information-based Dealer

In this section, I present another type of dealer who uses information about the market rather than his available resources. In fact, information-based dealers usually disregard the responsibilities of the inventory-based model such as maintaining stock to serve the incoming orders. Information-based dealers are assumed to be capable of producing infinite amount of options to satisfy accepted asks, and borrow infinite amount of cash to realise accepted bids. The key aspect of the information-based dealer is the way how he balances the inevitable loss caused by informed traders, or so called *insiders*, by setting up a corresponding bid-ask spread. In Section 4.2, I have already reviewed how agent can find out implicit information such as the proportion of the informed traders in the market from the incoming orders and other public information using Bayesian learning technique.

I use Copeland and Galai's [33, 125] model to implement an information-based dealers. In this model, the dealer assumes that there are certain number of speculators in the market who have an insider information which enables them to estimate the future more accurately. This information is unknown to other dealers and liquidity traders (i.e. uninformed traders), so they cannot take advantage from it. However the informed trader possessing the private information can monetize it by engaging in trade with other traders in the market. This is the main reason why the information-based dealer has to be cautious about informed traders in the market, and should be reluctant to trade with them because ultimately they are going to beat the dealer. However the orders are anonymous in the market, and the dealer cannot recognise the orders coming from an informed trader. The dealer's objective is to find optimal bid-ask spread that would maximise his profits. Similar to inventory-based dealer, if he sets too wide bid-ask spread he might loose liquidity traders from whom he can make profit while shielding himself from informed traders. The optimal bid-ask spread must attract enough liquidity traders that generate profit to compensate the loss caused by informed traders. Note that the traders are anonymous in both cases, so the dealer has no way to distinguish the informed traders from liquidity ones.

The dealer sets bid-ask spread for limited quantity of options to secure himself from informed traders. Also the dealer can revise his prices after fulfilling the order. This is because dealer can update his posterior belief about the true valuation of the item from the new orders arrived having a prior belief on the proportion of informed traders in the market. So we can say that each new order coming into the market reveals the private information about the option's true price.

As we have already discussed in Section 4.2.2 of this chapter that trader can incorporate newly arrived information into his prior belief using Bayesian inference. I showed how dealer can find out the posterior distribution of θ by simply plugging the variables into Beta posterior distribution given in (4.11). He can then estimate the value $\hat{\theta} = \frac{\mathbb{E}[X'|X=x]}{N'}$ by using Beta-Binomial distribution given in (4.13). I have also mentioned how dealer distinguishes the informed traders from uninformed traders. Dealer takes orders anonymously from traders, but once the actual price of the asset is realised, he can look back to the orders he accepted and identify those who correctly 'guessed' the true price of the option. I stated that this is a very naive approach, and it has a bias from other liquidity traders who accidentally submitted the correct prices. However in order to keep this part simply, I will use this way of identify the orders for information-based dealer. In long run, this approach is going to give us a good estimate on which portion of total orders accepted are going to hit the expected asset price, leaving the real population of informed and uninformed irrelevant for the dealer.

Once the dealer has $\hat{\theta}$, he can use this probability to compute his expected payoff. According to Copeland and Galai's approach, the expected loss from accepting informed orders can be computed using following formula:

$$\pi_I = \int_{p_a}^{\infty} (p - p_a) f(p) dp + \int_{0}^{p_b} (p_b - p) f(p) dp \tag{4.41}$$

where $f(p)$ is the PDF of option prices.

In equation (4.41), we can see that the dealer the expected loss from informed orders involving the sales at price p_a for all possible cases above it, and buying at price p_b for all possible cases below it. This is because dealer thinks that the loss is certain if he accepts an order from informed trader. The main reason for this is that if the trader is informed and he is engaging in trader with the dealer, the informed trader certainly knows that the option price is either underpriced or overpriced, so it is beyond the dealer's bid-ask spread. Therefore the rational informed trader would participate in trader with the dealer only if it is attractive to him.

The most important aspect of the equation (4.41) is the PDF for the option price. Initially this has been defined for the asset prices by Copeland and Galai, but in this research, I re-implement it for options. If p were asset prices, we could use log-normal distribution $\log \mathcal{N}(r, \sigma\sqrt{T})$ for $f(p)$, so it could specify the probability of asset prices at time T with risk-neutral measure r and volatility σ. In this case, we can see that term $\int_{p_a}^{\infty}(p - p_a)f(p)dp$ replicates the expected payoff of European call option with strike p_a, and $\int_0^{p_b}(p_b - p)f(p)dp$ is European put option with strike p_b. Together the expected loss of the dealer from informed traders would be equal to the sum of 2 option prices: one call at p_a and one put at p_b. This also resembles *Long Straddle* option portfolio discussed in Chapter 2, although it requires both call and put to have the same strike price. Below formula shows how the cost of dealer from informed traders can be computed using the risk-neutral price of the long straddle:

$$\pi_I = C(T_1, p_a) + P(T_1, p_b) \tag{4.42}$$

where $C(T_1, p_a)$ is Black-Scholes call price with strike p_a and $P(T_1, p_b)$ is Black-Scholes put price with strike p_b.

However in our current case, we are not dealing with asset prices, but with option prices. Therefore the informed trader evaluating the option prices, must virtually create another type of option which takes as an underlying European option contract itself. This type of option is commonly referred as *compound options*, and its payoff involves the value of another option at the time of expiration. Hence it has 2 expiration times, one for the compound option itself T_1, to enable its holder to buy or sell European option at indicated price, and the expiration time of the underlying option T_2, for which its holder can buy or sell an asset. It is clear that the expiration time of the compound option T_1 must come first than the expiration time of underlying option T_2, so $T_1 < T_2$. Similarly, there are 2 strike prices for compound option, K_1 is the exercise price at which the underlying option is traded at maturity, and K_2 is the strike price of an underlying option. In this way, at expiration time T_1 trader decides whether to exercise the compound option, and if it is exercised, the trader will have the right to buy, say, call option at strike K_2 for K_1 dollars. This describes '*Call on Call*' type of compound option. In total, we can have 4 variations of such compound options, define their intrinsic values as follows:

1. *Call on Call* - $CoC(T, K)$: Right to buy call option $C@(T_2, K_2)$ at time T_1 at price K_1. The bounds of the call on call option are defined as:

$$\max\{C(K_2, T_1) - K_1, 0\} \leq CoC(T_1, K_1) \leq C(K_2, T_1) \tag{4.43}$$

2. *Call on Put* - $CoP(T, K)$: Right to buy put option $P@(T_2, K_2)$ at time T_1 at price K_2. The bounds of the call on call option are defined as:

$$\max\{P(K_2, T_1) - K_1, 0\} \leq CoP(T_1, K_1) \leq P(K_2, T_1) \tag{4.44}$$

3. *Put on Call* - $PoC(T, K)$: Right to sell call option $C@(T_2, K_2)$ at time

T_1 for price K_2. The bounds of the call on call option are defined as:

$$\max\{K_1 - C(K_2, T_1), 0\} \leq PoC(T_1, K_1) \leq C(K_2, T_1) \tag{4.45}$$

4. *Put on Put* - $PoP(T, K)$: Right to sell put option $P@(T_2, K_2)$ at time T_1 for price K_2. The bounds of the call on call option are defined as:

$$\max\{K_1 - P(K_2, T_1), 0\} \leq PoP(T_1, K_1) \leq P(K_2, T_1) \tag{4.46}$$

where $C(T, K)$ denote the market price of European call option with strike K at time T, and we use $P(T, K)$ for put option.

Geske has derived the risk-neutral valuation of compound options using the Black-Scholes framework [66, 87]. All 4 variations of compound options have closed form solutions that return the risk-neutral price for given set of parameters T_1, T_2, K_1, K_2 and the rest of the market parameters such as risk-free rate r, asset spot price S_0, asset price volatility σ, etc. Below are these solutions, as they were written in Geske's paper [66].

1. *Call on Call*: Risk-neutral price for Call on Call option is:

$$\begin{aligned} CoC(T_1, K_1) =& S_0 M\left(a_1, b_1, \sqrt{\frac{T_1}{T_2}}\right) \\ &- K_2 e^{-rT_2} M\left(a_2, b_2, \sqrt{\frac{T_1}{T_2}}\right) - e^{-rT_1} K_1 N(a_2) \end{aligned} \tag{4.47}$$

2. *Call on Put*: Risk-neutral price for Call on Put option is:

$$\begin{aligned} CoP(T_1, K_1) =& K_2 e^{-rT_2} M\left(-a_2, -b_2, \sqrt{\frac{T_1}{T_2}}\right) \\ &- S_0 M\left(-a_1, -b_1, \sqrt{\frac{T_1}{T_2}}\right) - e^{-rT_1} K_1 N(-a_2) \end{aligned} \tag{4.48}$$

3. *Put on Call*: Risk-neutral price for Put on Call option is:

$$\begin{aligned} PoC(T_1, K_1) =& K_2 e^{-rT_2} M\left(-a_2, b_2, -\sqrt{\frac{T_1}{T_2}}\right) \\ &- S_0 M\left(-a_1, b_1, -\sqrt{\frac{T_1}{T_2}}\right) + e^{-rT_1} K_1 N(-a_2) \end{aligned} \tag{4.49}$$

4. *Put on Put*: Risk-neutral price for Put on Put option is:

$$PoP(T_1, K_1) = S_0 M\left(a_1, -b_1, -\sqrt{\frac{T_1}{T_2}}\right) \tag{4.50}$$

$$- K_2 e^{-rT_2} M\left(a_2, -b_2, -\sqrt{\frac{T_1}{T_2}}\right) + e^{-rT_1} K_1 N(-a_2) \tag{4.51}$$

where

$$a_1 = \frac{\ln\left(\frac{S_0}{S^*_{T_1}}\right) + \left(r + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}}$$

$$a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{\ln\left(\frac{S_0}{K_2}\right) + \left(r + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}}$$

$$b_2 = b_1 - \sigma\sqrt{T_1}$$

T_1 and K_1 are the expiration time and strike price of compound option

T_2 and K_2 are the expiration time and strike price of underlying European option

S_0 is the stock price at the time when compound option is issued

$S^*_{T_1}$ is the root asset price which makes the Black-Schole's value of the underlying option with parameters (T_2, K_2) equal to K_1 at time T_1.

$M(a, b, \rho)$ is standard normal bivariate cumulative distribution for values a and b, and correlation coefficient ρ.

$N(a)$ is standard normal cumulative distribution for value a.

Hence the total loss from informed traders (4.41) is equal to the cost of straddle strategy with call and put options, if the asset being traded is an option itself the value of which is derived from the asset price behaviour. Therefore we can go one step further, and use call option, without loss of generality, as an underlying, to construct straddle strategy using compound options. In case of call option $C@(T_2, K_2)$, our straddle strategy would look as follows:

$$\pi^c_I = CoC(T_1, p_a) + PoC(T_1, p_b) \tag{4.52}$$

where p_a is the dealer's ask price for call option $C@(T_2, K_2)$ and p_b is the dealer's bid price for the same option.

And for put option $P@(T_2, K_2)$, the straddle with compound options:

$$\pi_I^p = CoP(T_1, p_a) + PoP(T_1, p_b) \tag{4.53}$$

where p_a is the dealer's ask price for put option $P@(T_2, K_2)$ and p_b is the dealer's bid price for the same option.

Using formulas (4.52) and (4.53), we can easily compute the expected loss of the dealer from informed traders for given bid-ask spread on either call or put option. The rational bidder has to find such bid-ask spread which would minimise his expected loss, while at the same time increase his profit from liquidity traders. Let us assume that the dealer wants to know the expected loss from the informed traders for trading a European call $C@(0.5, \$105)$ (i.e with time to maturity 6 months, and strike \$105). Also let us assume that the dealer finds out the true value of the option only at the maturity of the option, and not before. In this way, he can clearly know who was the informed trader and how much loss he made by selling lower or buying higher than given true price. For sell-side it can be equivalent to the cost of 'Call on Call' price and for the buy side it is 'Put on Call' price. Note that in both cases $T_1 = T_2$, meaning that both compound options are exercisable only at the maturity date of the underlying option. So T_1 is the time when the dealer finds out the true value of the option, T_2 is the underlying option's expiry date, and they are both equal. So in order to compute the expected loss from both selling and buying options, dealer uses the (4.52) formula and computes $CoC(0.5, p_a) + PoC(0.5, p_b)$ for different values of p_a and p_b. I have plotted this in a colour map shown in Figure 4.15:

One cautionary note for the loss in case $p_a < p_b$ is that it is equal to infinity for obvious reasons, although in Figure 4.15 colorbar it looks as if that it is equal to \$10. We can also see in Figure 4.15 that the dealer's loss is minimised when his bid-ask spread is wide, while the loss is almost approximately \$8.70 if the bid-ask spread is zero at price \$10. The dealer needs to minimise his loss by maintaining wider bid-ask spread as much as possible without loosing more profit from liquidity traders.

Information-based dealer uses option pricing technique to evaluate the expected loss from informed traders in underlying market with given parameters, not the value of the option being traded. Figure 4.15 illustrates the expected loss for arbitrary bid and ask prices p_a and p_b. The algorithm chooses such

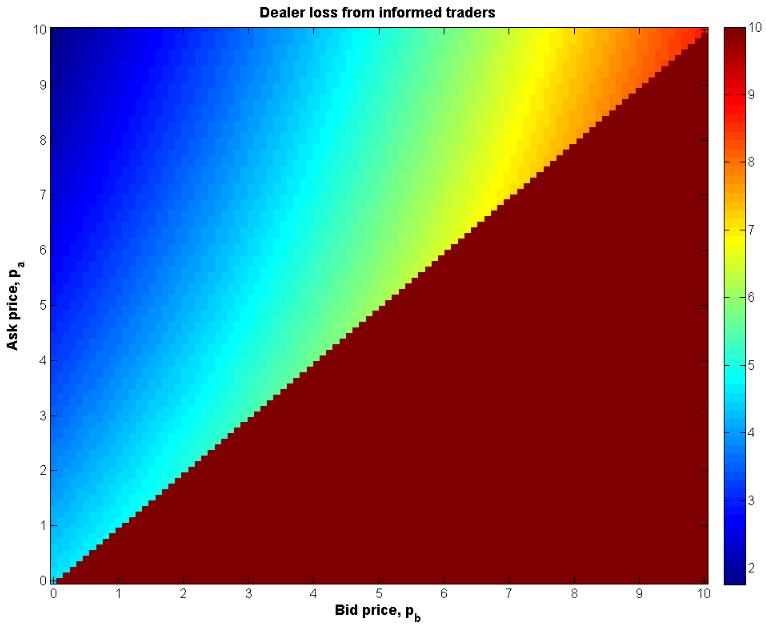


Figure 4.15: Information-based dealer's expected loss from informed traders for given bid and ask prices

p_a and p_b around computed option price $\hat{p}$ which minimises the expected loss. But we can see from the Figure 4.15 that maximum p_a and minimum p_b has the lowest expected loss. This would mean that the dealer will simply maintain the widest possible bid-ask spread in order to avert any other trader from trading with him.

In contrast to the cost incurred by informed traders, the dealer needs to make profit too. Otherwise his bid-ask spread is always set to the maximum breadth. So we can assume that the dealer makes profit by trading options with other uninformed traders. The gain from trading with the liquidity traders can be computed given the prior knowledge about the probability distribution of liquidity trader's actions. Liquidity traders can engage in three types of actions: 1) buying φ_B, 2) selling φ_A or 3) staying silent φ_S, while the probability for the informed trader's action is always 1 on either of the actions. This is because informed traders are rational, and always take action which is most beneficial to them. Liquidity trader on the other hand, due to the lack of the information cannot exhibit such rationality. Dealer can find out the PMF of these actions by observing the orders coming from uninformed traders. For example, dealer knows how many traders are involved in the market, he can find out if the order was informed/uninformed after the asset price is realised at time T. He, of course, knows how many of them were bids, and how many were asks. In such way, he can find the PMF from given sample of historic orders: $\varphi_B = \frac{N_B}{N}$, $\varphi_A = \frac{N_A}{N}$ and $\varphi_S = \frac{N_S}{N}$.

Similar to inventory-based dealer, information-based dealer delegates the option pricing to respective module which decides the true value of the option for himself. It can be Black-Scholes price or any other pricing method we discussed in Section 4.3. Let this price be $\hat{p}$. Then the dealer can determine his expected profit from filling uninformed orders:

$$\pi_U = \varphi_B(p_a - \hat{p}) + \varphi_A(\hat{p} - p_b) + \varphi_S * 0 \tag{4.54}$$

Note that in equation (4.54), the last term means that the dealer does not make any profit from silent traders, therefore it is equal to zero. We can combine equations (4.54) and either (4.52) or (4.53) depending on what is being traded to one equation which denotes the total expected payoff of information-based dealer. It is given below:

$$\pi = (1 - \theta)\pi_U - \theta\pi_I \tag{4.55}$$

I have simulated several occasions of dealer setting bid-ask spreads based on different beliefs and factors to show what are the important aspects that affect the width of the spread. In particular, I set the proportion of silent liquidity traders to $\varphi_S = 0$, and ranged $\varphi_B \in [0.9, 0.07]$. Obviously, we can figure out by $\varphi_A = 1 - \varphi_B$. I also changed the ratio of informed traders θ from 0.9 to 0.07, and observed how it may affect dealer's payoff if he sets different bid-ask spreads. The dealer is given with $\hat{p} = 5$ (i.e. evaluated value of the option). Figure 4.16 illustrates how θ and φ_B may affect the dealer's expected payoff.

In Figure 4.16, we can see that the payoff of the dealer significantly depends on θ. When 90% of the traders are informed, the dealer cannot compensate his loss even if he sets a spread $p_a = 10$ and $p_b = 0$ for the option worth \$5. This tells us that there is an informed traders who knows the fact that the asset price is going to be above \$115 at maturity, and it is very likely that the dealer will encounter this informed trader and sell call $C@(0.6, 105)$ at cheaper \$10 price. In such case, the dealer's expected payoff is negative meaning he will loose about \$1 per deal. The loss gets even bigger, once dealer narrows his bid-ask spread. However, once the ratio of informed traders is below 50%, there is chance that the dealer can make profit. According to Figure 4.16, the

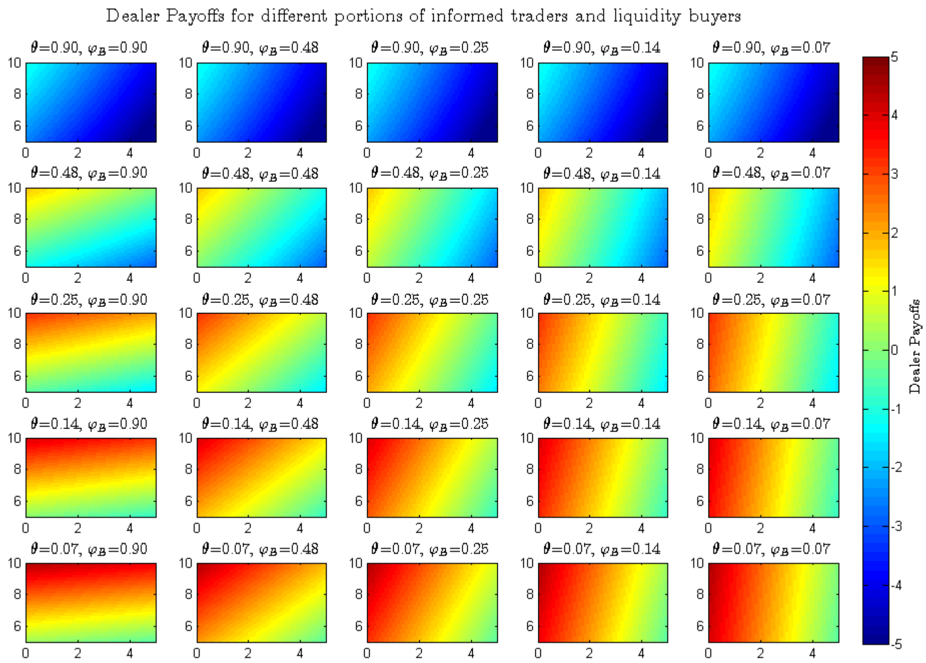


Figure 4.16: Information-based dealer's expected payoff for given bid-ask spread

optimal ratio of informed traders for the dealer to break even is somewhere around 0.48.

Once dealer starts to make profit, φ_B becomes an important factor. The bottom line of color subplots in Figure 4.16 shows that the change in ask price p_a can significantly affect the total expected payoff when there are many liquidity buyers $\varphi_B = 0.9$. It can be also noticed that the change in bid price p_b is less important for dealer's profit when there are many buyers. It is different when the proportion of liquidity buyers is less than sellers, and $\varphi_B = 0.07$. We can see now that the payoff is mostly affected by the change in bid price p_b, and the dealer is less sensitive to the changes in ask price p_a. Hence, the probability of liquidity trader's action is key parameter in optimising the dealer's payoff.

From Figure 4.16, we can also conclude that the best bid-ask spread in all cases is the widest one, which is (0,10). However this is not realistic, because the volume of orders depends directly on the prices set by the dealer. If the dealer sets too wide bid-ask spread, he might loose the volume from liquidity traders. The volume from informed traders is assumed to be the same as from liquidity traders, because by setting higher volume, informed traders might reveal themselves and can significantly impact the prices. Therefore it is commonly assumed that the informed traders conceal their orders as if they are coming from liquidity traders, and hence set the same average quantity as the liquidity traders.

Dealer can model the demand and supply functions depending on the conditions of the market, however for the simplicity sake I will use linear function as I did for inventory-based dealer in Section 4.5.1. The method that I accepted for modelling the linear supply and demand functions is equivalent to arrival rate functions in inventory-based trader case. So for this experiment, I use $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$ for depicting the order arrivals to buy and sell to the dealer respectively. We can incorporate these functions with the expected payoff function (4.55), and make one consolidated optimisation problem for the information-based dealer.

$$\max_{p_a, p_b} \pi = [(1-\theta)\boldsymbol{\pi}_U - \theta\boldsymbol{\pi}_I]^\intercal \boldsymbol{\lambda} \tag{4.56}$$

where

$\boldsymbol{\lambda} = (\lambda_a(p_a), \lambda_b(p_b))$

$\boldsymbol{\pi}_U = (\varphi_B(p_a - \hat{p}), \varphi_A(\hat{p} - p_b))$

$\boldsymbol{\pi}_I \in \{(CoC(T_1, p_a), PoC(T_1, p_b)), (CoP(T_1, p_a), PoP(T_1, p_b))\}$

$\lambda_a(p_a)$ arrival of buy orders at dealer's ask price p_a.

$\lambda_b(p_b)$ arrival of sell orders at dealer's bid price p_b.

I have simulated the objection function given in (4.56) to see how the arrival of orders affect the dealer's expected payoff. We used the same parameters for the linear arrival functions $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$ that were defined for the inventory-based dealer. So the dealer assumes that arrival functions intersect on $\hat{p}$ and the equilibrium volume is 4. Second, the dealer assumes that no one sells any option at zero price, hence making the supply equal to zero. And third, the demand function $\lambda_a(\cdot)$ takes the negated slope of the supply function $\lambda_b(\cdot)$. Figure 4.17 illustrates the results.

Solving (4.56) would give us the optimal bid-ask spread for given option, and allow the Copeland-Galai dealer to set optimal bid-ask spread based on his belief in the proportion of informed/uninformed traders, probability of buy/sell/silent actions of uninformed traders and the actual given option price $\hat{p}$ and dealer's model on supply and demand functions. From Figure 4.17 it can be seen that such optimum exists for any combination of informed traders and

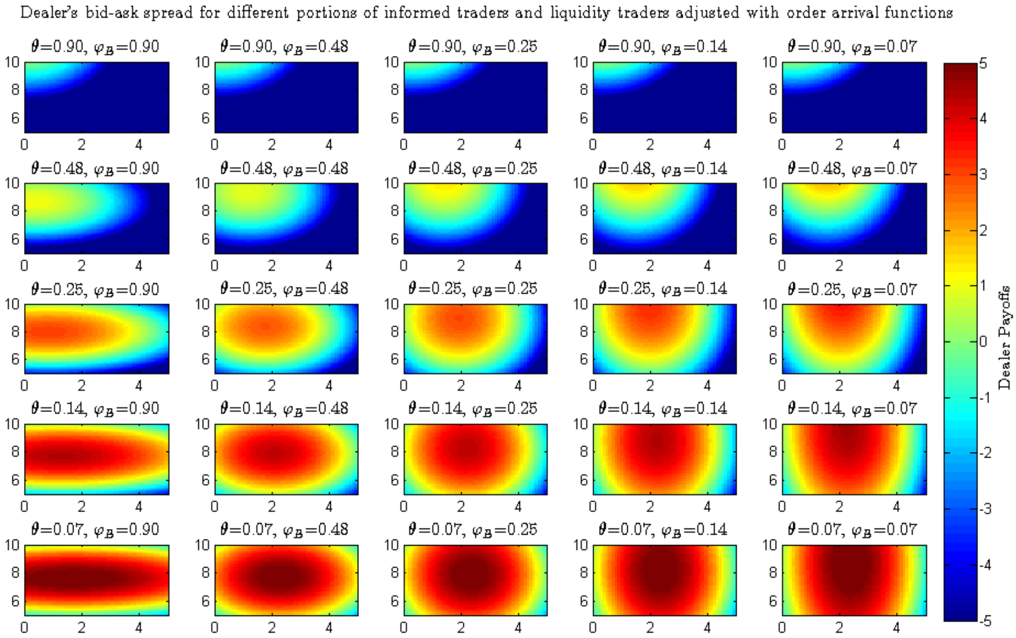


Figure 4.17: Information-based dealer's expected payoff for given bid-ask spread involving the order arrival functions

buy/sell behaviour of liquidity traders. Moreover we can also notice that this optimum is uniquely determined for linear arrival functions: $\lambda_a(\cdot)$ and $\lambda_b(\cdot)$ which strictly decrease and increase respectively.

In Figure 4.17, it can be noticed that the population of informed traders can significantly affect the expected income of the dealer as it was shown in Figure 4.16 too. It also stresses that sticking to the maximum bid-ask spread is the optimal strategy for the dealer in case there are more than 90% informed traders in the market. However the optimal point shifts downwards as the number of informed traders decreases, making the optimal bid-ask spread more narrow. For example, in case if there are only 7% of informed traders in the market, the dealer is better off with the bid-ask spread of (2,8) for estimated option price $\hat{p} = 5$. On the other hand, the liquidity trader's buying behaviour can skew the dealer's expected payoff range horizontally giving the dealer wider range of ask prices maintaining approximately the same payoff. Similarly, the lean on the seller side would increase the range for the potential bid prices for the dealer.

To conclude, I use information-based dealer which takes Copeland and Galai's algorithm as a base for trading options as one of the potential market participants in simulated option market environment because it adapts its bid-ask range based on the beliefs in the population of informed traders, liquidity traders' behaviour and order arrival functions. This is a reasonable model which allows dealer's to minimise their potential losses and gain some profit. I use `COP-` prefix to denote this proxy trading algorithm.

4.5.2 Traders

In this section, I employ proxy trading algorithms that continuously adapt to the changes in the market and can be used by traders to update their orders. These trading methods are mostly used in simulating CDAs due to their simplicity in implementation, computational tractability and effectiveness in trading performance.

Zero-Intelligence Plus Trader

One famous example of such algorithm is ZIP developed by Cliff and Bruten in 1997 [29]. Gode and Sander tested random-like Zero Intelligence (ZI) trading agent which does not take into account any conditions in the market and does not follow any profit while making offers [74]. He also introduced budget constraints for ZI trader (i.e. ZIC trader) which prevented the trader from incurring into loss. ZIC traders randomly pick prices to quote within the range of minimum allowed bid in the CDA and agents private limit price. Thus ZIC agents have profit margin determined by the difference between maximum (if buyer) or minimum (if seller) price the agents can afford and the current market price. If this difference turns negative agent does not participate in any trade. Cliff, on the other hand, improved this kind of trader with little more intelligence toward predicting the future prices. It uses previous prices and its past experience to forecast the future price. Based on this future price, ZIP algorithm adapts its internal profit margin to current market trend. Whenever ZIP buyer receives an offer which is lower than its last order, it will decrease its profit margin. For ZIP seller if the offer it receives exceeds the price that he submitted it increases its profit margin. On the other hand, ZIP traders will also adapt their quotes if the market is highly competitive and their orders are continuously unsuccessful.

We talked about ZI traders in previous section, and these type of traders which are normally regarded as noise traders, mostly engage in speculation with the market. And of course one of such speculation techniques can be described using ZIP algorithm. According to this algorithm, trader starts the bidding process with some arbitrarily low profit margin which he later increases or decreases based on the events happening in the market. These events include the successful execution of bids and asks submitted to the market. Let us denote the last submitted order as $s(t)$, trader's current bid $p_b(t)$, and current ask $p_a(t)$. Then we can describe the ZIP algorithms for generating asks and bids as follows in Algorithms 2 and 3:

Algorithm 2 ZIP Generation of Asks

Require: $s(t)$
  if $s(t)$ is accepted **then**
    if $p_a(t) \leq s(t)$ **then**
      $p_a(t+1) = increaseAsk(s(t))$
    end if
    if $s(t)$ is bid **and** $p_a(t) \geq s(t)$ **then**
      $p_a(t+1) = decreaseAsk(s(t))$
    end if
  else if $s(t)$ is ask **and** $p_a(t) \geq s(t)$ **then**
    $p_a(t+1) = decreaseAsk(s(t))$
  end if

Algorithm 3 ZIP Generation of Bids

Require: $s(t)$
  if $s(t)$ is accepted **then**
    if $p_b(t) \geq s(t)$ **then**
      $p_b(t+1) = decreaseBid(s(t))$
    end if
    if $s(t)$ is ask **and** $p_b(t) \leq s(t)$ **then**
      $p_b(t+1) = increaseBid(s(t))$
    end if
  else if $s(t)$ is bid **and** $p_b(t) \leq s(t)$ **then**
    $p_b(t+1) = increaseBid(s(t))$
  end if

For decreasing and increasing the prices gradually so they follow the last submitted order and the estimated price of the option $\hat{p}$, I have to implement an adaptive `decrease` and `increase` functions. This can be achieved by using simple technique implemented by Cliff and Bruten[29] using adaptive profit margin $\mu(t) \in [0.1, 0.5]$ between some arbitrary range. In this way the bid or ask prices $p_b(t)$ and $p_a(t)$ can be computed using corresponding profit margins $\mu_b(t)$ and $\mu_a(t)$. Below is the function which ZIP trader computes for determining the prices.

$$p_b(t) = \hat{p}(1 + \mu_b(t)) \tag{4.57}$$
$$p_a(t) = \hat{p}(1 + \mu_a(t)) \tag{4.58}$$

Because the formulas for $p_b(t)$ and $p_a(t)$ are symmetric, I describe how increase and decrease operations are done for one of them, namely for the bid. Trader adapts its margin function to the events happening in the market. Below are the list of recursive functions that make the profit margin of ZIP trader:

$$\mu_b(t+1) = \frac{p_b(t) + \Gamma_b(t+1)}{\hat{p}} - 1 \tag{4.59}$$
$$\Gamma_b(t+1) = \gamma\Gamma_b(t) + (1-\gamma)\Delta \quad \Gamma(0) = 0 \tag{4.60}$$
$$\Delta = \beta(\tau - p_b(t)) \tag{4.61}$$
$$\tau = Rs(t) + A \tag{4.62}$$

where

$\mu_b(t), \mu_a(t)$ are profit margins for both bid and ask. Initial values for the profit margins are drawn from uniform distribution $\mu_b(0), \mu_a(0) \sim U(0.1, 0.5)$.

$\Gamma(t)$ recursively defined autoregressive function which imposes the effect of previous price on current bid.

Δ variable representing the convergence of current bid to target price τ through learning coefficient β

τ variable representing the private target price of the trader. It is a perturbed value drawn from uniformly distributed random variables. R is relative perturbation factor where $R \sim U(1, 1.05)$ if target price should be raised, and $R \sim U(0.95, 1)$ if dropped. A represents absolute perturbation, where $A \sim U(0, 0.05)$ if target price should be raised, and $A \sim U(-0.05, 0)$ if it should decreased. R and A are random variables uniformly distributed in an arbitrary range. The bigger the range is, the more volatile the trader prices are. By quick analysis, we can conclude that the target price may vary within $\tau \in [1.05 * s(t) + 0.05, 0.95 * s(t) - 0.05]$ based on above given ranges. So it will not go far from the last order $s(t)$.

β is the learning coefficient of the trader which affects the speed of convergence with the trader's private target price τ. It is drawn from $\beta \sim U(0.1, 0.5)$ while initialising the trader.

γ is the momentum coefficient, or the lag effect of the last price on current price. It is drawn from $\gamma \sim U(0.2, 0.6)$.

In order to simulate ZIP trader, it must operate within some mechanism, because its orders depend on the outcome of the mechanism. I run CDA in Chapter 6 to observe how option prices adapt to the changes inside the mechanism using ZIP algorithm.

However, in this section, in order to demonstrate how ZIP trader can adjust his bids and asks to changing market parameters, I run historic records of option prices on NASDAQ index as accepted orders. We have to be very careful while using the term '*historic records of option prices*', because they are not the actually traded or even quoted prices of the options. It is impossible to find a certain option contract with a continuous open interest in the market over the period of long time. This is due to the multiplicity of different variations of option contracts available in the market, and not everyone willing to take part in trading each of these contracts in his daily trading routine. Therefore bids and asks on certain options are occasional, and the data on their real historical values is very limited. Fortunately, we have talked about volatility surface in Section 4.2.1, and how we can use it to approximate the option's value from given market data using implied volatility parameter discussed in Section 4.3.2. Hence I use these tools to evaluate the theoretical Black-Scholes price of the option and make a fixed bid-ask spread around it to simulate the historical prices of the option.

For example, I took an option contract registered as `'NDX-161216C04320000'`, indicating call expiring on 16 December 2016 with strike $4320, and plotted its Black-Scholes price between 02.01.2014 and 31.12.2014 based on the implied volatility obtained from constructing the volatility surfaces for each day of above period. While plotting the volatility surface, the whole option chain traded on given day is used. Historical prices of the `'NDX-161216C04320000'` option contract and NASDAQ-100 index itself are listed in Tables A.3 and A.2 in the Appendix A. The risk-free rate is taken from the return of 2-year T-Bill [2] within the same period of time which is listed in Table A.1 in the Appendix A. A Figure 4.18 shows the fixed bid-ask spread around theoretical historical

[2]source: http://www.treasury.gov/

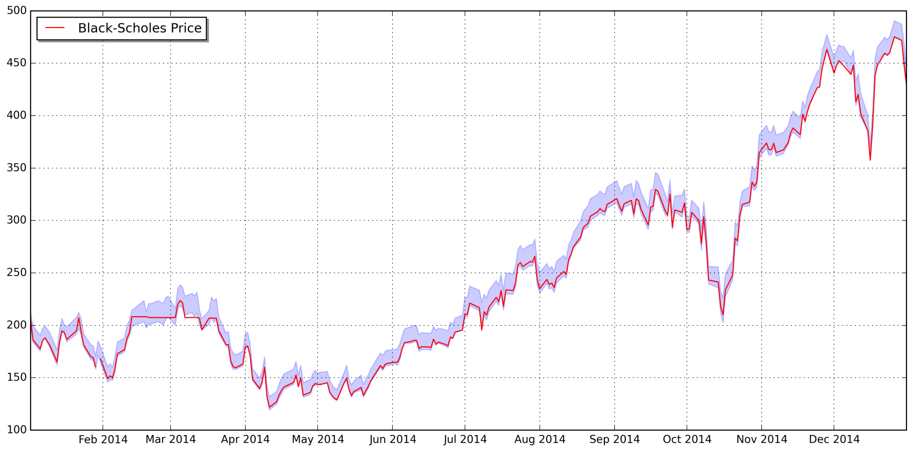


Figure 4.18: Historical prices of NDX-161216C04320000 from 02.01.2014 to 31.12.2014

prices in a shaded area. The historical prices in this case is nothing but Black-Scholes prices fed with the approximated historical implied volatility.

Next I simulated ZIP trader having Black-Scholes prices as his evaluated $\hat{p}$ price, and bids and asks given in Figure 4.18 as accepted bids and asks for each trading day. ZIP trader takes different values of learning coefficient β and momentum γ. Figure 4.19 illustrates my results for these settings.

It can be seen from Figure 4.19, the learning coefficient β controls the adaptive behaviour of the trader to new option price. The higher is the β, the faster the trader gets adapted to new prices which are in given case Black-Scholes prices. These prices can be replaced with any option pricing technique we have discussed in Section 4.3, and ZIP applied upon it. On the other hand, we can also notice that γ determines the momentum of traders direction. This highlights the fact that the trader is likely to follow his previous direction, rather than fully adapting to new price. An example of ($\beta = 0.5, \gamma = 0.9$) illustrates that trader's ask price goes beyond the Black-Scholes price once it starts to grow at the end of the year, and results in a short-term peak in mid-December 2014. This is the result of the accumulated momentum, and its significant effect in determining the next ask price.

Because the current ZIP did not interact with above market, it is hard to evaluate his expected profit, because it depends on whether of his orders have been cleared by the mechanism. Therefore we could not present the statistics about its performance based on historic data. I shall provide more details while simulating ZIP trader in online DAs. I will refer to traders who use ZIP

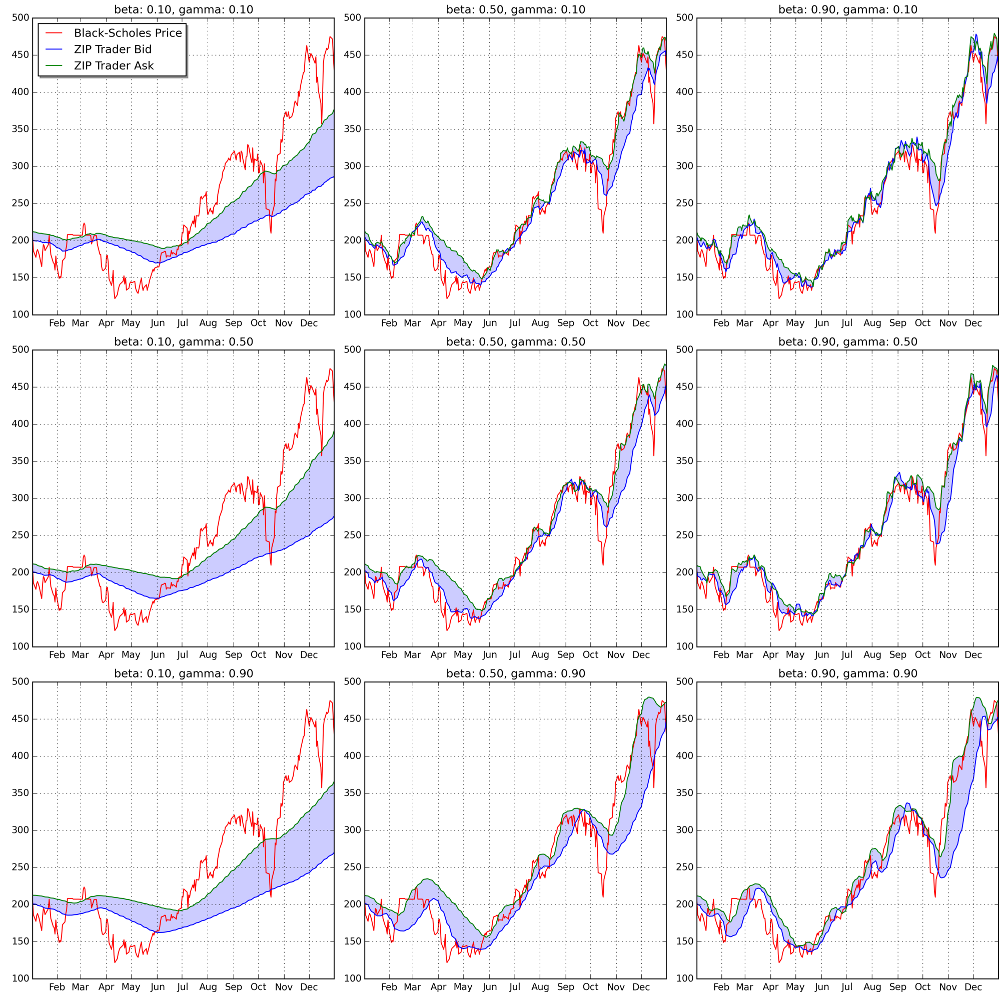


Figure 4.19: ZIP bids and asks on NDX-161216C04320000 from 02.01.2014 to 31.12.2014

proxy algorithm with the prefix `ZIP-` onwards.

Gjerstad and Dickhaut Trader

Gjerstad and Dickhaut proposed different approach for trading agents which is based on a belief function that agent develops to estimate if a particular order is likely to be accepted in the market [69]. It uses Bayes rule for calculating the probability of the success of the order to be submitted. In other words, the traders build their beliefs based on the frequencies of submitted bids and asks and the frequencies of the accepted bids and asks succeeded in transaction. GD strategy also takes into account a memory length for using only recently occurred data. So GD strategy will always place most likely and most profitable quotes.

The quotes submitted to the market must be broken into corresponding bins, so we could compute the histogram for each listed case below.

$TA(p)$ - total number of accepted asks for price p

$RA(p)$ - total number of rejected asks for price p

$A(p)$ - total number of submitted asks for price p

$TB(p)$ - total number of accepted bids for price p

$RB(p)$ - total number of rejected bids for price p

$B(p)$ - total number of submitted bids for price p

In GD algorithm, the probabilities are computed for given price p. However the option price subjects to change over time as it approaches maturity. Hence the sample becomes biased towards fixed prices. Therefore instead of keeping the statistics of TA, RA, A, TB, RB and B for prices, I better do this for the ratios of submitted bid or ask price with Black-Scholes price. In this way, we can keep track with constantly changing option prices as they approach maturity, and at the same time have the posterior probability unbiased towards fixed prices. So in GD, p stands for the ratio of p/p_{BS} where p_{BS} is the corresponding Black-Scholes price.

GD algorithm defines formulas for updating the beliefs of sellers and buyers. As we did for ZIP trader, we will also combine both buying and selling behaviour into a single trader. Trader's sell-side belief in the success of his ask price p_a will be measured with posterior PMF $f_a(p_a)$. For any $p'_a < p_a$ such that

p'_a is rejected by market-maker, the trader assumes that the ask p_a will also be rejected. And similarly, for any $p'_a > p_a$ such that p'_a is accepted, then p_a should also be accepted. The reason for that is the buyer is always interested in buying for less, and if his bid is cleared at higher p'_a price, it will be surely cleared at less p_a. We can write these rules more formally as given below:

$$f_a(p_a | p'_a \text{ rejected}; p'_a < p_a) = 0 \tag{4.63}$$

$$f_a(p_a | p'_a \text{ accepted}; p'_a > p_a) = 1 \tag{4.64}$$

Trader can determine the optimal p_a between the lowest rejected and highest accepted asks by maximising the probability of success based on his posterior belief. We can write seller's belief in p_a as given below:

$$f_a(p_a) = \frac{\sum_{p \geq p_a} TA(p) + \sum_{p \geq p_a} B(p)}{\sum_{p \geq p_a} TA(p) + \sum_{p \geq p_a} B(p) + \sum_{p \leq p_a} RA(p)} \tag{4.65}$$

Similarly for buyer-side, we can draw the same rules symmetrically. If p'_b such that $p'_b > p_b$ is rejected, then the lower bid p_b will be rejected too. If p'_b such that $p'_b < p_b$ is accepted, then the higher bid p_b will be accepted too. Formally, it is as follows:

$$f_b(p_b | p'_b \text{ rejected}; p'_b > p_b) = 0 \tag{4.66}$$

$$f_b(p_b | p'_b \text{ accepted}; p'_b < p_b) = 1 \tag{4.67}$$

$$f_b(p_b) = \frac{\sum_{p \leq p_b} TB(p) + \sum_{p \leq p_b} A(p)}{\sum_{p \leq p_b} TB(p) + \sum_{p \leq p_b} A(p) + \sum_{p \geq p_b} RB(p)} \tag{4.68}$$

Of course, above belief functions (4.65) and (4.68) depend on the data that has been accumulated from past bids and asks. The length of the history data to be used can define the memory of the trader. Trader can become more myopic by reducing the number of bids and asks he uses in order to compute his posterior beliefs. Trader can even fit an approximation function which replicates the last couple of bids and asks, and use it as one of his belief functions. Gjerstad and Dickhaut proved that $f_a(\cdot)$ and $f_b(\cdot)$ functions must be monotonically non-increasing and non-decreasing respectively. For that purpose, traders can use a polynomial function with the order of at least 3.

Based on obtained probabilities, we can obtain the expected payoff of the strategy, by simply finding the optimal bid-ask spread around estimated option price $\hat{p}$.

$$p_a^* = \underset{p_a \in (p_{b_{min}}, p_{a_{max}})}{\arg\max} f_a(p_a)(p_a - \hat{p})^+ \tag{4.69}$$

$$p_b^* = \underset{p_b \in (p_{b_{min}}, p_{a_{max}})}{\arg\max} f_b(p_b)(\hat{p} - p_b)^+ \tag{4.70}$$

where $\hat{p}$ is evaluated option price, $p_{b_{min}}$ is the minimum outstanding bid, and $p_{a_{max}}$ is the maximum outstanding ask.

Because the algorithm requires direct interaction with the mechanism itself, we cannot simulate it without running a CDA. Moreover the real life data on intraday submissions of bids and asks for options is too limited, or not available at all. Therefore it is difficult to test the performance of GD algorithm on trading options separately. I will take more closer look on its performance while simulating it in online DAs in Chapter 6. I refer to the trader who use GD algorithm for trading with the prefix `GD-` later on in this thesis.

There are other improvements of GD algorithm, and one has been proposed by the original authors. GDX algorithm is the optimization of GD which performs much faster and uses dynamic programming methods for optimally using resources available [104, 105]. However they are not used in this paper.

4.6 Summary and Contribution

In this chapter, I have discussed the important aspects of building option trading agents starting from the simulation of ZI traders to actual trading algorithms they use in online mechanisms. Also I have covered how agents can use different option pricing techniques in order to create liquidity in the market and work around the 'no-trade' outcome. I have also described how agents obtain important knowledge about the market using two main techniques: a) calibration method for determining the implicit volatility in the market, and b) Bayesian learning in order to use new information to update the agent's prior beliefs in the example of informed traders. To sum up, I can summarise the main contributions of the chapter as follows:

- *IIKB Framework* - I have extended the existing IKB framework developed by [160] with an additional layer responsible for inventory-based agents and demonstrated its importance for LMSR, inventory-based or delta-hedging traders. I have explained the modules and their relationship across the elements of the layers while describing the behavioural layer.

- *Volatility Surface* - I have proposed an approximation rule for constructing a volatility surface with incomplete data. It used existing market prices in the option chain to determine the implied volatility for different strikes and expiration dates, and then interpolated the missing data between existing data points. The interpolated implied volatility is then used to price options and option portfolios using volatility pricing technique.

- *Zero-Intelligence Option Pricing* - Similar to Gode and Sunder's research [74] in zero-intelligence pricing of assets, I have introduced zero-intelligence pricing for options which returns random option prices within the defined bounds of option's possible intrinsic value. The option prices are obtained from simulating 2 different asset pricing models till the date of expiry of the option. This allows us to test ZI traders in different mechanisms and measure their performance towards established option pricing rules such as Black-Scholes.

- *Indifference Option Pricing* - I proposed to use two comparatively new option pricing techniques as part of option trading agent's pricing method:

  1. the exponential utility which generates risk-averse price based on agent's utility indifference to the prices submitted in the market. This allows agents to exhibit fear of trading at wrong price, and thus charge more than expected for particular type of options.

  2. LMSR based option pricing which is a new option pricing method proposed by Othman *et al.* [42]. I incorporated LMSR pricing rule into portfolio of options while pricing given option contract. It allows traders holding different option portfolios to price given option from the perspective of the overall payoff they can get by selling or buying this option.

- *Proxy Trading Algorithms for Options* - I have implemented two types of proxy trading algorithms: one for dealers whose main objective is to maintain the market without running into loss and one for traders whose main objective is to get the better deal in the market. I used two algorithms for each, and adapted some of them uniquely for options.

Below are the important contributions made to each of them:

1. Garman's Model - This method involves computing the optimal bid-ask spread for the dealer based on the probability of failure given there is a model for the arrival of orders to the bidder and it depends on the prices submitted by the dealer. I have incorporated Garman's model into proposed IIKB framework. I have reformulated the Garman's method into LP for the inventory-based trader, and provided an example regarding the arrival functions. I simulated the risk of failure from holding a certain bid-ask spread based on given example, and presented it as colormap chart. I also proposed that inventory-based trader picks the narrowest possible bid-ask spread which has minimum probability of failure.

2. Copeland and Galai's Model - This method involves setting up bid-ask spread based on dealer's belief in the population of informed traders in the market. I showed how agent can learn about the population of informed traders using Bayesian technique when the dealer realises the true intrinsic value of the option at the date of its expiry. Dealer can induce the posterior probability of accepting informed orders from this fact. Later, I have introduced Geske's formulas [66] for pricing compound options to calculate the expected loss from informed traders, as it has been shown that it simulates the straddle option portfolio. I used these components to formulate an objective function for the information-based dealer. Then I computed the optimal bid-ask spreads for various proportions of informed traders in the market. I also took into account different ratios on buying and selling behaviour of liquidity traders.

3. ZIP and GD Traders - I have stacked up the option pricing mechanism on top of well-known proxy trading algorithms such as ZIP and GD, and explained how they can interact together through evaluated $\hat{p}$ option price. I have also simulated ZIP algorithm for historical prices of the option to see trader's bid-ask spread adapts to continuously changing environment under various factors ZIP trader's learning and momentum characteristics. ZIP trader had to adapt its bid-ask spread to the option's Black-Scholes price $\hat{p}$ and market bids and asks. As for GD trader, I used risk-neutral ratio of the quoted price to compute the probabilities, instead of using the

actual price as it was initially proposed by Gjerstad and Dickhaut.

Although I use above asset pricing, option pricing and proxy trading techniques in testing proposed mechanisms in next chapters, I admit that the whole architecture should be put to live testing and measured for the performance in real option markets. However option markets are not very liquid for particular type of an option, and moreover direct access to the exchange is limited to brokers only. Hence it is difficult and costly to undertake experiments on real-time option markets. Moreover above algorithms oversimplify the market microstructure based on certain assumptions. For example, I simplified the arrival functions to linear functions whereas in real-world these functions are not static and continuously take different shapes. The idea of informed traders in the market is also oversimplification of the market structure where ones hold valuable insider information and others are simply trading out of their private interests. This kind of binary classification of traders is inappropriate in real-life scenarios because information itself is incomplete due to future uncertainty, and one can only have limited information which can affect the option price to some extent but not fully predict its true value. To summarise, considering the said assumptions, described methods employ most of the key aspects of option trading and covers both classic and cutting-edge methods involved in option pricing. It uses proxy trading algorithms from the perspective of a dealer and trader taking advantage of new information arriving to the market.

Chapter 5

Direct Double Auction

5.1 Introduction

There has been a growing interest in the research of markets as complex game-theoretic systems since Myerson coined *mechanism design* as a framework for strategic interactions between self-interested agents [119]. A new discipline of *auction theory* emerged as a part of mechanism design, and it found its applications in solving many of well-known problems such as resource allocation, scheduling, supply chain optimization, operations control and multi-agent system implementation [36]. The ultimate goal of any auction is the allocation of resources to agents through determining their true value otherwise unknown to the seller or buyer. The space of auction types can be extended way beyond its classic format. They may vary in their initial settings, bidding rules, market clearing methods etc. Parsons describes more than 30 variations of auctions based on properties such as dimensionality, quantity and heterogeneity of traded items; direction, sidedness, openness of accepted bids; and kth order prices in determining winners[134].

Standard financial theory provides a number of methods for calculating option prices based on the market performance of an underlying asset. But there are few models that take into account strategic interactions carried out by automated agents, and their role in forming the prices. I have designed option trading agents in previous Chapter 4, and now I am going to use these automated traders in auction settings set up for different scenarios in both underlying and option markets. I provide a testable environment where various market mechanisms and trading behaviours can be simulated and analysed.

In this chapter, I present the design and simulation of a direct Double Auction (DA) which can be used for trading options. The main aspects of proposed DA are that it holds Dominant Strategy Incentive Compatible (DSIC) and individual rationality properties, along with approximate efficiency in the allocation of multi-unit orders. The key aspect of the mechanism is that the orders are considered atomic, which is an important requirement for traders wishing to take option portfolios. However the mechanism lacks budget-balance and therefore requires an additional agent who would cover the costs created from facilitating the truthful trades.

The chapter is organised in following way. In Section 5.2, I start with the definitions of DSIC mechanisms and other desired properties of the mechanisms. Section 5.3 starts with the idea of McAfee's DA [113] which matches single-unit bids and asks with DSIC property and then extends it to multi-unit DA with atomic orders. Then I follow up with the use of revelation principle towards certain agents such as ZI traders. I also introduce the idea of simultaneous multi-unit DAs for trading option portfolios. In Section 5.4, I define the main components of the experiments to run, and highlight the main indicators collected from the simulations. I analyse the allocative efficiency and budget-balance of the DA from obtained results in different experimental scenarios. I investigate the analytical parameters of options such as Greeks obtained from simulation results and compare them with Black-Scholes results. Section 5.5 does the important sanity check on the mechanism's ability to simulate option prices when all traders are set to be Black-Scholes traders. In Section 5.6 I present my important findings from simulating different trading agents in proposed mechanism through analysing the error in efficiency, budget-deficit that each scenario imposes, along with the key indicators of option pricing methodology such as option prices, trade volumes, accepted orders, delta, gamma and theta. Section 5.7 draws the summary of the chapter by emphasising the key contributions and suggesting the further improvement that can be done to proposed methodology.

5.2 DSIC Mechanisms

In designing economic mechanisms, there are two major considerations that we should take into account: the maximisation of the gains of individuals participating in the game, and the maximisation of the social surplus. Most of the economic systems such as markets, continuous DAs already accomplish these goals according to competitive equilibrium result. However the processes that govern such systems are very complex, and often misunderstood. The traders employ sophisticated trading strategies to reach the desired efficiency, while exchanges have to continuously adjust their policies to meet the market demands and safeguard them against unpredicted crushes. From game theorist perspective, we should address these systems by trying to analyse the interactions and predict the final outcome that is likely to happen if all the participants of the system keep on to their corresponding strategies. However as I noted earlier, the decision making processes of market participants are extremely hard to understand and subject to constant change, which makes any attempt to predict the future hopeless.

However what if we change our perspective by posing different type of questions such as if we should investigate the game itself and try to improve it by knowing the preferences of its participants, if we can design such game without knowing the secret preferences of its participants, but guarantee that in an equilibrium state the desired properties are held. If I apply this thought to markets, the question is if it is possible to build such markets which are much simplified for traders to interact with and also hold the desired properties of currently established economic systems. How economically efficient would be such mechanism? How computationally feasible is its implementation and operation in real life? Often dubbed as '*inverse game theory*' [148], *mechanism design* studies the implementation of protocols for distributed systems, not necessarily cooperative, to attain specified goals. In other words, it is about making rules for given game setting in such way that the equilibrium result is almost the same as the expected result.

Before I start answering the above stated questions, let me define some of the important concepts that I use throughout this chapter. This involves understanding how we can formulate problems correctly so the proposed mechanism becomes its solution. First let us define what is a game setting for which the mechanisms are developed.

Definition 5.2.1. *Game Setting* is described as a setup consisting of N agents, O outcomes, Θ strategies (i.e. choice of actions) and a utility function $u_i : O \times \Theta \to \mathbb{R}$.

Definition 5.2.2. *Quasi-linear Game Setting* is a game setting where the outcome is $O = X \times \mathbb{R}^n$ and utility function is defined as shown below:

$$u_i(o, \theta) = v_i(x, \theta) - f_i(p_i) \tag{5.1}$$

where $o = (x, p)$ is the mechanism outcome, $x \in X$ non-monetary outcome, $p \in \mathbb{R}^n$ is the monetary outcome, $v_i : X \to \mathbb{R}$ is the valuation function and $f_i : \mathbb{R} \to \mathbb{R}$ is strictly monotonically increasing function.

Definition 5.2.3. *Mechanism* for a game setting is generally described as the set of actions A available to agents, and a mapping function $M : A \to \Pi(O)$ which turns actions into distribution of outcomes. Mechanism is called deterministic if there is a bijective relationship between actions and outcomes.

So the job of the mechanism designer is to determine these list of actions available to agents, and to design a function which would map these actions into desired outcomes. The mechanism can be implemented using a dominant strategy equilibrium.

Definition 5.2.4. *Dominant strategy implementation* is the implementation of a mechanism which makes the desired outcomes the dominant strategy equilibrium for participating agents.

In other words, the dominant strategy implementation means that the mechanism provides agents a dominant strategy game and it assumes that the agents are rational to follow this strategy. Once every agent plays his dominant strategy, this results in dominant strategy equilibrium outcome which is the same as the desired outcome. This type of mechanisms are also referred as '*strategy-proof*' mechanisms.

One type of strategy-proof mechanisms is DSIC mechanism where the *truthfulness* is the dominant strategy for the agent. It emerges from *revelation principle* theorem [148] which states that if there is a strategy-proof mechanism, then there is also a strategy-proof and truthful mechanism. The main idea is that once there is an agent who has private information in his valuation method v_i which he uses to compute his utility for certain outcomes caused by his corresponding actions, and he has set of actions that are dominant to him, the agent can truthfully reveal his private information and his valuation method to the mechanism which would run these dominant actions on his behalf and reach the same dominant strategy equilibrium. This enables us to simplify the complex systems where there is a dominant strategy, but it involves series of sophisticated actions to achieve maximised utility. Agents can truthfully reveal their private methods to the mechanism, instead of carrying out those actions themselves.

I review DSIC mechanisms from the perspective of a quasilinear mechanism where the outcome consists of two types non-monetary and monetary values. This game setting is incredibly good for viewing DAs as mechanisms. In terms of DA, the outcome of the mechanism can be viewed as q_i the quantity of goods allocated to agent i and p anonymous clearing prices. The agent i will buy (sell) the item q_i is positive (negative). So the quantities for a pure seller will be all negative, and for a pure buyer positive. I assume that the valuation function $v_i(q_i)$ will also reflect this relationship. Quasilinear utility assumption also implies that agents are risk-neutral, because his utility changes linearly for given quantity and price. In other words, agent is indifferent between the outcomes of 5 and 10 goods allocated to him, or 1000 and 1005 goods allocated to him. The changes in his utility are exactly the same in both cases, and so the actual amount of the quantity does not affect his utility. However, risk-neutrality in the context of option pricing must not be confused, as it involves pricing options based on risk-free investment rates. Hence I re-formulate the quasilinear function for DA case:

$$u_i(o_i, \theta_i) = v_i(q_i, \theta_i) - pq_i \tag{5.2}$$

where $o_i = (q_i, p)$ is the DA outcome, q_i is the allocation result for agent i, and p is the anonymous price, and v_i is the valuation function of the agent.

We can also see that the second term of (5.2) pq_i satisfies the monotonically increasing requirement of f_i function defined for quasilinear utility. Also another assumption that I introduced into this quasilinear function is the *conditional independence* of agent's utility. This would mean that the agent's utility accepts θ_i instead of θ, meaning that it only depends on the agent's own choice of actions, and not the other agents involved in the DA. This would make the agent's valuation function v_i independent on the actions of the other agents.

Now let us define the DA itself from the perspective of mechanisms.

Definition 5.2.5. *Double Auction* is a mechanism in a quasilinear setting which is described as a tuple of three elements:

1. A_i is the set of actions available to the agent i in the form of a bid or ask defined as a tuple $(\hat{q}_i, \hat{v}_i)$ where $\hat{q}_i$ denotes the declared quantity, and $\hat{v}_i$ the declared price. Negative $\hat{q}_i$ implies sell action, and the positive $\hat{q}_i$

implies the buy action. θ_i is the particular choice of actions (bids/asks) that agent submits to the mechanism.

2. $\chi : A \to Q$ is the allocation rule which maps the agents' actions to quantities of goods they are allocated with.

3. $\rho : A \to \mathbb{R}$ is the payment rule which maps the agents' actions to the price they receive/pay for the goods

So the problem of the DA boils down to the generation of an outcome which determines how many items the traders must buy or sell and at what price this should happen. The outcome must be the result of the respective bids and asks of the traders, and should possess certain desired properties that we talk about later in this chapter.

Definition 5.2.6. *Direct Double Auction* is a DA where there is a dominant strategy for each agent, so the choice of his actions is restricted to a single dominant action which he always uses. As the actions in DA are bids and asks of the agent, this would mean that agent has only one bid or ask to submit.

Thus the agent has only one bid or ask to submit in direct DA, this must be the truthful bid or ask based on the revelation principle. In other words, out of many possible values for $(\hat{q}_i, \hat{v}_i)$, the agent only chooses the truthful one making $\hat{v}_i = v_i(\hat{q}_i)$.

Definition 5.2.7. The DA is called *efficient* if its allocation result q satisfies $\sum_i v_i(q_i) \geq \sum_i v_i(q'_i)$ for any feasible allocations q and q' in an equilibrium.

The feasible allocation implies that the mechanism should not allocate more than offered. Formally this would mean $\sum_i q_i \geq 0$. It can also be seen that the equilibrium efficiency is measure using the true valuation of the allocation result, not the declared one.

Definition 5.2.8. The DA is called strongly *budget-balanced* if its payment rule satisfies $\sum_i \rho(\theta_i) = 0$ for any equilibrium profile of actions θ. The DA is called weakly budget-balanced if its payment rule satisfies $\sum_i \rho(\theta_i) \geq 0$ for any equilibrium profile of actions θ.

The strong budget-balance of the DA would mean that it does not generate any income, hence all its collected payments are paid back to participants. The weaker version of budget-balance states that the mechanism can make profit, but it should never take loss.

Although budget-balance and efficiency of the mechanism are highly desired qualities, they cannot come along with DSIC property according to Green-Laffont, Hurwicz's impossibility theorem [75, 90]. The theorem states that there is no DSIC mechanism which both efficient and weakly budget-balanced even in a single-unit DA. This would mean that there is no direct DA which would hold both qualities at the same time. This result has been further generalised by Myerson-Satterthwaite [119] in their theorem which states that there is not even a BIC DA which simultaneously holds efficiency, weak budget-balance and ex-interim individual rationality, even for the agents using quasi-linear utility. BIC is weaker notion of incentive compatibility compared to DSIC, because it does not guarantee the equilibrium outcome matching the desired outcome, but it only attains the desired outcome on average. Ex-interim individual rationality would mean that on average the trader knowing his own valuation method, but not knowing the valuation methods of other traders, is not worse off from participating in the market compared to staying idle. In other words, if trader decides to enter the market, he must not be charged more than what he quoted. Formally it can be defined as follows:

$$\mathbb{E}_{v_{-i}|v_i}[v_i(\chi(\theta_i, \theta_{-i})) - \rho(\theta_i, \theta_{-i})] \geq 0 \quad \forall i, v_i \tag{5.3}$$

where θ is the equilibrium profile, v_{-i} valuation of traders other than i. Note that the results of allocation function $\chi(\theta_i, \cdot) = q_i$ and payment function $\rho(\theta_i, \cdot) = p * q_i$ for particular trader in direct DA.

From equation (5.3) we can see that the quasilinear utility of agent i when applied both allocation and payment rules of the mechanism must not be less than zero on average. In terms of options, the notion of individual rationality of the mechanism must not be confused with the potential loss the trader may have from buying or selling option contract. This type of loss is incurred by the uncertainty in the underlying market and has no relation to the mechanism where the options are traded.

Definition 5.2.9. The direct DA is called *tractable* if its allocation rule $\chi(\theta)$ and payment rule $\rho(\theta)$ can be computed in polynomial time.

To sum up this section, we have learned what DSIC mechanism is and how this notion is translated to the definition of a direct DA. I have also defined key aspects of DAs such as efficiency, budget-balance and individual rationality. However the findings of Green-Laffont and Myerson-Satterthwaite have shown that it is impossible to design a direct DA with these properties. But it should not ruin our hope in designing a direct DA at least having the approximations of these properties with acceptable errors. We shall look at how the direct DA can be designed, and what would be the worst case error in its efficiency and budget-balance in the next section.

5.3 Design of Direct Double Auction

As I mentioned earlier in previous chapter, there are two major economic concerns in developing mechanisms, they are the maximisation of individual utilities and the social surplus. The question that comes next is how we can design direct DA which would satisfy these needs. What is involved in designing a direct mechanism in the first place and how should we choose allocation and payment rules in order build such mechanisms? Myerson and Satterthwaite proved that there are specific requirements for choosing such functions in order to make the mechanism DSIC[119]. Here are these requirements:

1. Mechanism must be in a quasilinear game setting
2. Mechanism must not charge or reward the agents for not participating
3. Mechanism must have non-decreasing monotonic allocation rule

The first requirement stems from the definition of DA as a mechanism having both monetary and non-monetary outcome. The second requirement ensures that if the agents order is not allocated, he must not be charged or reward with any payment, so his payment must be equal to zero. The most important and the remarkable finding of Myerson and Satterthwaite is the third requirement which specifically describes what the allocation rule must be chosen. Non-decreasing monotonic allocation rule requires that the buyer (seller) gets weakly more allocations for higher bid (lower ask).

Hence we know that the allocation rule must be non-decreasingly monotonic, we can align this aspect with the initially desired property of established markets - surplus maximisation. This type of mechanisms are generally classified as *Groves mechanisms* and formulated as follows:

$$\chi(\theta) = \arg\max_q \sum_i \hat{v}_i(q) \tag{5.4}$$

$$\rho_i(\theta) = h_i(\hat{v}_{-i}) - \sum_{j \neq i} \hat{v}_j(\chi(\theta)) \tag{5.5}$$

where $h_i(\hat{v}_{-i})$ is an arbitrary function which does not depend on agent i's declared valuation.

It has been formally proven that surplus maximising allocation rule is non-decreasingly monotonic [117], but it is also intuitive to see that this is true. In nutshell, let us assume that the trader gets more allocation $q_i' > q_i$ for $\hat{v}_i' < \hat{v}_i$. In this case, the allocation is not surplus maximisation because the result of equation (5.4) could be improved by giving the q_i' to $\hat{v}_i$ bid. This would mean that the surplus maximisation allocation rule can be used to design a DSIC mechanism. In fact, the whole family of Groves mechanisms are DSIC mechanisms [117].

Once the concrete implementation of allocation rule of DSIC mechanism is clarified, the next step is the determination of the payment rule. Fortunately, for above stated requirements, Myerson's Lemma [119] defines unique and explicit formulation of the payment rule that makes the mechanism DSIC. This payment rule includes the *critical values* of each agent who has been allocated with goods. Critical value of an agent is the value that the agent needs to beat in order to get the good. For example, in terms of single-item auction, the payment rule corresponds to the second price, because agents must beat the second price to be the winner of the auction. It can be written as follows:

Definition 5.3.1. For the mechanism satisfying the above requirements, the *Myerson's payment rule* is

$$\rho_i(\theta) = \int_0^{v_i} z \frac{\partial \chi_i(z, \theta_{-i})}{\partial z} dz \tag{5.6}$$

where θ_i denote the agent i's order, θ_{-i} the orders of the rest of the agents, and $\frac{\partial \chi_i(z,\theta_{-i})}{\partial z}$ is the marginal allocation rule for the agent i.

The marginal allocation rule used in equation (5.6) determines some critical value z at which the allocation of the agent changes from 0 to 1 in a single-unit DA case. In multi-unit case, there could be many critical values that cause the allocation monotonically increase from q_i to $q_i' > q_i$. In this way, Myerson's payment rule states that the price agent has to pay must be equal to the sum of the agent i's critical values. However the critical values of the agent i are not dependent on agent i's own valuation, because they are formed from the submitted orders of the other agents. This is the main reason why there is no v_i involved in computing the payment for agent i in Groves family of efficient mechanisms. However there is still undefined function $h_i(\hat{v}_{-i})$ to compute the DSIC payments. In 1971, Clarke proposed the implementation of this function must be such that the payment paid by the agent i is equal to the social cost imposed by the agent i to the market [28]. It is evident that if the agent i is allocated with the good, this would mean that someone else in the market had to loose this good for this agent i. Thus agent i created a cost to the mechanism which otherwise would not be the case. The social cost is obtained by computing the difference between two terms: 1) *Clarke tax* - social surplus of the mechanism without agent i's participation:

$$h_i(\hat{v}_{-i}) = \sum_{j \neq i} \hat{v}_j(\chi(\theta_{-i}) \tag{5.7}$$

and 2) the social surplus of the mechanism with agent i's participation but his bid not included in it. This gives us the explicit form of the payment rule also known as Vickrey-Clarke-Groves (VCG) payment. It can be fully written as:

$$\rho_i(\theta) = \sum_{j \neq i} \hat{v}_j(\chi(\theta_{-i}) - \sum_{j \neq i} \hat{v}_j(\chi(\theta)) \tag{5.8}$$

Now we know how to build direct mechanisms based on surplus maximising allocation rule and VCG payments. This mechanism is also efficient because it is in the class of Groves mechanisms. However the VCG mechanism does not hold individual rationality unless two additional assumptions are imposed, which are also realistically plausible. First is the *choice-set monotonicity* which makes an assumption that removing an agent from the mechanism weakly decreases the number of possible choices for the remaining agents. In other words, the entrance of a new agent into the market does not eliminate the potential matchings between existing agents, but otherwise creates new matchings involving the new agent. Second assumption is the absence of *negative externalities* which means that every agent has non-negative utility for any choice that mechanism can make without his participation. In the example of DA, this would mean that agent makes no payment for the allocations of other agents whether he does not participates in the market or participates but gets zero allocation. If these assumptions are hold, then VCG mechanism is not only efficient, but it is also ex-post individually rational.

Also VCG mechanism is not weakly budget-balanced even if above two assumptions are true. From (5.8) we can see that the equation can result in negative payment if the first term (i.e Clarke tax) is greater than the second term. So the assumption that must hold for VCG to be weakly budget-balanced is the *no single-agent effect* assumption, which states that the Clarke tax must be weakly greater than maximised surplus without agent i's valuation. In other words, this would mean that other traders are weakly better off from the absence of agent i. Although this assumption holds for single-side auctions, it is not true for DA because the mechanism has to subsidise the difference between bids and asks.

However the savvy reader might notice that VCG payments are unique for each agent i, while the direct DA that I defined earlier uses anonymous prices. Because the direct DA that I propose is for trading options, and the options are identical, most intuitively their prices must be the same too when traded in multiple units. After all, the ultimate goal of the research is to price options using DAs, and it should be an anonymous price even if the mechanism is subjected to loose one of its desired properties such as budget-balance. Unfortunately, if we wish to preserve the strategy-proofness of the mechanism, we cannot have fully anonymous prices all the time. In a certain case, the prices are different towards buyers and sellers. I shall talk about this in next section.

5.3.1 McAfee's Double Auction

In this section, I review McAfee's DA and reformulate its allocation rule into LP problem. This would facilitate the design of proposed multi-unit DA in the next section. McAfee's DA is a direct DA which uses surplus maximisation as its allocation rule. However it makes trade reductions to achieve individual rationality and weak budget-balance by preserving the strategy-proofness of the mechanism. Because it can generate positive income, it requires an additional agent who takes the surplus, but never buys or sells anything. It is also computationally tractable and approximately efficient. The loss in efficiency is bounded by $1/\min(m, n)$, the number of buyers and sellers respectively. McAfee's DA is designed for single-unit bids and asks. The payment rule employs both anonymous and bid-ask pricing.

McAfee's allocation rule [113] (in the context of DAs, i.e. matching rule) can be written as an order statistics of bids $b_{(1)} \geq b_{(2)} \geq \cdots \geq b_{(m)}$ and asks

$a_{(1)} \leq a_{(2)} \leq \cdots \leq a_{(n)}$ matched as $k \leq \min(m, n)$ such that

$$b_{(k)} \geq a_{(k)} \tag{5.9}$$
$$b_{(k+1)} < a_{(k+1)} \tag{5.10}$$

.

We can reformulate this rule into a LP problem defined below:

Definition 5.3.2. For a given vector of valuations v, McAfee's allocation rule is

$$\max_{\lambda} \sum_i v_i q_i \lambda_i \tag{5.11}$$
$$s.t. \quad \lambda_i \in \{0, 1\} \quad \forall i \tag{5.12}$$
$$q_i \in \{-1, 1\} \quad \forall i \tag{5.13}$$
$$\sum_i q_i \lambda_i = 0 \tag{5.14}$$

where represents sell/buy action by trader i, λ_i is an allocation decision variable.

Theorem 5.3.1. *For McAfee's allocation rule, LP formulation* (5.11) *is correct.*

Proof. From objective function (5.11), it can be seen that optimiser only finds efficient trades. Given that the supply and demand is matched in constraint (5.14), we can assume that the number of trades is $k' = (\sum_i \lambda_i)/2$, hence we have to prove $k' = k$. Let's assume that $k' < k$, then it means that there is $b_{(k'+1)} - a_{(k'+1)} > 0$ and optimiser could have added this difference to result greater surplus. So it is not the maximum surplus. Let's assume that $k' > k$, then it would mean that $b_{(k')} - a_{(k')} < 0$, and optimiser would be better off not including this match into allocation, as it decreases the objective. Therefore $k' = k$. □ □

McAfee's pricing rule is applied for one of two cases:

- Anonymous Price: After allocation rule identifies k efficient trades, $p^0 = (b_{(k+1)} + a_{(k+1)})/2$ is computed from highest rejected bid and lowest rejected ask. If $p^0 \in [a_{(k)}, b_{(k)}]$, all k efficient trades are cleared at p^0, the budget is balanced and ex-post individual rationality is hold.
- Two Prices: If $p^0 \notin [a_{(k)}, b_{(k)}]$, top $k - 1$ efficient trades are matched,

buyers pay $b_{(k)}$, sellers receive $a_{(k)}$. kth efficient trade is rejected. Mechanism makes $(k-1)(b_{(k)} - a_{(k)}$ profit. Individual rationality is also held, because $\cdots \geq b_{(k-2)} \geq b_{(k-1)} > a_{(k)}$ and vice versa for sellers.

The 2-price case can occur when the $b_{(k)} = b_{(k+1)}$, because $p^0 = (b_{(k+1)} + a_{(k+1)})/2 > b_{(k+1)} = b_{(k)}$, and thus k efficient trades cannot be cleared at p^0 because it violates the individual rationality of kth trader. Therefore McAfee mechanism clears the bids with the highest rejected bid, which is kth efficient bid. This simulates the second-price Vickrey auction for bidders. Similarly this thought can be applied to seller side as well.

Another important aspect is the tie-breaking for $b_{(k)} = b_{(k+1)}$ where mechanism has to choose whose bid to include, and whose do not. In McAfee's DA this is done through randomization, but in my case the numerical optimiser does this operation on mechanism's behalf.

5.3.2 Proposed Multi-Unit Double Auction

In this section, I extend McAfee's DA to a multi-unit auction but in the process I have to give up its weakly budget-balanced property and introduce an agent who has to subsidise the exposed multi-unit bid or ask in order to preserve strategy-proofness of the mechanism and the atomicity of orders. There have been a number of multi-unit DA designs proposed previously [86, 102] which support weak budget-balance property. However these mechanism partially satisfy the orders to spread the excess demand (supply) to balance supply and demand. In economics, supply and demand can be balanced using classic Walrasian mechanism which pushes the price of good up and down until demand equals supply. But this requires the assumption that the law of demand (supply) (i.e. marginal utility decreases (increases) with every transaction) hold for every trader. However, because there are combinatorially many possible option contracts available in the market, the demand and the supply are scattered, and thus less competitive. I have explained `RND-` method of choosing quantities for traders, and this clearly does not conform to the law of demand(supply). Therefore DSIC version of a Walrasian mechanism (i.e. clinching auction) is out of consideration for this purpose.

In Huang *et al.* 's design [86], multi-unit DA solves a LP for surplus maximising objective subject to the constraints on no buyer has to buy more than he needs and no seller has to sell more than he wants. If there is an overdemand, each buyer's demand is diminished evenly to eliminated the excess demand. The same is applied to sellers if there is oversupply. There are separate clearing prices for buyer and seller. The least winning bid and the greatest winning ask are accepted as clearing prices for the rest of winning buyers and sellers correspondingly and rejected for those buyer and seller who submitted them. This mechanism is proven to be DSIC, ex-post individual rational, weakly budget-balanced and asymptotically efficient.

Similarly, in Loertscher *et al.* 's [102] mechanism reduces the overdemand or oversupply through applying different pricing and allocation rules in each given case. For example, if supply and demand balance then every winning agent trades the quantity it requested at a reserve price r which is determined inside the gap between the least winning bid and the greatest winning ask. If there is overdemand, all winning sellers trade the quantities they posted at r, but the clearing prices and the quantities for the buyers are discounted using VCG. The same rule applies if there is an oversupply. This mechanism is proven to be DSIC, ex-post individual rational, weakly budget-balanced and asymptotically efficient.

In my design, I propose a multi-unit DA that preserves the *atomicity of orders* at the expense of budget-balance. The mechanism is said to support the atomicity of orders only if it either fully satisfies the multi-unit order or fully rejects it. The key reason for an atomicity of orders in option market is that it is crucial for the option trader who uses an option portfolio, because option portfolio determines exactly at which quantity each type of option needs to be sold or bought. Trader cannot take quantity less than requested, because the violation of atomicity of orders would result in the distortion of option portfolio as a whole. This in its effect change the trader's payoff structure and may go against his belief in certain future events. Both Huang *et al.* and Loertscher *et al.* provide solutions to cut the overdemand/oversupply in various ways to preserve the budget-balance, but my proposed mechanism gives up budget-balance in favour of atomicity of orders. In order to accomplish that, my mechanism will also have the right to trade with participating agents, and its trade will be limited to the amount that partially satisfies certain agent's winning order.

Now let us extend McAfee's mechanism to multi-unit mechanism. Then consider multi-unit bid as a tuple $\mathbf{b}_i = (b_i, q_i)$ where b_i is per unit bid, and q_i is the amount demanded. The same is defined for multi-unit ask. We can split this tuple into set of equally-valued single-unit bids $\mathbf{b}_i = \bigcup_{t=1}^{q_i} b_{i,t}$. This can be done to asks as well. Then we have complete set of bids $\mathbf{b} = \bigcup_{i=1}^{n} \mathbf{b}_i$ and asks $\mathbf{a} = \bigcup_{i=1}^{n} \mathbf{a}_i$. We can use single-unit McAfee's mechanism to find the allocation and payment. However, we can observe below that not all bids/asks can be fully satisfied.

Lemma 5.3.1. *In multi-unit McAfee's mechanism, there exists at most one multi-unit bid/ask which is partially satisfied, and the remaining winning bids/asks are fully satisfied.*

Proof. Let us assume that we use McAfee's matching rule for expanded set of single-unit bids $\mathbf{b}$ and asks $\mathbf{a}$ ordered subsequently by its host multi-unit bid or ask. Then we should have some k such that $b_{(k)} \geq a_{(k)}$ and $b_{(k+1)} < a_{(k+1)}$ for their constituent single-unit bids and asks. We can also claim, without loss of generality, that there exists such a multi-unit bid $\mathbf{b}_i$ such that two of its bids $b_{(k)}, b_{(k+1)} \in \mathbf{b}_i$. This would imply that $b_{(k)} = b_{(k+1)}$ because multi-unit bidder sets single price per unit, and this price is the same for all bids inside multi-unit bid. However, there cannot be some multi-unit ask $\mathbf{a}_j$ having asks such that $a_{(k)} = a_{(k+1)}$, because it contradicts with $b_{(k)} \geq a_{(k)}$ and $b_{(k+1)} < a_{(k+1)}$. Hence, $a_{(k)}$ and $a_{(k+1)}$ must belong to different multi-unit asks. It must also be the case that the multi-unit ask which owns $a_{(k)}$ is fully satisfied, and so do other preceding winning multi-unit asks. This is also true for all multi-unit bids preceding the multi-unit bid which owns $b_{(k)}$. We can conclude that multi-unit bid which owns $b_{(k)}$ is partially satisfied, and it is the only such multi-unit order among other winning multi-unit orders. $\square$

I can formulate an LP problem for for multi-unit bids and asks where $\lambda_i \in [0, 1]$ now. So it is not binary any more, and takes any value between 0 and 1. When it takes 1, the multi-unit bid/ask is fully satisfied, zero means it is rejected. But when $\lambda_i \in (0, 1)$, the agent i is partially satisfied.

Definition 5.3.3. For given vectors of valuations and quantities (v, q), allocation rule for multi-unit DA is:

$$\max_{\lambda} \sum_i q_i \lambda_i v_i \tag{5.15}$$

$$s.t. \quad \lambda_i \in [0, 1] \quad \forall i \tag{5.16}$$

$$\sum_i q_i \lambda_i = 0 \tag{5.17}$$

$$q_i \in \mathbb{Z} \tag{5.18}$$

where q_i represents quantities, v_i is the agent's valuation, λ_i is an allocation decision variable.

The solution of above allocation problem can be used to find the volume demanded and supplied. Below are the formulas for computing the volumes of matched multi-unit bids V_b and asks V_a.

$$V_b = \sum_i q_i \quad \text{s.t. } q_i > 0, \lambda_i > 0 \tag{5.19}$$

$$V_a = \sum_i |q_i| \quad \text{s.t. } q_i < 0, \lambda_i > 0 \tag{5.20}$$

Let us denote the number of multi-unit bids matched (both fully and partially) as K, and for multi-unit asks L. Also Kth multi-unit bid would mean the lowest bid matched, and Lth multi-unit ask would mean highest ask matched. I denote their quoted valuations as b_K and a_L, and quantities as bq_K and aq_L respectively. From Lemma 5.3.1, we know that there is at most one $\lambda_i \in (0, 1)$ exists, so let us denote this as λ^*. It can also be noted that if such λ^* exists, it either belongs to Kth multi-unit bid, or Lth multi-unit ask. Now depending on whether λ^* exists, and if it exists, to whom it is assigned to, we apply appropriate payment rule. There are 3 cases that can emerge in this mechanism:

1. No λ^*: This would mean that supply and demand is matched exactly, hence $V_a = V_b$. In this case, buyers pay at b_{K+1}, sellers receive at good a_{L+1}. Because $b_{K+1} < a_{L+1}$, mechanism subsidises the deficit of $V_a(a_{L+1} - b_{K+1})$. Figure 5.1 illustrates supply and demand lines and corresponding clearing prices for bids and asks. It also shows deficit mechanism subsidises in blue-shaded A box.

2. λ^* is assigned to buyer: This means that there is an over-demand, hence $V_b > V_a$. In this case, mechanism rejects Kth multi-unit bid. If there is a tie, it is randomly resolved. The remaining $K - 1$ buyers pay b_K per unit, L sellers receive a_{L+1} per unit. As the implication of Kth buyer rejection, a number of sellers at the bottom of the list can be exposed to $V_a - V_b + bq_K$ number of goods unmatched. So mechanism pays out $a_{L+1}(V_a - V_b + bq_K)$ to them. Because $b_K < a_{L+1}$ and number of full matches is $V_b - bq_K$, mechanism subsidises in total the deficit of $(V_b - bq_K)(a_{L+1} - b_K) + a_{L+1}(V_a - V_b + bq_K)$. Figure 5.2 illustrates the supply and demand lines and corresponding clearing prices for bids

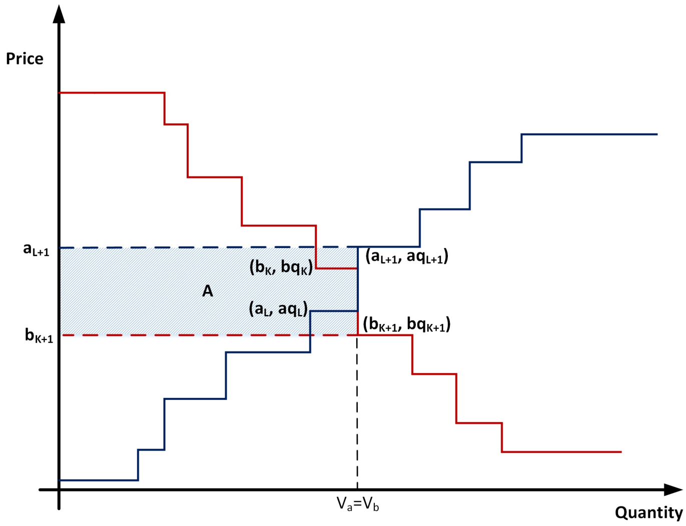


Figure 5.1: Multi-unit DA pricing rule when supply and demand match

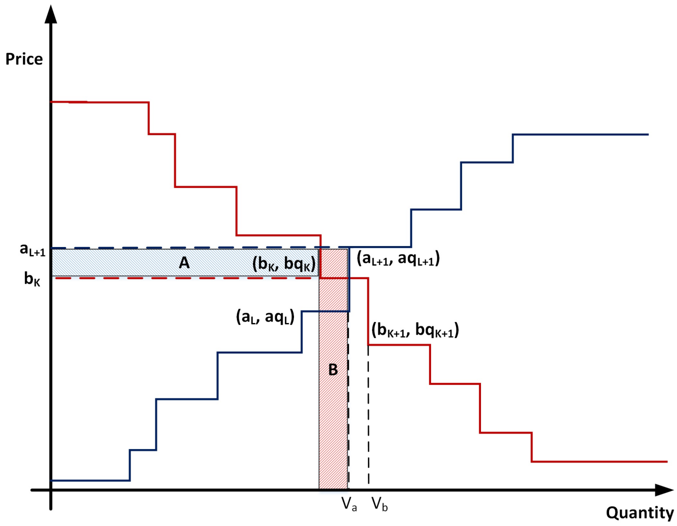


Figure 5.2: Multi-unit DA pricing rule when there is over-demand

and asks. It shows two deficits subsidised by mechanism: A denotes the deficit from the difference of clearing bid and ask, and B denotes the deficit created from mechanism buying from exposed asks.

3. λ^* is assigned to seller: This means that there is an over-supply, hence $V_b < V_a$. In this case, mechanism rejects Lth multi-unit ask. If there is a tie, it is randomly resolved. The remaining $L-1$ sellers receive a_L per unit, K buyers pay b_{K+1} per unit. As the implication of Lth seller rejection, a number of buyers at the bottom of the list can be exposed to $V_b - V_a + aq_L$ number of goods unmatched. So mechanism sells out in total $b_{K+1}(V_b - V_a + aq_L)$ worth of goods, and generates income. Because

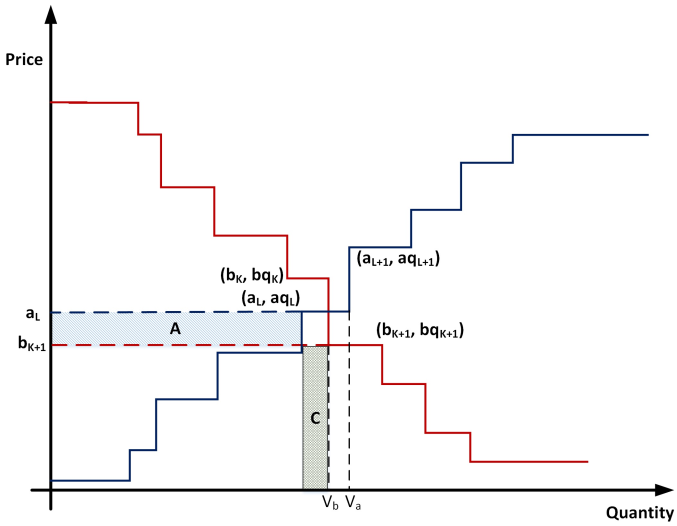


Figure 5.3: Multi-unit DA pricing rule when there is over-supply

$b_{K+1} < a_L$ and number of full matches is $V_a - aq_L$, mechanism subsidises in total the deficit of $(V_a - aq_L)(a_{L+1} - b_K) - b_{K+1}(V_b - V_a + aq_L)$. Figure 5.3 illustrates the supply and demand lines and corresponding clearing prices for bids and asks. It shows one deficit and one gain taken by mechanism: A denotes the deficit from the difference of clearing bid and ask, and C denotes the income caused by mechanism selling to exposed bids.

In above payment rules, mechanism is not only taking loss from clearing bids and asks at their offsetting prices, but also covering the exposed bids and asks resulting from the rejection of least efficient traders. Although the first part of the mechanism's loss can be insignificant in competitive markets due to narrow difference between inefficient bid and ask, the second part contributes the large portion of it, as the mechanism takes the responsibility to cover the exposed bids or asks. Given that the difference between $a_{L+1} - b_K$ is insignificant, the worst case budget-deficit for the mechanism is given below:

$$\bar{q}(K-1)(a_{L+1} - b_K) + a_{L+1}(\bar{q} - 1) \tag{5.21}$$

In worst case budget-deficit scenario, all buyers submit cap quantities $\bar{q}$, and Kth multi-unit bid is covered for $\bar{q} - 1$ of its bid. The mechanism rejects Kth bid, and leaves $\bar{q} - 1$ quantities for matched asks exposed. Mechanism spends extra $a_{L+1}(\bar{q} - 1)$ to cover these exposed asks. Hence it is the incentive of the mechanism to keep $\bar{q}$ as low possible to minimise its loss. For this purpose, I shall also empirically analyse the budget-balance of the mechanism in different experimental cases in Section 5.6.

Theorem 5.3.2. *Proposed multi-unit DA is DSIC and individual rational.*

Proof. Proof is done using Vickrey's argument. Without loss of generality, let us assume buyer i submits multi-unit bid (b_i, q_i) and $b_i > v_i$.

1. No λ^*: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. If buyer gets fully satisfied, then $b_i \geq b_{K+1}$. So buyer's utility is $v_i - b_{K+1}$, and in case if it is $v_i < b_{K+1}$ buyer gets negative utility, while if he posted v_i he would not trade and his utility would be zero. If $v_i \geq b_{K+1}$, the utility is indifferent to truthful bidding. If his bid is rejected, buyer is also indifferent, because his utility is zero.

2. λ^* assigned to buyers: Then the clearing price is b_K, $K-1$ buyers trade and there is one partially satisfied bid. If buyer gets fully satisfied, the above Vickrey's argument applies for critical bid b_K. If buyer gets rejected, he is indifferent to truthful bidding. However if buyer is partially satisfied, then $b_K = b_i > v_i$, he is rejected and he would be rejected for submitting v_i. So he is indifferent.

3. λ^* assigned to sellers: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. The same argument for no λ^* case applies here.

In case if bidder submits $b_i < v_i$.

1. No λ^*: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. If buyer gets fully satisfied, then $v_i > b_i \geq b_{K+1}$ and buyer has the same positive utility. If buyer gets rejected, and $v_i > b_{K+1}$, buyer misses the positive utility, otherwise he is indifferent.

2. λ^* assigned to buyers: Then the clearing price is b_K, $K-1$ buyers trade and there is one partially satisfied bid. If buyer gets fully satisfied, he is indifferent. If buyer gets rejected, the above Vickrey's argument applies for critical bid b_K. However if buyer is partially satisfied, then $b_K = b_i < v_i$, he is rejected and misses a positive utility.

3. λ^* assigned to sellers: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. The same argument for no λ^* case applies here.

So there is a dominant strategy for buyer i, and it is $b_i = v_i$.

If rationality of all buyers assumption holds, then the buyer must play his dominant strategy. I have shown that in this case, buyer's utility is always non-negative. Hence, buyer is ex-post individual rational. Same argument applies to sellers. □

Because the mechanism runs only once on a specific trading day, DSIC is supported only within the span of this trading day. Hence it does not apply for the series of trading rounds throughout the option's lifespan. In other words, each simulation of a trading day is a separate instance of a mechanism with heterogeneous traders truthfully submitting the results of their evaluations of option price.

There two ways of looking at the efficiency of the mechanism I proposed. First way is computing the efficient trades happened within the mechanism. Because mechanism takes the place of Kth (Lth) rejected partially satisfied buyer (seller), the efficient trades are not lost. Hence mechanism can be considered efficient. However there is a partially satisfied bid (ask) rejected from the trade. In second way of looking at mechanism's efficiency, we can consider this rejected partially satisfied bid (ask) as the lost efficiency, because the traders are not benefiting from it. In this case, at most $\bar{q}-1$ units of goods supposed for trade can be lost. I shall analyse this loss in my experiments.

Unfortunately, proposed multi-unit DA like any other Vickrey mechanism is highly susceptible for colluding. To show this, we can assume following hypothetical situation. In option pricing, the option's price is bounded by its intrinsic value and the spot price of the underlying asset. Hence any trader valuation is between $[\max(S_0 - F, 0), S_0]$ for calls, and $[\max(F - S_0, 0), S_0]$ for puts [1]. Consider all K and L traders report minimum possible value $\max(S_0 - F, 0)$, however $a_{L+1} = S_0$. Let us assume that 'λ^* is assigned to buyers. There are $K-1$ buyers and L sellers who trade successfully. The Kth bid is partially satisfied, so it is rejected. In this situation, using the formula for worst-case budget-deficit (5.21), we can compute what mechanism pays out:

$$\bar{q}(S_0 - \max(S_0 - F, 0))(K-1) + S_0(\bar{q}-1) \leq \bar{q}S_0(K-1) + S_0(\bar{q}-1) = S_0(\bar{q}K-1) \tag{5.22}$$

The result of the equation (5.22) is, indeed, the same as the mechanism's efficiency, because all the winning asks are paid at S_0, and mechanism received nothing from buyers. This example shows that the mechanism can be the only

[1]We changed the strike K to F to avoid confusion with K efficient buyers

contributor of the efficiency created. There are many known ways of preventing collusion in Vickrey mechanism, and one simple way of discouraging it can be the introduction of posted reserve prices on both sides of the market. However I do not use this rule in my experiments for simplicity reason, because computing the actual value of the reserve price requires prior information about the type distribution of traders. In our mechanism, traders use sophisticated option pricing methods and it is difficult to find the analytical forms of their respective distributions.

Also it is worthwhile to mention that the proposed mechanism is tractable, because it uses LP for determining the allocation which is polynomially solvable, and the payment rule is $O(1)$.

5.3.3 Revelation of ZI Traders

In this section, I propose different allocation and payment rules for ZI traders who are also considered as forecasting traders. ZI traders simulate the underlying market and come up with random path of the asset prices. ZI trader believes that this asset price path is his forecast, and computes the option's intrinsic value based on this information. Once the information about the asset price is provided, computing the option's intrinsic value becomes deterministic and can be safely delegated to some other agent. I have already mentioned in early sections that any dominant strategy mechanism can be converted to direct mechanism using the revelation principle. So I can also convert proposed multi-unit DA into a mechanism where ZI trader can truthfully reveal not only his bid or ask, but his prediction too. In fact, if he reveals his prediction, there is no need for him to reveal his actual bid or ask quote, other than reporting the quantity he needed. This significantly simplifies the job of the mechanism when it comes to pricing different option types such as OTM call or ATM put, etc. Instead of collecting quotes for each option type separately, mechanism can simply solicit the predictions, and corresponding quantities for each type of option. I shall discuss about this in next chapter.

Given that the intrinsic value of the call option is $\max(S_0 - K, 0)$, and ZI trader has his prediction of $\hat{S}_{T,i}$, the value of the option for the trader is $e^{-rT}\max(\hat{S}_{T,i} - K, 0)$. Thus trader can reveal his prediction, and let the mechanism to do the evaluation. Also the traders option valuation is linearly increasing for his prediction. Hence the higher is the prediction, the more is the option valued by the trader. Combining these facts together, we can formulate following allocation problem for ZI traders:

Definition 5.3.4. For given vectors of predictions and quantities (v, q) of ZI traders, allocation rule for multi-unit DA is:

$$\max_{\lambda} \sum_i q_i \lambda_i S_{T,i} \tag{5.23}$$

$$s.t. \quad \lambda_i \in [0,1] \quad \forall i \tag{5.24}$$

$$\sum_i q_i \lambda_i = 0 \tag{5.25}$$

where $q_i \in \mathbb{Z}$ represents quantities, $S_{T,i}$ is the ZI agent's prediction, λ_i is an allocation decision variable.

As it can be observed from above allocation rule, now mechanism uses predicted asset price $\hat{S}_{T,i}$ for each agent, instead of the option price. In previous model, the ZI traders would have submitted 0 if their predictions where below strike K. Although the price of the option is zero anyway, the predictions below K could be different too, and this information is lost while the option price is being computed. Now mechanism knows who has the worst and who has optimistic predictions, and based on this factor it can decide to whom allocate the option. Of course, sellers expecting the asset price fall are matched with the buyers expecting the rise in asset price. This is much better perspective of matching orders, because otherwise sellers would have submitted zero making them indistinguishable from each other, and most importantly, making buyers want to freely acquire options in maximum amounts. Above revelation helps sellers who expect drop in price to generate more profit, even though their valuation is zero.

To see how the option price is computed from resolved predictions, I use the same payment rule described for multi-unit DA. After figuring out the clearing predictions $S_{T,K}$ for buyers, and $S_{T,L}$ for sellers, the option's intrinsic values for both are found, and traders are cleared at these computed prices respectively.

The same methodology can be used for other traders such as VOL (i.e. volatility) trader who uses Black-Scholes method after projecting the volatility surface from market information available. VOL traders can quote their corresponding implied volatilities like ZI traders do their predictions. There is a monotonically increasing relationship between implied volatility σ_i of the agent i and his computed option price. LMSR trader can reveal his portfolio to the mechanism, because this is main factor used to price the option. However, traders like MC and EXP that we proposed in Chapter 4, use an array of stochastic simulations of asset prices to compute the option prices. So their pricing methods are not deterministic and the mechanism is unlikely to replicate exactly the same simulation paths used by these traders even if they reveal their key parameters to the mechanism. I shall provide simulation results for the revelation of ZI traders later in this chapter.

5.3.4 Parallel Multi-Unit Double Auctions

In this section, I extend multi-unit DA into parallel multi-unit double-auctions to accommodate multi-unit multi-item bids and asks of traders. This involves trading different option types in parallel, so the traders can obtain desired option portfolios through this mechanism. I shall assume that the traders must indicate the quantities for the exact type of option listed in the option chain. For example, bearish spread option portfolio requires the trader to buy one OTM call and sell one ITM call, but it does not specify at which strike price the OTM and ITM must be. The trader has to choose one of the available OTMs or ITMs and use it as part of his strategy. This concept is further generalised to combinatorial exchanges in Chapter 7, where OTM and ITM options are considered as substitutes and the trader does not need to specify the strike price of the OTM or ITM option, as the mechanism determine which OTM or ITM to provide based the traders valuation.

Simultaneous multi-unit DA can be viewed as multi-unit multi-item DA where traders can disclose their linear valuations of options to market maker where traders would want to have their orders satisfied fully or nothing. In this setup, the valuation and the quantities matrices (V, Q) represent the trader preferences for each option $o \in G$ option chain. So let us construct LP allocation rule for this mechanism:

Definition 5.3.5. For given valuations and quantities (V, Q), allocation rule for simultaneous multi-unit DA is

$$\max_{\lambda} \sum_i \sum_j v_{ij} q_{ij} \lambda_{ij} \tag{5.26}$$

$$s.t. \quad \lambda_{ij} \in [0, 1] \quad \forall i \in N, \forall j \in G \tag{5.27}$$

$$\sum_i q_{ij} \lambda_{ij} = 0, \quad \forall j \in G \tag{5.28}$$

where λ_{ij} determines the allocation of each option to each trader.

However it follows from Lemma 5.3.1 that there must be at most G number of partially satisfied bids/asks. In order to preserve the atomicity of the orders, mechanism must discard all of the other multi-unit bid/ask of partially satisfied traders even if they are fully satisfied in other DA. It uses the same pricing method per DA as it was described previously. So the mechanism has to cover the exposed bids/asks at its expense. Hence it can also inherit DSIC and individual rationality from single-item multi-unit DA.

In the example of ZI traders, the mechanism can avoid the complexity of matching multiple valuations of options in parallel DAs. It can simply use McAfee's (5.9) and (5.10) rules, which sorts the buyers and sellers based on their predictions $\hat{S}_{T,i}$, and then reject partially satisfied bids/asks for each DA and globally. Mechanism as usual covers the exposed bids and asks, applies the same payment rule. I shall provide an experimental results for option portfolio trading agents using this mechanism.

5.4 Design of Experiments

In this section, I present the structure and parameters of the experiments that I conduct in order to observe the option prices reported by traders and the clearing prices that mechanism has chosen. First, I provide the instances of traders that will be used as the participants of my proposed DA. Then I talk about the market simulation involving the parameters of the underlying market simulation. I also provide the instances of options that are priced in proposed mechanism, and determine the aspects that are analysed for each market simulation. Finally, I highlight the details of the experiments run for determining the option Greeks such as delta, gamma and theta.

5.4.1 Traders

We have looked at different ways of obtaining option prices for traders in Chapter 4, but we are yet to observe how they are going to be used in proposed mechanism. I have to split the traders into two major categories. The ones who interact with a multi-unit DA, and trade one type of option there, and the others as option portfolio traders who submit their preferences to simultaneous multi-unit DA. The first category of traders can be further broken down into following groups:

- risk-neutral traders
- risk-averse traders
- portfolio holders
- mixed traders

Risk-neutral traders use VOL (Volatility pricing) and MC (Monte-Carlo) option pricing methods and their results are compared with BS (Black-Scholes) prices. MC traders can come in different variations depending on the asset pricing model they use to simulate the possible trajectories of the underlying market. I use two of the proposed asset pricing method: GBM and JD models with parameters found earlier. I denote MC trader using GBM asset pricing model as MC, and the one who uses JD model as MC*. Also all of the traders can use one of the quantity choosing methods proposed: random integers (RND) and linear quantities (LIN). The quantities for each trader are capped at $\bar{q} = 2000$, because of the mechanism's limitation. Also MC traders can simulate the interest rates using one of three approaches: a) interest rate is constant, b) interest rate follows Vasicek's model, c) interest rate follows Vasicek Jump model. Agents use constant annualised risk-free interest rate $r = 0.12$ to simulate the their asset price trajectories. If MC trader uses the risk-free rate as a stochastic process, then for Vasicek process it uses following parameters: drift $\mu_r = 0.46$, volatility $\sigma_r = 0.09$, mean-reversion $\theta = 0.46$ which where found from calibrating to historic US T-Bills in Chapter 3. For simulating risk-free rates as Vasicek Jump process it uses the same parameters as Vasicek process, but for jumps: arrival rate $\lambda = 4$, mean jump $\mu_\lambda = 0.1$ and jump standard deviation $\sigma_\lambda = 0.2$. These parameters were arbitrarily set as they were simulated in earlier chapter.

I run series of experiments with risk-neutral traders, where some cases include solely the same trader, while others involve mixed traders. This way we can observe how option prices may change depending on the population of traders in the market. Table 5.1 shows the instances of risk-neutral traders involved in each experimental case.

Risk-averse traders involve traders with exponential utility. These type of traders normally overprice the options when selling, and underprice when buy-

Experimental Case	**Traders**	**Population**
Monte-Carlo 1	MC-RND	100
Monte-Carlo 2	MC-LIN	100
Monte-Carlo JD 1	MC*-RND	100
Monte-Carlo JD 2	MC*-LIN	100
Volatility 1	VOL-RND	100
Volatility 2	VOL-LIN	100
Mixed Risk Neutral	MC-RND MC*-RND VOL-RND	30 30 40

Table 5.1: Experiments with Risk-Neutral Traders

ing because of the perceived fear of loosing. Hence, if the market population is made of only risk-averse traders, there should not be any trade due to missing match. The risk-aversion of EXP traders depends on parameter a. For each option setting, there is a threshold for this parameter which sets EXP buyer and EXP seller submit such orders, so they do not match. From the conducted experiments, I found out that this parameter is $a = 0.03$. If $a < 0.03$, then EXP sellers and buyers match, and basically simulate MC-RND trader. Therefore I set the risk-aversion to threshold value. I also use -RND for choosing quantities.

In order to facilitate trade among risk-averse traders, there should traders who bid or ask even worse than them. If the market is populated with one of the risk-neutral traders, say, MC-RND, these traders trade mutually with each other constituting Black-Scholes prices and leaving most EXP-RND traders rejected. In other words, because MC-RND submit very small bid-ask spread, EXP-RND traders cannot beat them and mostly stay out from the trade. Therefore I use ZI-RND (Zero-Intelligence) traders who provide liquidity for risk-averse traders and enable them to participate in trade. ZI-RND traders are meant to produce any possible option price within legitimate range. This is, of course, provide chance for EXP-RND traders to clear their orders. However it is important to note that if there are other traders, such as risk-neutral traders in the market, and they are not less than ZI-RND traders, they take ZI-RND's cheap deals because of their competitive orders and leave EXP-RND traders outside this deal.

Therefore I simulate two experiments involving EXP-RND traders: one with 80 EXP-RND traders and 20 ZI traders, and the other 40 EXP-RND, 40 ZI-RND and 20 MC-RND. In the first experiment, we observe what prices EXP-RND

Experimental Case	Traders	Population
Mixed Exp 1	EXP-RND ZI-RND	80 20
Mixed Exp 2	EXP-RND ZI-RND VOL-RND	40 40 20

Table 5.2: Experiments with Risk-Averse Traders

traders bid and ask, and how their orders are matched with ZI-RND traders. The second experiment introduces few MC-RND traders in to the mechanism, and check how MC-RND traders contribute in bringing down the prices in the market. Table 5.2 shows the experimental cases using EXP traders.

The portfolio holding traders use LMSR pricing method and as it was said before their option pricing emerges from the portfolio they already hold. Similar to EXP-RND traders, LMSR traders also create a positive bid-ask spread, which forbids them from trading if the market is uniformly populated with LMSR traders holding the same portfolio. Therefore LMSR trader should be also simulated in mixed groups each holding different set of portfolios, and thus produce different prices. It is also important to note that LMSR trader unlike other trader are deterministic in their pricing, because the only factor which affects their pricing decision is their portfolio and fixed range of events that can occur. Therefore two LMSR traders holding the same portfolio produce same bids or same asks. To make market more heterogeneous, I use most of the option portfolios given in Table 4.12to simulate traders from neutral, non-neutral, bullish and bearish perspectives. The full list of LMSR traders with the portfolios they hold is given in Table 5.3.

LMSR trader's bid-ask spread width is determined using his liquidity parameter b. After running several experiments with LMSR trader, I found out that $b = 100$ provides reasonable range of bids and asks which are likely to match. LMSR use only -RND method to pick quantities for options. Table 5.4 lists these experimental scenarios using different LMSR traders together.

And finally, I run all proposed option traders together to observe the market when the options are priced truly from different perspectives. The traders that I use for 'Mixed All' case experiment are shown in Table 5.5. I have included 10 traders for each type of option pricing method proposed, and in total the trader population is 100. This experiment should mimic the heterogeneity of trader's pricing policies, and determine the equilibrium price of options when

Trader Name	Belief	Portfolio
LMSR-NEUT1	Neutral	Butterfly Call Spread
LMSR-NEUT2	Neutral	Iron Butterfly
LMSR-NEUT3	Neutral	Long Call Ladder
LMSR-NEUT4	Neutral	Short Strangle
LMSR-NON-NEUT1	Non-Neutral	Short Call Ladder
LMSR-NON-NEUT2	Non-Neutral	Long Straddle
LMSR-NON-NEUT3	Non-Neutral	Long Strangle
LMSR-NON-NEUT4	Non-Neutral	Strip
LMSR-BULL	Bullish	Bullish Call Spread
LMSR-BEAR	Bearish	Bearish Call Spread

Table 5.3: LMSR Traders and their portfolios

Experimental Case	Traders	Population
LMSR Neutral And Non-Neutral	LMSR-NEUT1 LMSR-NEUT2 LMSR-NON-NEUT1 LMSR-NON-NEUT2	25 25 25 25
LMSR All	LMSR-NEUT1 LMSR-BULL LMSR-BEAR LMSR-NON-NEUT1	25 25 25 25
LMSR More Bull	LMSR-NEUT3 LMSR-BULL LMSR-BEAR LMSR-NON-NEUT3	10 70 10 10
LMSR More Bear	LMSR-NEUT4 LMSR-BULL LMSR-BEAR LMSR-NON-NEUT4	10 10 70 10

Table 5.4: Experiments with LMSR Traders

Traders	**Population**
MC-RND	10
MC*-RND	10
MC-LIN	10
MC*-LIN	10
VOL-RND	10
EXP-RND	10
LMSR-NEUT1	10
LMSR-NON-NEUT1	10
LMSR-BULL	10
LMSR-BEAR	10

Table 5.5: Experiment with All Traders - 'Mixed All'

each of the traders are evenly distributed in the market.

Besides the above given traders, I also simulate ZI traders and option portfolio traders. Because ZI traders submit their predictions instead of the option prices, the simulation setup involve different settings for these agents. Along with the option prices, I also plot the traders aggregated forecasts and observe how they changed based on cases when the traders exhibit bullish, bearish or neutral forecasts. ZI traders interact with multi-unit DA similar to other agents. I set at the core of the option portfolio trader - the ZI trader who choose corresponding option portfolio based on his forecast. Also the structure of option portfolio bids differ from multi-unit option bids. Such trader needs simultaneous multi-unit DAs to post his quotes on different types of options. For that reason, I also run such mechanism and observe how traders taking different option portfolios collaboratively affect the aggregated predictions by picking bullish, bearish or neutral portfolios.

5.4.2 Market Simulation

I describe how both underlying and option markets are simulated in proposed experiments. I simulated the asset prices using GBM with parameters that I found by calibrating NASDAQ-100 indices. So daily drift is $\mu = 0.0007$, and volatility is $\sigma = 0.0089$. The figure 5.4 shows the instance of simulated asset prices that I use for all experiments. Using the asset prices given in Figure 5.4 enables to compare different experimental setups and their impact on pricing option. It can be seen that the initial asset price is the same as NASDAQ-100 on 2 January 2014, $S_0 = \$3563.57$. And the asset price ends up at $S_T = \$3597.59$ at the end of the year. So it is slightly increased due to

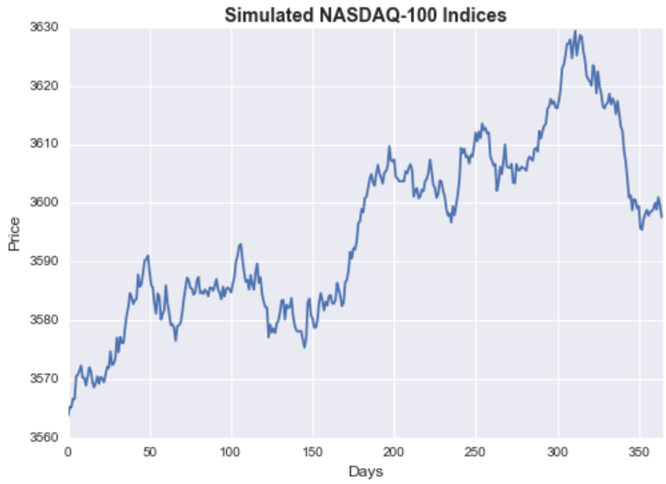


Figure 5.4: Simulated NASDAQ-100 Indices

Option Type	**Strike** K	**Maturity Date** T
OTM Call	\$ 3603.00	1
ATM Call	\$ 3563.00	1
ITM Call	\$ 3523.00	1

Table 5.6: Options Traded in DA

small mean drift. This particular instance of asset price is interesting because it includes dramatic fluctuation near the end of the year. This should enable us to stress test the option pricing methods that I proposed.

The option market is run using proposed multi-unit DA. I create 100 traders for each trading day, so they submit their corresponding pricing bids and asks to the mechanism which then matches them and clears the orders at DSIC prices. We can observer how different set of traders defined in previous section result in different option prices. I also analyse three types of options in proposed mechanism and simulate them separately with given set of traders. Table 5.6 shows these three option that I use in my simulations. It can be seen in Table 5.6, that I use \$40 interval between strikes to determine OTM, ATM and ITM options. Note that OTM Call option ends up out-of-the-money when the option expires at $T = 0$, because $S_T = \$3597.59 < \$3603.00$. In this way, it is possible to observe if the traders also end up pricing this option at zero price, because its intrinsic value is, indeed, zero. Also I use only call options for the simulation, because put prices can be directly computed from the call price using call-put parity relationship.

Mechanism simulates 365 trading days, going up to the point the option expires. For each trading day, I capture key outcomes of the mechanism for further analysis. This involves the clearing prices, volumes traded and some other relevant statistics. The mechanism is analysed using following parameters for each trading day.

- Black-Scholes Price: This shows the current Black-Scholes price for comparison. In this way, we can see how the option prices differ from theoretical price.

- DA Ask Price: The clearing ask price resulted from the trades. This is the lowest rejected ask according the proposed mechanisms payment rule.

- DA Bid Price: The clearing bid price resulted from the trades. This is the highest rejected bid according the proposed mechanisms payment rule.

- Trade Volume: This shows how many options have been traded, so we can observe how volumes for ATM, OTM and ITM options differ, and at which settings these differences happen.

- Efficient Trades Rejected: This gives us an indicator on what proportion of efficient trades have been rejected by the mechanism due to being partially satisfied. This parameter can also differ by given mechanism setting.

- Budget Balance: This is an important figure which describes the gains and losses of the mechanism from participating in the trading process in order to cover exposed bids and asks that are resulted from rejecting efficient trades.

- Min Accepted Ask: This factor shows the lower boundary for all accepted orders in the mechanism. We shall analyse how this boundary changes as the option approaches its expiry date.

- Max Accepted Bid: This factor shows the upper boundary for all accepted orders in the mechanism. We shall analyse how this boundary changes as the option approaches its expiry date.

- Mean Accepted Order: This factor shows mean price for accepted orders in the mechanism. Although the mechanism is not cleared at this price, it can be used for analysing other parameters such as option delta, gamma and vega.

From captured details, I plot corresponding charts and analyse the simulation results. The market prices are plotted as line graph, indicating trading days on horizontal axis, and the prices on vertical axis. It could be presented as a candlestick graph like most of the price charts in real markets having ask and bid prices shown separately, but due to very narrow spread between cleared bid and ask prices, the candlestick graph looked less comprehensive. I show the range of accepted orders using shaded area around cleared asset prices. I also display the volume information parallel to the horizontal timeline axis show how many option have been traded for each particular day. I shall present the efficiency error and budget-balance in histograms to observe the magnitude and the distribution of these parameters for each experimental case.

The experimental setup for simulating the market of ZI traders and consequently the option portfolio traders is different from the previous setting. Because the prices are compared with the aggregated forecasts, it would be easier to $S_0 = 100$, so the magnitude of forecasts are clearer. Below Table 5.7 provides the key parameters that are used to simulate underlying asset market and simultaneous multi-unit DAs.

Name	Value
Initial Asset Price	$S_0 = 100$
Strike Price	$K = 100$
Strike Intervals	$\epsilon = 10$
Asset Price Volatility	$\sigma = 0.05$
Risk-free rate	$r = 0$
Time to maturity	$T = 1$
Number of trading days	100
Random Quantities Scaler Range	[-15, 15]
Number of agents	$N = 100$
Number of option types	$G = 3$

Table 5.7: Market setup for ZI traders and option portfolio traders

It can be seen from the Table 5.7 that I run 3 simultaneous mechanisms for calls with different strikes in $K \pm 10$. These mechanisms are used by option portfolio traders to take their positions based on their beliefs. The increased trade in OTM options aggregate to higher forecasts, and for ITM, it means that traders expecting the prices to fall. Along with aggregated forecasts, I present the corresponding valuations for ATM call and its corresponding Black-Schole value. I test cases where the ZI traders have different volatility in simulating the asset prices and where supply and demand are unbalanced.

These experiments also show the effect of market volume in determining the option prices.

5.4.3 Greeks Simulation

In Greeks simulations, I analyse the change of option prices generated by the mechanism with respect to changes in 3 relevant factors such as asset price, delta and time-to-maturity. These are also called as option's delta, gamma and theta. These parameters play key role in risk-neutral pricing of any financial instrument. For example, option's delta can be applied in constructing self-financing portfolios, while gamma is used in making portfolio more stable over a longer period of time through a gamma-hedging strategy. Theta is also an important indicator of option's performance, as it shows how option's value decreases as it approaches its maturity. It determines the likelihood of option ending up in-the-money or out-of-the-money for any given moment of option's lifespan. As a results, option price is less likely to change by the end of its maturity date. This fact should be also observed in the experiments. The formulas for option's Greeks analysis are given in Chapter 2, so I do not write them here.

I shall simulate each Greek parameters with respect to two parameters. In first case, I change the asset price linearly from \$3465 to \$3665 covering moneyness range of all three initialised options listed in Table 5.6. For example, at asset price \$3665 even the initially created OTM ($K = 3603.00$) option ends up in-the-money. Also at asset price \$3465, even initially declared ITM ($K = 3523.00$) option is out-of-the-money. So it covers every possible case for all three options, and let us observe how their Greeks change with the asset prices. These linearly increasing prices are used as asset prices and traded in proposed multi-unit DA. Note that the time-to-maturity while changing the asset prices is fixed at $T = 1$.

I also run the simulation for Greeks by linearly decreasing the time-to-maturity parameter, and fixing the asset price at ATM $S_0 = K$. In this way, we can observe how option Greeks change according to the time from $T = 1$ to $T = 0$. As the option nears its maturity date, Greeks exhibit an interesting behaviour showing the sensitivity of option price for changes in asset price, delta and time-to-maturity. For example, delta for OTM approaches zero as the option nears its maturity, meaning that the slight changes in the asset price are less likely to result in option price to change from zero. While for ITM option, the asset price changes are almost at the same magnitude as the option price when option is near its maturity date. It is interesting to observe how mechanism is able to simulate these properties of option pricing.

I present the results of the simulation of Greeks along with theoretical Black-Scholes Greeks for each type of option. This should enable us to see how these aspects of option pricing can be different from theoretically computed ones. It is also important to point out the similarities between two of them, such if they take similar shapes or not. For example, Black-Schole's delta for call over asset prices resembles a sigmoid function between 0 and 1. We should observe if the mechanism's results will correspond with such shape.

5.5 Verification of Proposed Mechanism

In this section, I present the first results from the mechanism's both market and Greeks simulations in order to verify if it gives expected results in a staged environment, or generate absurd option prices. I populate the market with Black-Scholes (BS-RND) traders with a slight perturbation in their main parameters such as volatility. This is done in order to facilitate trade, which otherwise would not happen because every one ends up at the same price. This insignificant perturbation should facilitate the trade, but also limit the range of valuations submitted to the mechanism. Also it can be seen from the name of the trader that he uses random integer to determine the option quantity. The main goal of simulating such scenario is to see if the cleared prices correspond with Black-Scholes price when all traders are using the same pricing technique. This is an important factor because it exhibits the several aspects of the proposed mechanism:

- Efficiency (Unanimity): The prices obtained by the mechanism must be the overall representation of submitted orders, and when all traders in the market are BS traders, the price should correspond with Black-Scholes price. This shows the efficiency of the mechanism, as it represents the unanimity of quotes.
- Consistency: The prices obtained by the mechanism must also be consistent with option's definition. This involves option's final payoff according to its strike and asset price at expiry date. For example, OTM option

price should end up at zero at its maturity, because it generates no payoff.

- Robustness: The sensitivity of option prices to changes in asset prices, delta and time-to-maturity must be consistent when the Greeks are simulated. This exhibits how option's delta, gamma and theta correspond to Black-Schole's delta, gamma and theta when the mechanism is put to simulation of one parameters while fixing the others. In the context of this research, these control factors are the asset prices and time-to-maturity.

- Budget-Deficit: This also shows the minimal loss the mechanism generates because all traders are almost unanimous at their bids and asks. Hence the difference between prices is insignificant and mechanism has to cover less.

Figure 5.5 shows the option pricing results for 3 options: ATM, OTM and ITM options in three consecutive charts. The prices are the results of orders of BS traders cleared by multi-unit DA. They are indicated with red line. The blue dash line represents the actual Black-Schole price for given option. Yellow shaded area shows the boundaries of accepted quotes. The lower chart indicates the volume traded for given date. The blue shade on top of the volume line denotes the volume of the rejected efficient trade.

We can see in Figure 5.5 that the option prices completely overlap with the Black-Scholes prices. This is the same for all ATM, OTM and ITM options. We can also note that the volumes of traded options are the same too. The range of accepted quotes, however, vary for OTM at the beginning of the option's life, but it narrows down at the end of the option's life. Also we can observe that the OTM ends up at zero at its maturity, which shows the consistency of the mechanism as well. ATM and ITM options expire in-the-money, hence their prices are positive and equal to their corresponding payoffs. Also we can notice the general pattern of valuations narrowing down to concrete value once option's payoff date approaches. Figure 5.6 illustrates the mechanism's error in efficiency and the distribution of mechanism's gain and deficit. We can see that most of the time, mechanism rejects about 0% to 5% of overall efficient trades for 100 traders with quantities capped at 2000. This indicator should go down as the number of traders increase, and the quantity quoted per each trader decreases. I have talked about this factor while describing the mechanism. One interesting aspect of error in efficiency is that it is mostly efficient for OTM option, because the range of accepted orders is wider, hence

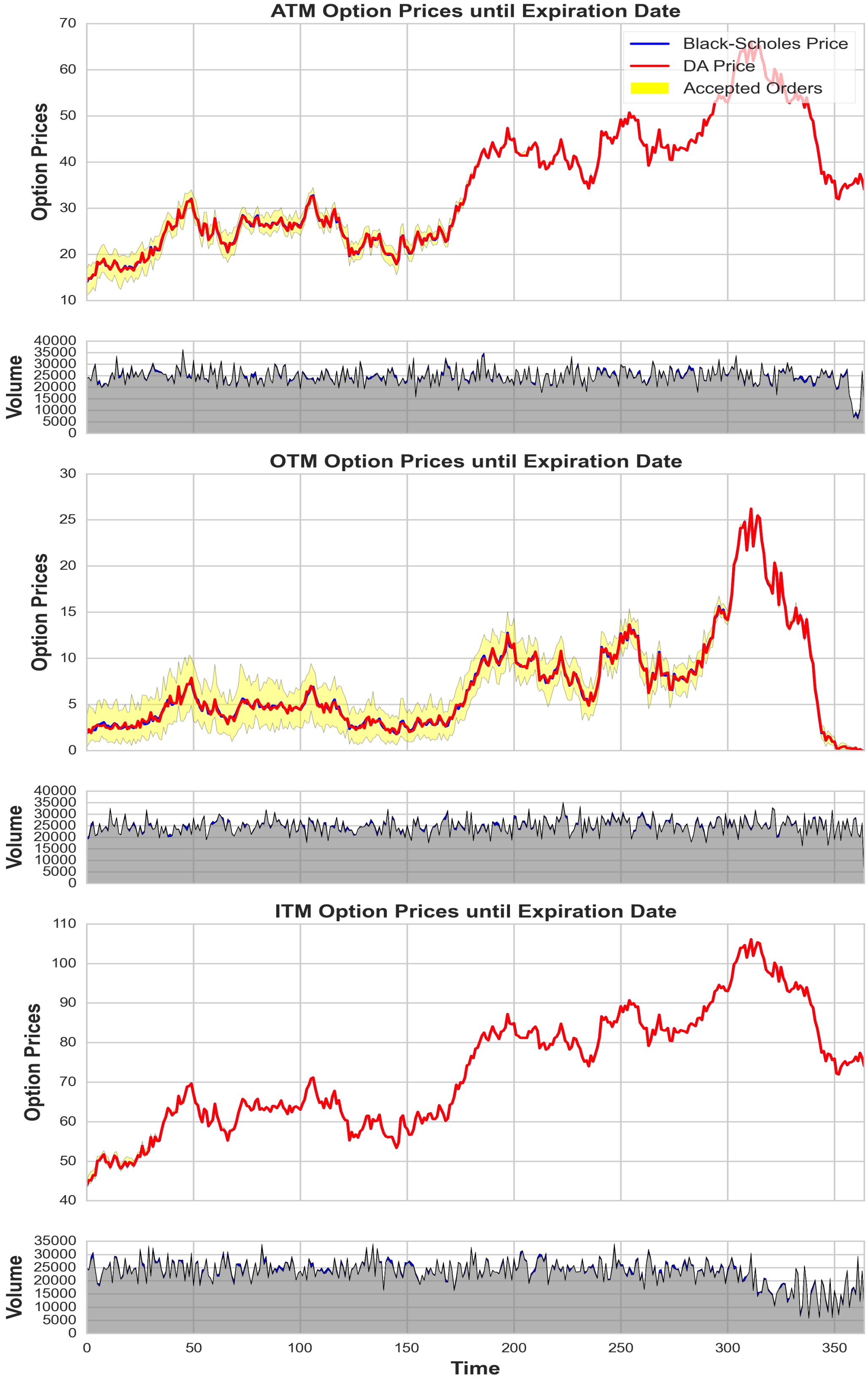


Figure 5.5: DA prices of ATM, OTM and ITM options for BS-RND traders

there are more matching orders and less partially satisfied orders.

Observing the budget balance histogram, we can see that it is evenly distributed around zero mean. Also because of more efficient trades, less partially satisfied orders and of course lower prices, mechanism has to cover less deficit resulting from OTM option. Therefore the budget-balance for OTM is centred around zero, while the budget-balance for ITM and ATM options are comparatively flatter.

We can also observe the option Greeks in Figure 5.6. As expected, the mechanism is robust for changes in parameters such asset price and time-to-maturity, because it replicates the same pattern as Black-Scholes does. The sensitivity of DA simulated option prices on asset prices, or delta, is equal to the same value obtained through Black-Scholes analytical solution. It can be seen in left figure, that the option price becomes more sensitive to asset price as the asset price grows, and the option goes deeper into moneyness, while the sensitivity is less when option is outside moneyness. This sigmoid-like delta curve becomes even steeper if the option is near its maturity date. While for ATM option it is 0.5 of asset price change, the OTM's delta is nearly zero when it is near its maturity date. And the opposite is true for the ITM option.

Option's DA gamma indicates the sensitivity of delta on asset prices, and it fully corresponds with Black-Scholes analytical gamma. It can be seen that the delta is at its maxima when the asset price is equal to strike price. The can be seen from delta chart too, where the delta grows the fastest around its strike. Also the gamma is more centralised if the option is near its maturity. This is because delta also become steeper when option approaches its maturity date. We can see from the right hand gamma chart, the ATM gamma grows upward as the option approaches its expiry. This is because there is an equal likelihood that the option can end up in-the-money or out-of-the-money, hence the delta is uncertain up until the expiry. If option ends up in-the-money, delta jumps to 1, and if out-of-the-money then it drops to zero. This dramatic change in delta, therefore is described by upward moving gamma. However the gamma for both ITM and OTM options approaches zero, because there is a least possibility the option near its maturity may end up in opposite moneyness range.

And finally, option's theta shows the sensitivity of option price to time-to-maturity and it also fully complies with Black-Scholes analytical solution. This is a negative indicator because mostly option looses its value as its payoff becomes more realistic with the time. We can see in the last pair of charts

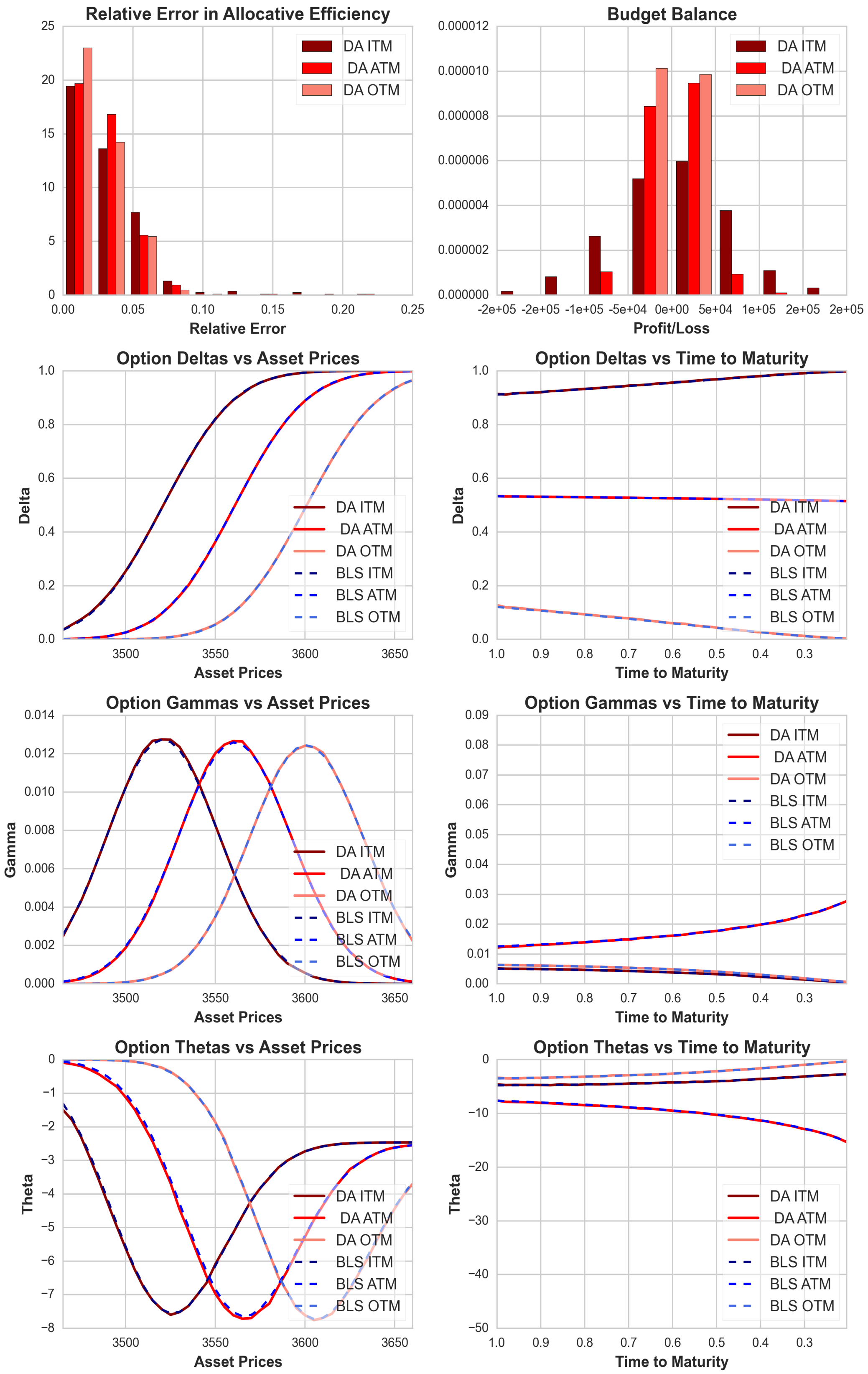


Figure 5.6: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for BS-RND traders

of Figure 5.6 that the option price is most sensitive to change in time when the asset price is equal to its strike. Because this is the very moment which determines if the option is going to be OTM or ITM onwards, and based on that fact, the time affects the further changes in option price. It can be seen that when option is deep out-of-the-money, the change in time has no effect in option's price, as it is always zero. However when the option goes deeper in-the-money, the theta steadily decreases, making the option yielding a payoff more realistic. Also it should be noted that the theta is more centralised for option near its maturity, because it is more certain at which side of the moneyness spectrum the option ends up. We can see in the right hand chart, that the theta goes down for the ATM option approaching its maturity. This justifies the point that the option price is more likely to loose lot when it is still uncertain about its moneyness and it is near its expiry date. However for OTM and ITM options the near to maturity timeframe is less sensitive, because there is certainty about option's moneyness and the payoff it is going to end up with.

Above presented results prove my point that the proposed mechanism populated with BS-RND traders can replicate the Black-Scholes formula and its analytical solutions for option's sensitivity on various factors. This should make the proposed mechanism reliable ground for testing other types of traders playing in the market, and analysing option prices and Greeks obtained from their simulation.

5.6 Experimental Results

In this section, I present the important findings from simulation of different traders in proposed DA. I enclosed all results of the simulation in Appendix D. Although I reference these results throughout this section, I do not include all of them into this chapter. I review the results from simulating risk-neutral, risk-averse, portfolio holding and mixed traders and point out their important characteristics. I also discuss about the results obtained from simulating ZI traders and multi-unit multi-item DA by observing the aggregated forecasts they collectively produced.

5.6.1 Risk-Neutral Traders

The risk-neutral traders rely on simulating the asset prices using risk-neutral drift or by solving Black-Scholes formula for implied volatility to price options. There are two types of risk-neutral traders that I simulated: ones that use random integer (-RND) to choose the quantity and ones that use linear function (-LIN). Although the pricing results for both types of risk-neutral traders are not significantly different from Black-Scholes price and analysis, the volumes traded for each are intuitively different.

Beside their prices are similar, there are certain differences in the workings of risk-neutral agents too. For example, VOL-RND trader produces results very close to Black-Scholes, because it actually uses Black-Scholes formula after figuring out the implied volatility from the previously traded price. Therefore there is a very little divergence in quoted prices. Although MC-RND resulted overall the same price as the VOL-RND and Black-Scholes formula, the range of accepted orders is wider meaning that there were more surplus, and hence more potential efficiency lost. Figure D.6 shows that the prices almost overlap, but the range of accepted orders in yellow shade is comparatively wider than in Figure D.2 for VOL-RND traders. Note that in both cases the range of accepted orders becomes smaller as the option approaches its maturity. The bigger range of orders produced by MC-RND traders are due to randomised numerical simulations that each MC-RND trader runs. Figures D.3 and D.7 show that there is also a difference in efficiency error between two types of traders. MC-RND traders generate more rejected partial orders due to the more diversity in their pricing, while VOL-RND efficiency error is bounded around 2% from overall efficiency. The other Greek indicators as well as budget-balance appear similar and correspond to Black-Scholes' analytical solutions.

However, we can see that MC*-RND traders price options slightly higher than Black-Scholes price due to potential expected jumps in the asset price. Although there is an analytical solution for such case where the asset price is a jump-diffusion process, and it is known as Black-Scholes-Merton model [115], I leave the comparison with this model outside this research scope. Figure D.10 illustrates this difference in prices. The difference appears to be even bigger for OTM options. Also we can notice that the range of accepted orders shifted upwards too along with the DA price. However DA and Black-Scholes prices converge as they approach maturity. For OTM option, we can see that they both end up at zero. We can see in Figure D.11 that the error in efficiency is also distributed with wider discrepancy, so the mechanism rejects about 8% of efficient trades partially satisfied due to mismatch in bid and ask quantities. The key differences are in Greeks. It can be seen in Figure D.11 that the delta is slightly skewed horizontally, which makes already over-priced options less sensitive to changes in asset price. This can be seen in delta's relationship with time to maturity, as the ITM line is less sensitive when the option approaches maturity, and vice versa OTM is more sensitive. This is because MC*-RND traders expect jumps and the change of option's moneyness is more likely than the asset price was following GBM. Similar differences can be observed in gamma and theta too.

Another category of risk-neutral traders are the traders that use linear functions for choosing quantities. Figures D.4 and D.5 show how VOL-LIN traders match their demand and supplies to obtain option prices. Although the prices are similar to VOL-RND, the error in efficiency is considerably different. The efficiency error for OTM option is almost evenly spread upto 6%, while for ITM and ATM options it is either no error or 4% error. Also we can see the difference in volume, while VOL-LIN traders generate around 60K trade volume, the result for VOL-RND is more than twice less that amount. It can also be seen that the volumes either drop or become unstable when the option is near its maturity and the prices are certain. This is because VOL-LIN traders on each side of the DA come up with almost the same price, and the same demand and supply. When there is a bid-ask spread, the mechanism clears them all, if there is no mechanism rejects everything. Therefore the volumes become so unstable. The Greeks for both VOL-LIN, VOL-RND and Black-Scholes are the same.

For MC-LIN and MC*-LIN traders presented in Figures D.8, D.9,D.12 and D.13 the results look similar too. However the volumes and efficiency error are different. For example, mechanism's error in efficiency is insignificant for MC*-LIN traders due to more diverse quotes and bigger volumes. The ATM and ITM options were traded at higher volumes, but OTM option resulted in 3 times lower volume. This because the MC*-LIN traders' supply decreases as the price of the option decreases too, hence there is less options available for buyers. This also affects the option Greeks. For example, comparing the deltas, we can see that for MC*-LIN traders, OTM prices are less sensitive to asset price changes than for MC*-RND traders.

5.6.2 Risk-Averse Traders

The pricing method of risk-averse traders is fundamentally different from risk-neutral pricing, because this looks at the option pricing from the perspective of a risk-averse agent who has more fear in loosing rather than gaining. As it was said earlier, the utility of this agent is modelled as a concave function with a parameter a indicating the concavity, or risk-averseness factor, of the trader. Therefore it is natural for EXP traders to overprice when selling or underprice when buying the option. Therefore market populated uniformly with risk averse traders does not produce any trade.

The results of the simulation of EXP-RND traders with ZI-RND trader shown in Figures D.16and D.17 display higher volatility in prices. We can also see that the range of accepted orders are considerably bigger which takes its toll on efficiency loss upto 30% from overall trade. This also affects the distribution of budget-balance which is in magnitudes higher than what was produced for risk-neutral traders. This wide acceptance range is caused by ZI-RND traders posting uncompetitive prices which are then matched with EXP-RND traders. We can also observe that there is a very small volume for each type of option, because there are only 20 traders per day. These 20 trades are basically the matches between all 20 ZI-RND traders and best 20 EXP-RND traders.

The Greeks analysis shows that the OTM and ATM options are highly sensitive to changes in asset price, while the sensitivity of ITM option price corresponds with Black-Scholes solution. Also the sensitivity to time-to-maturity is the same for all options and almost overlap with theoretical solution. The option's theta analysis is jagged up due to the volatility of OTM option price. The deeper OTM is the option, the more likely that its price is zero. However in proposed scenario there are EXP-RND traders selling deep OTM option for positive amount, and there are ZI-RND traders who wish to buy it for that amount. So the deep OTM's price is not zero, but it emerged from the random opportunity for EXP-RND trader by ZI-RND trader. This, of course, shows its impact on determining theta of the option in given experimental case.

In Figures D.18 and D.19, I display another set of results where there is small portion of risk-neutral traders in the market who are always matched with some portion of ZI-RND traders. Fortunately, the number of ZI-RND traders is 40, and VOL-RND traders 20, so this means that 20 trades are matched between ZI-RND and VOL-RND traders. However the remaining 20 ZI-RND trader should be matched with the best 20 EXP-RND traders. This experiment shows the role of risk-neutral traders in influencing the option prices, and the mechanism's efficiency and budget-balance. We can see from the Figure D.18 that the prices are indeed more stable, although the range of accepted order is almost the same. These are the bids and asks of ZI-RND traders. Also we can see that the volumes are now increase twice, because there are 40 trades matched. From Figure D.19, it can be observed that the efficiency loss and budget-balance are also decreased due to VOL-RND traders. The Greeks show that the sensitivity of the option prices on asset prices are still high on ATM and OTM option prices, while for ITM it corresponds with Black-Scholes solution.

5.6.3 Portfolio Holders

Portfolio holders using LMSR traders produce different results depending the population. As it has been said earlier, if the market is solely populated with LMSR traders holding the same portfolio, they produce a positive bid-ask spread and do not match their order. Moreover the prices they bid or ask would be the same. However if we mix these traders with traders holding an opposite portfolio, say, LMSR trader with a neutral portfolio is matched against LMSR trader with a non-neutral portfolio, then there will be a trade in the market. Therefore I simulate the cases where traders have corresponding opponents to match their orders.

Figures D.20 and D.21 show the trade between neutral and non-neutral portfolio holders. It can be seen that the prices are volatile around Black-Scholes prices. This is explained using the deterministic nature of LMSR traders. The whole market consists of 2 neutral LMSR traders and 2 non-neutral LMSR traders who output all together 8 different pricing quotes, 4 for bids and 4 for asks. We can even see this from the range of accepted orders, which virtually sticks to the clearing prices. So naturally, one of 4 bids and one of 4 asks is used as the clearing price for the matched orders. Because mechanism has very few options to determine the clearing price among mostly homogeneous quotes, the option price for each trading day differs significantly. Also depending on the side of the market where the quantities match fully, the mechanism pushes the clearing price to one bid (ask) further.

In Figure D.20, we can also see that the OTM prices start underpriced, and once the option turns ITM when asset price hits above its strike \$ 3603.00, the traders start overpricing it until it ends up OTM again, where the option's value is zero. The neutral trader clearly underprice OTM option, and non-neutral traders match their higher orders with them. Because non-neutral traders make profit when the asset price moves in either direction, while neutral traders make loss in that case. However when OTM is near turning ATM, it becomes profitable for neutral traders so their price is higher. The price stops from growing upwards when it is not profitable for neutral traders, and it levels off on top of Black-Scholes price as long as it is ITM.

Figure D.21 shows that the efficiency loss for the trades is about 10%. The Greeks exhibit more interesting behaviour. Deltas can be seen very steep compared to Black-Scholes delta, indicating the option price's volatility per change in asset price. Delta with respect to time to maturity shows that change in asset price has no effect on option's delta for OTM, and bears full effect for ITM option. This can be interpreted using the determinism of LMSR traders where option prices maintain certain window of prices on every spot of option's life. So even when the option approaches its maturity date, its price is not around its likely payoff, until the payoff is actually realised. We can observe the similar behaviour for gamma as well. The theta shows that the option price looses less than Black-Scholes when asset price is around its strike. This is because of the balance between neutral and non-neutral traders who view the option from the perspective of the portfolio they hold.

The similar results have been generated from other experiments with different populations of LMSR traders. Figures D.22, D.24,D.25, D.26, D.27 and D.23 display my results for the experiments listed in Table 5.4.

5.6.4 Mixed Traders

In this simulation, I run all types of traders introduced previously. All of them constitute the equal portion of the overall population. Although most of the results almost overlap with Black-Scholes prices, there are occasional spikes that happen due to one of the non-traditional traders is allocated with an option. This is especially evident in OTM option prices shown in Figure D.28. Also the range of accepted orders is upward shifted denoting that most of the traders posted higher prices for the option.

Greeks given in Figure D.29 show that the ATM and OTM option prices while they are all ITM are less sensitive to changes in asset price from skewed deltas. This relationship is also shown in gammas. However theta shows that the option prices are more sensitive to time to maturity throughout the timeline. As usual, this sensitivity peaks when option is around its strike.

5.6.5 ZI Traders and Option Portfolios

I conducted series of experiments to see how estimated predictions of ZI traders, and consequently the option prices change in proposed multi-unit DA and how the choice of option portfolios in simultaneous multi-unit DAs affect the overall predictions. These experiments are different from previous one because here I analyse the forecasts given by the traders, and then derive the option prices. I gave the simulation parameters in Table 5.7, so these parameters are considered as fixed. Also in this experiment I can control the quantity of the options traders cab bid or ask, so they are not randomly picked. The main goal of these experiments is to determine how the aggregation of forecast can be used to derive option prices, and how option the choice option portfolios affects these predictions.

In first set of experiments with revealed multi-unit DA, ZI simulate asset prices as GBM, and then use the maturity asset price as his forecast. While simulating asset prices traders can use the same volatility parameter as in underlying market, or use their own volatility which is random. The assumption is that the forecasting agents have their own methodology of forecast, and it might not necessarily align with the parameters of the underlying market. I create this diversity through assigning different volatility parameters for each trader. To sum up, I simulate 4 scenarios for revealed ZI traders:

- $Vol = Vol, Supply = Demand$: In this setting the asset price volatility, and the volatility for agents are the same. Also supply and demand scalers are taken from a random variable $\lfloor 15 * z \rfloor$ where $z \sim \mathcal{N}(0, 1)$. This balances the supply and demand the market around zero.
- $Vol = Vol, Supply > Demand$: The same as above, except supply and demand scalers are taken from $\lfloor 15 * z - 5 \rfloor$ where $z \sim \mathcal{N}(0, 1)$. This balances the market around 5 oversupply.
- $Vol = Vol, Supply < Demand$: The same as above, except supply and demand scalers are taken from $\lfloor 15 * z + 5 \rfloor$ where $z \sim \mathcal{N}(0, 1)$. This

balances the market around 5 overdemand.

- $Vol \neq Vol, Supply = Demand$: The same as above, except implied volatility of traders differ around real asset price volatility with lognormal standard deviation of 0.5.

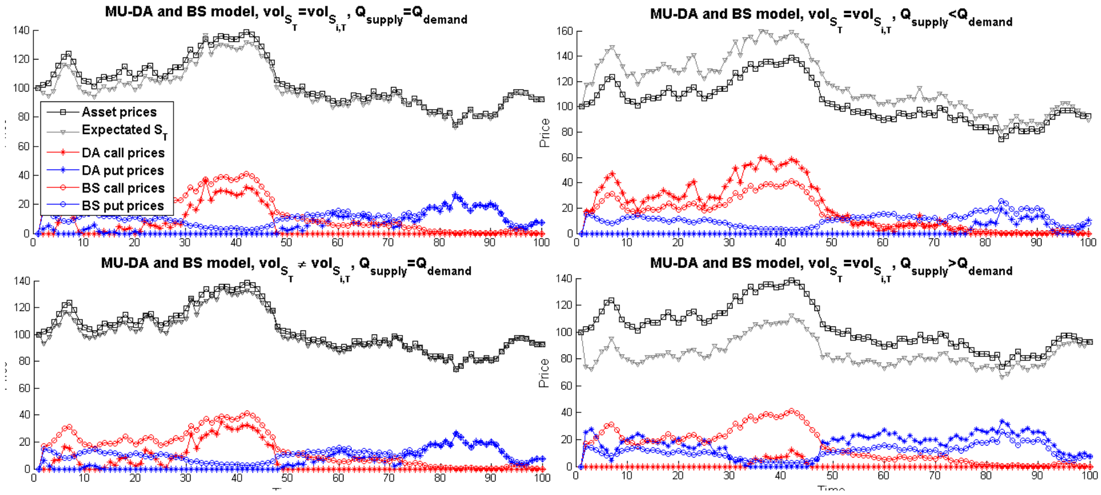


Figure 5.7: DA predictions of ZI traders

In Figure 5.7 we can see estimated predictions change when implied volatility, supply and demand are different. It illustrates that revealed DA can effectively simulate Black-Scholes prices, as long as the supply and demand are equal. However we can see that call prices drop blow Black-Scholes model when the supply exceeds demand, and vice versa. We can also observe that randomised agents' volatility around real asset price volatility can better approximate Black-Scholes option prices due to wider range of orders submitted to the mechanism.

Another set of experiments reveals the key aspect of simultaneous multi-unit DA exhibiting the effect of option portfolios on estimated predictions. In this experiment, I calculate the estimated predictions as weighted average of aggregate predictions obtained for different option types through simultaneously executed multi-unit DAs. In this set, I consider following cases:

- *Balanced Bullish, Bearish and Neutral Traders*: In this setup, traders submit orders for option portfolios with equal distribution.

- *More Bullish Traders*: Traders use more bullish option portfolios compared to other option portfolios.

- *More Bearish Traders*: Traders use more bearish option portfolios compared to other option portfolios.

- *More Neutral Traders*: Traders use more neutral option portfolios compared to other option portfolios.

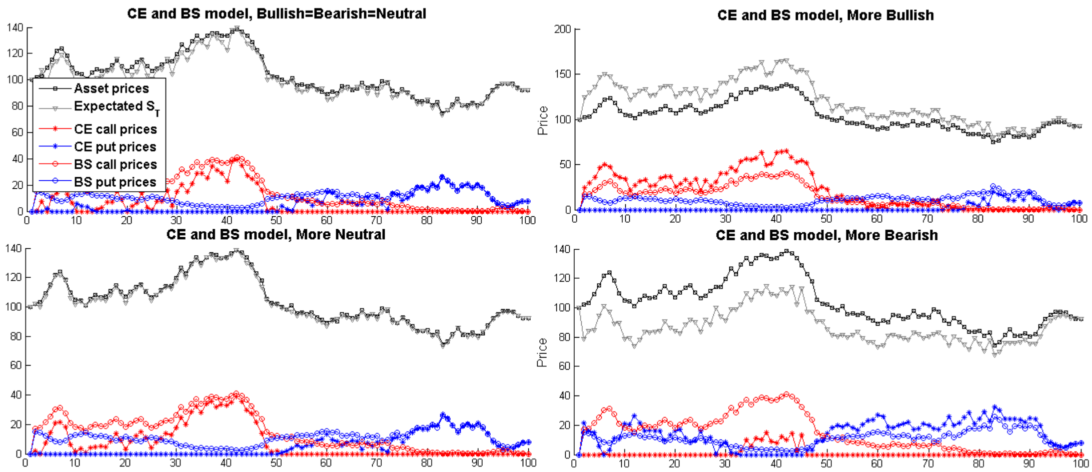


Figure 5.8: Simultaneous multi-unit DA predictions of Option Portfolio traders

Figure 5.8 illustrates the estimated predictions obtained from simulating simultaneous multi-unit DA where traders use option portfolios to interpret their predictions. It also shows the corresponding option prices compared to Black-Scholes model. As it was expected, we can observe that estimated predictions are higher when traders are more bullish, and lower if they are more bearish. Also we can see that estimated predictions stick up well with the asset prices when traders are more neutral. This clearly shows that option prices are affected by the choice of option portfolios in the market, although option portfolio is not purely a buy/ask order, but it is mixed combination of bids and asks for particular options.

As we have already mentioned, proposed mechanisms are not budget-balanced and it is worthwhile to view how they yield loss and profit from covering the partial bids/asks of rejected traders. Figure 5.9 shows the cumulative cost and revenue for revealed DA and simultaneous multi-unit DA mechanisms.

It can be seen from the Figure 5.9 that in cases of oversupply in multi-unit DA or more bullish traders in simultaneous multi-unit DA, the revenue of the mechanism is soaring, because there are fewer bids than asks, and the mechanism always ends up partially satisfying some seller. As a result it rejects that seller, and takes its role of selling options to exposed bidder. Hence mechanism increase its revenue day after day. The opposite phenomena happens when there are more bids than asks, and mechanism has to spend money on behalf of rejected bidder to buy out exposed asks. Mechanisms are somewhat stabilised

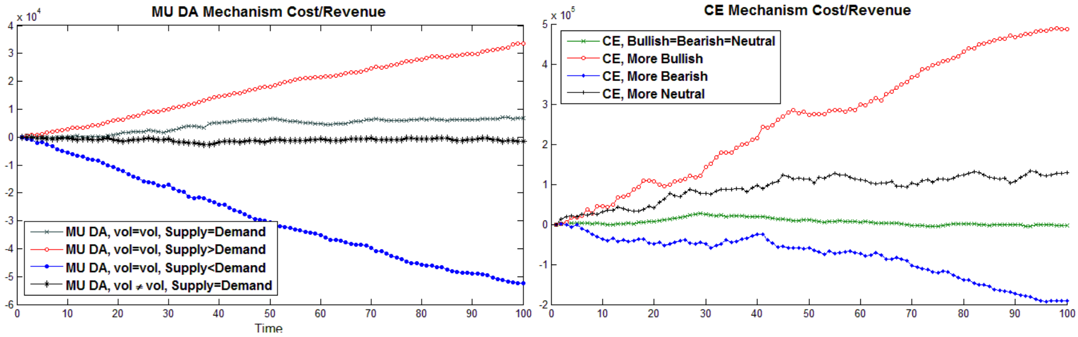


Figure 5.9: Cost/Revenue for multi-unit DA and CE mechanisms

around zero when the balance of supply and demand is maintained. Also it is interesting to observe that in simultaneous multi-unit DA, the mechanism revenue/cost is more volatile and enormous because the volumes of options traded are at least G times bigger.

5.7 Summary and Contribution

In this chapter, I defined the important properties of DSIC mechanisms and review McAfee's DA as a starting point for my design of multi-unit DA. Then I proposed the design of direct multi-unit DA preserving the atomicity of orders and simulated previously developed option traders using it. I also suggested the idea of using revelation principle for ZI option traders to aggregate their forecasts. Then I used this forecast to select option portfolios and generated aggregated forecasts in simultaneous DAs. I designed the experiments based on the characteristics of the option traders. The experiments were also meant to highlight the important aspects of the option pricing such as Greeks. I also emphasised the performance of the mechanism through the number of rejected partial efficient orders and budget-balance. The verification of the mechanism is achieved through simulating uniform Black-Scholes traders producing the same results as Black-Scholes' analytical solution.

I can list the main contributions of the chapter in following list:

- *Design of a direct multi-unit DA with atomic orders.* I used McAfee's single-unit DA to extend the proposed mechanism into multi-unit DA. This mechanism allows option traders to submit their orders truthfully knowing that their orders either get fully satisfied or rejected. The atomicity of orders is important in the scenario where option trader wants to take an option portfolio which dictates the exact amount of option types he needs to hold or write. I analysed the important properties of proposed mechanism such as efficiency, individual rationality and budget-balance.

- *Design of a revealed multi-unit DA for ZI traders.* I suggested the use of revealed mechanism where ZI traders can post their predictions instead of their valuations on options. I also mentioned that this mechanism can be used for BLS traders who reveal their implied volatility instead.
- *Design of a simultaneous multi-unit DA for option portfolio traders.* I formulated an allocation rule which replicate running multi-unit DAs. I also suggested that ZI traders taking option portfolios can submit only their forecasts, instead of posting the valuation for each option type.
- *Verification of results.* I tested the mechanism with the traders using Black-Scholes pricing method with little perturbation to facilitate the trade, and obtained results close to Black-Scholes analytical result. This showed several aspects of the mechanism such as efficiency, consistency and robustness. It also provided an optimistic view on the budget-deficit that mechanism can run into.
- Simulation of heterogeneous traders in proposed multi-unit DA. Below is my important findings from the simulation:
  - *Risk-Neutral Traders:* Although most of the prices generated by risk-neutral traders are similar to Black-Scholes' solution, the range of orders accepted in the market exhibit the discrepancies in pricing options. For example, despite MC-RND traders submit wider range of quotes for the option, they all cleared at near Black-Scholes price. Moreover, I found that the risk-neutral traders generate least error in efficiency of the mechanism, and hence keep budget-balance somehow controlled. The DA clearing prices were close to Black-Scholes prices regardless of the method of choosing quantities both random and linear. The only difference in clearing prices was exhibited by MC*-RND traders posting higher option prices, as expected. However their prices also converged to Black-Scholes price as the option approached its maturity. I also found that MC*-LIN traders generate less error in efficiency because of reduced quantities for winning orders. The Greeks analysis of MC*-LIN traders

showed that OTM option was less sensitive to changes in asset price, because of its higher prices.

– *Risk-Averse Traders:* Simulating EXP-RND traders solely could not give any results, because of their risk-averseness, sellers always overprice the option, and buyers always underprice. Hence I used ZI-RND traders to provide liquidity for EXP-RND traders, as other traders such as risk-neutral ones would always beat them. This resulted in volatile prices and enormous range of accepted orders. The efficiency error constituted almost 30% due to fewer accepted orders submitted by only 20 ZI-RND traders. Also Greeks analysis has shown that the ATM and OTM options were highly sensitive to the changes in asset price due to EXP-RND traders posting lower prices fearing from option ending up out-of-the-money. The OTM option's theta near its maturity was unstable because EXP-RND trader submits overpriced quote, and ZI-RND trader buys it at that price, making OTM option's value positive even if it becomes not exercisable soon. Introduction of small group of VOL-RND traders helped to stabilise the market around Black-Scholes price, although the range of accepted orders remained the same.

– *Portfolio Holders:* Similar to EXP-RND traders, portfolio holding LMSR traders cannot trade with each other if they constitute the only population of the market. Hence LMSR traders with diverse portfolios have been used to run the simulations. The deterministic nature of LMSR traders create limited amount of diverse bids and asks, and therefore mechanism experiences sharp jumps in option prices. I explained in the example of neutral and non-neutral LMSR traders that the OTM option prices are initially underpriced, and when option enters moneyness it becomes overpriced because of higher likelihood of generating payoff. The Greeks analysis has shown that the option price is highly sensitive to the changes in asset price, and due to fixed range of events horizon at any given time of the option's life, the delta with respect to time to maturity is constant for all OTM, ATM and ITM options.

– *Mixed Traders:* Mixed traders resulted near Black-Scholes prices with occasional spikes caused by EXP-RND traders. The Greeks analysis is also approximated to the analytical solution using Black-

Scholes model.

- *Simulation of ZI traders and option portfolio traders for aggregated predictions.* In these series of simulations I used revealed multi-unit DA for aggregating the predictions of ZI traders, and tested if the derived option prices simulate Black-Scholes prices. In case of equal supply and demand, and same volatility parameters, ZI agents aggregated forecasts aligned with the asset prices and hence gave the Black-Scholes price. However the overdemand and oversupply resulted in rise and fall in aggregated predictions, and produced lower prices for options. I have also tested how the choice option portfolios affect the aggregated predictions, and as expected, bullish traders resulted in higher forecasts, and bearish traders gave lower forecasts. The mechanism's cumulative budget-balance appeared to be stable around zero when the supply and demand are matched, and hence the aggregated forecasts aligned with the current asset prices.

From above contributions, we can derive the key messages of the chapter as well as compare them to the previous works done in multi-unit DA design and option pricing:

- Multi-unit DA, holding DSIC, individual rationality and efficiency and preserving the atomicity of orders, has to give up budget-balance. This is because there should be an additional trading agent who supplements the exposed orders resulted from rejecting partially satisfied order. In such mechanism, there is at most one partially satisfied order which has to be rejected to preserve its atomicity. This result compares to the previous works [86, 102] proposing the design of multi-unit DA and also supporting the weak budget-balance with asymptotic efficiency. These mechanisms in contrast do not preserve the atomicity of orders, because the burden from excess demand or supply needs to be redistributed to affect the requested quantities of traders. Note that the atomicity of orders has been chosen as a necessary condition to preserve the quantities required by an option portfolio.

- Multi-unit DA populated with Black-Scholes traders will result in Black-Scholes prices. In fact, the simulation results approximate Black-Scholes prices if all traders use any of the risk-neutral pricing methods described. This compares with the previous research done in risk-neutral pricing of options, as it experimentally shows the emergence of Black-Scholes prices from the simulation of a market populated with risk-neutral option pricing traders.

- Multi-unit DA can also be designed for traders with revealed preferences. For example, ZI option traders can submit their forecasts to the mechanism, rely on the mechanism to allocate, compute option prices internally and clear the orders using any DSIC mechanism. Simulation results of ZI traders have shown that the options are overpriced if the aggregated forecasts in the market are above current spot price. The similar result is obtained for underpriced options using ZI traders.
- If multiple multi-unit DAs are simulated in parallel for different types of options, and traders requested to pick option portfolio from these parallel markets based on their ZI forecast, it resulted in aggregated forecasts which aligned with the direction of option portfolios chosen by trading agents. This, in turn, established a relationship between the choices of option portfolios by traders and their aggregate effect in forming option prices above (below) risk-neutral prices.
- Different option pricing methodologies such as risk-averse pricing, LMSR pricing can be mixed along with traditional risk-neutral methods in multi-unit DA environment in order to obtain option prices representing the aggregated beliefs of heterogeneous traders in the market. We have observed that in most cases the resulted option prices are aligned with Black-Scholes prices with certain level of discrepancies caused by differences in option pricing methods. Financial analysts can learn about such discrepancies from risk-neutral prices by hypothesising the population of traders and their methods of pricing options.

Besides accomplished work, there are several points that can be improved to make the implementation and the simulation of proposed mechanisms more applicable in real-life. I listed them below:

- *Budget-Balance:* In order to make the proposed mechanism practically applicable, the mechanism needs to be at least weakly budget-balanced. However if it is budget-balanced then the potential overdemand or oversupply must be cut at the cost of decreasing the quantities requested. The mechanism should be improved to coordinate the cut of excess demands and supplies in such way that this does not break the proportions given in option portfolios requested. For example, if there are parallel mechanisms running simultaneously for different options, and agent wants to take Butterfly spread which involves selling 2 ATM calls, and buying one OTM and one ITM call. So the agent has to participate 3 mechanisms to take Butterfly spread, and the quantities for OTM, ATM and ITM calls must maintain $1 : -2 : 1$ ratio respectively. There should be a coordinating agents across mechanisms who cuts excess demand/supply in such way that the ratio for each agent is preserved.

- *Collusion:* The proposed mechanism is susceptible to collusion because buyers can coordinate themselves to reduce the option price to zero, while sellers can achieve maximal price. This problem can be mitigated by introducing reserve prices computed from the prior distribution of seller or buyer valuations on both sides. In fact my mechanism already applies the validity check for the option price, so its value has to be not less than its intrinsic value, and not more than its underlying asset price. However, mechanism can even narrow down this interval by finding such reserve prices r_a for asks and r_b for bids which maximise the expected utility of sellers and buyers. r_a determines the upper-bound for sellers, and r_b determines the lower bound for buyers. In this way, colluders will have less room to manipulate the option prices, and at least pay the expected value of the option.

- *Extensive analysis of trader performance:* I have dropped the extensive analysis of the trader performance from the scope of this paper. It would be interesting to observe how each type of trader generates profit from participating in proposed mechanism or live trading. The traders make profit from trading and exercising options in the market, and these aspects of proposed traders will be extensively studied in future works.

- *Extended revealed mechanisms:* I have only proposed a revealed multi-unit DA for ZI traders, but this idea can be extended for other traders as well. Similarly Black-Scholes trader can quote option prices using implied volatility, and risk-averse agents can also reveal their expected payoff along with risk-averseness factor. LMSR trader can further reveal their current portfolios and the range of events horizon they use to measure the potential range of payoffs. Involving all these heterogeneous trader would require design of a hybrid mechanism which accepts these truthful revelations to find maximised allocation and compute DSIC payments.

- *Extended revealed mechanisms for trading option portfolios:* I analysed the effect of option portfolios on aggregated predictions through which I derived the option prices. At the core of these idea was ZI trader using GBM as their forecasting model. However I also proposed option pricing methods such as risk-neutral, risk-averse or portfolio based pricing, and they also use certain parameters to obtain their prices. In revealed mechanisms, these parameters can be used in choosing corresponding option portfolios and clearing the simultaneous multi-unit markets by adapted allocation rules. This type of mechanism generalise the proposed idea of revealed mechanisms for trading option portfolios.

To sum up, there have been several points proposed to build direct DA for trading options along with a number of its limitations that could be studied in future. In general, I have shown that the proposed mechanism can, in most cases, simulate the Black-Scholes model despite the fact that the traders are heterogeneously distributed.

Chapter 6

Online Double Auction

6.1 Introduction

In this chapter, I present the design and implementation of an online double auction. I use this mechanism for running the adaptive option trading algorithms that I developed in Chapter 4. I implemented the standard Limit Order Book (LOB) data structure for simulating the online double auction. As it was mentioned in Gode and Sunder [74], there are two motivations of using LOB as a mechanism to simulate markets. First, it is commonly used data structure in financial exchanges such as NYSE, CBOE and even Bitcoin exchanges. Secondly, it has been shown in laboratory experiments to yield approximately the same results as human traders would do in various economic environments. I would also add the triviality of its implementation and use. The presented implementation does not include the desired properties of mechanisms such as DSIC or even BIC and individual rationality. However, as it is common to most exchanges, the mechanism is efficient and budget-balanced.

Apart from Gode and Sunder's basic simulation results with CDAs, there have been a number of other CDA models proposed by many researchers. Smith *et al.* [149] proposed a model where orders arrive at random flows with the same frequency and fixed size. They simulated the prices from different distributions and analysed the aggregate effect of these orders in the orderbook. Chiarella and Iori [27] took an agent-based approach in simulating the orders for the CDA. They used fundamentalist, chartist and ZI traders to generate the orders, and have shown that the combination of such traders can mimic the real stock markets. But the core problem with this approach was its calibration to the historical prices. I also mentioned about the agent-based methodologies of Gjerstad and Dickhaut [69], Cliff [29], Cai *et al.* [23] in earlier chapters. Osterrieder *et al.* [103] proposed an approach where the prices modelled using GBM model and the order arrivals using the Poisson process the intensity of which depended on the quoted prices. They analysed the Swiss Exchange (SWX) using this model, and exhibited the relationships between three variables: time, order price and order size. Although I shall not model the order arrivals using a Poisson process [1], I use more simplified, but similar approach. However, similar to Osterrieder, my approach includes the simulated asset prices using stochastic models which can be easily calibrated to historical prices. Also I take advantage of an agent-based methodology in simulating the option market.

It is crucial to evaluate options using the existing mechanisms such as LOB, because the simulation results would be more likely to mimic the real market. Also it enables us to try different trading algorithms on a working testbed to evaluate their performance. Traders can interact live with the mechanism and observe the current orders to make their trading decisions. I use 4 types of options trading algorithms, 2 for option dealers, and 2 for option traders. 2 option dealers are Garman's portfolio-based algorithm and Copeland-Galai's information-based algorithm. For option traders, I use ZIP and GD algorithms. Besides above trading algorithms, I also need auxiliary traders such as informed trader who computes the option prices based on the information he has on the expiration price of the asset. I need this type of traders to simulate the performance of Copeland-Galai's information-based dealer. Another type of auxiliary trader is the Monte-Carlo trader who only posts the current Monte-Carlo simulation result of the option price in online double auction. I need this trader to verify if the implemented mechanism outputs results that simulate the Black-Scholes price when all traders post similar orders.

I organised the chapter as follows. Section 6.2 describe the design and implementation of the LOB. In Section 6.3, I present the experimental cases for the simulation and explain the reason for conducting them. Section 6.4 provides the verification results through the simulation of a hypothetical scenario of Monte-Carlo traders. Section 6.5 provides the experimental results and highlights the important findings. Finally, in Section 6.6, I summarise the chapter's main results and emphasise its key contributions.

[1] Information- and inventory-based dealers used in my model actually expect orders arriving as Poisson process.

Limit Order
order_id trader_id side ('bid', 'ask') price quantity
nextOrder() prevOrder()

Table 6.1: Limit Order Class

6.2 Design of Online Double Auction

In this section, I explain the inner workings of the mechanism I have developed including how it solicits orders from traders, how it places orders into corresponding bids or asks lists and how it clears the matched orders. The online double auction is going to run multiple trading days until the option gets expired. Also traders are initialised once at the beginning of the simulation, and then continuously involved in trading process. As it was mentioned in Chapter 2, CDA basically involves 2 types of orders: market order and limit order. The only difference of the market order from the limit order is that it does not include the price quote, and submitted by the impatient trader who is willing to trade immediately. Although such traders may use automated algorithms to detect when to enter the market, and when to exit, they are not involved in the actual pricing process, thus they are not the price-setters in the market. Hence I should omit such traders from the simulation, because the proposed traders involve the actual option pricing inside their trading algorithm and adjust their bids and asks to that value accordingly. So all the traders submit limit orders which get cleared by the mechanism as the crossing limit order arrives. Table 6.1 illustrates the class structure of the limit order.

I shall drop the word 'limit' while referring to limit order onwards. It can be seen from Table 6.1 that the order is multi-unit order. Unlike in direct double auction where the negative quantities represented asks, and positive number bids, in online auction the quantity is always a positive number. The side of the order is indicated separately in a different field. The methods `nextOrder()` and `prevOrder()` are used to maintain the depth of the orderbook, so once the order gets filled, mechanism knows which one is the next to be filled. Also it is worth noting that the 'order_id' gets incremented with the new order, and can be used for tie breaking.

Limit Order Book	
Bids	**Asks**
($5, 20)	($4, 30)
($3, 20)	($6, 20)
⋮	⋮

Table 6.2: LOB before filling the crossing order

Limit Order Book	
Bids	**Asks**
($3, 20)	($4, 10)
⋮	($6, 20)
⋮	⋮

Table 6.3: LOB after filling the crossing order

The LOB is the data structure where all orders are stored and cleared. It consists of two sides one for bids, and one for asks. Bids side of the LOB stores the bids in a descending order by their prices, while the asks side stores the asks in an ascending order. When the prices are equal, the 'order_id' is used to sort the orders giving priority for the earliest order. I refer the top bid and ask in both sides of LOB, as outstanding bid and ask. For an outstanding bid ($5, 20) with quote $5 and quantity 20, if a new ask ($4, 30) arrives, this is called the *crossing ask* and it gets cleared immediately for the 20 items, leaving the outstanding ask of ($4, 10) in LOB. This process is illustrated in following Tables 6.2 and 6.3. The clearing price the mechanism will for the above case is the average of both quotes, so it is $4.5. If the crossing ask covers multiple outstanding bids, then based on the bid it is covering the average price is going to be different. Hence the volume given by the seller is going to impact the sales prices, and he might have an incentive to split his big volume into smaller orders with discriminatory prices. This is thoroughly studied in *market impact theory*, and one of the common models that are used to understand the price impact of the volume is given by Kissel [96]. However I do not consider this factor in given trading algorithms, and simply assume that every trader expresses the quantities he wants in one order. Using the anonymous price for both buyer and seller makes the mechanism strongly budget-balanced.

The mechanism implements several lists of orders stored in balanced binary tree to process asks and bids in computationally efficient way. Order lists are tagged with the range of prices they contain inside, and the Red-Black (RB) tree is used to find them. RB tree is self-balancing binary tree that enables search in $O(\log n)$, and on average insertion and removal of nodes in $O(1)$ [34]. I used `Python`'s open source bintrees [2] implementation for the development of LOB data structure.

Similar to LOB, order lists also sort bids and asks according to their prices,

[2] https://pypi.python.org/pypi/bintrees/2.0.2

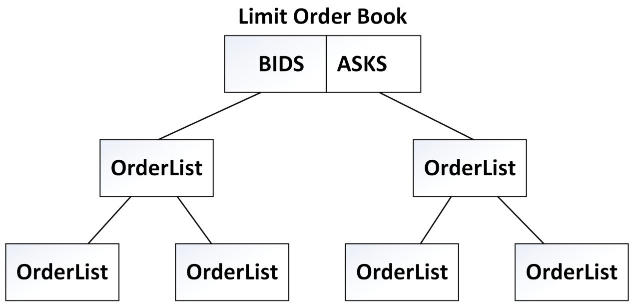


Figure 6.1: LOB using Order Tree

and use 'order_id' as a tie breaker giving priority to the earliest order. The Figure 6.1 illustrates the structure of the order tree. It can be seen from Figure 6.1 that there are two RB trees involved in building LOB, one for bids and one for asks. Each RB tree has the balanced list of order lists sorted by their most outstanding order. Hence at the top of the RB is the most outstanding bids and asks of the LOB which get cleared first when the crossing order arrives. Once the top orders list is empty, it is removed from the tree and the next order list is bubbled upwards. Inside the order list, the orders are structured as a linked list, so the insertion or removal of a node is a $O(1)$.

After clearing the crossing order, the mechanism generates two objects: `trades` and `order`. The `order` object is simply the remnant of the original order after processing. If order is crossing order, and gets fully satisfied, then mechanism returns nothing. If it gets partially satisfied, then the mechanism returns the order with a decreased quantity equivalent to the amount which is unsatisfied. And finally if the order is not cleared, but written in LOB, it gets assigned with 'order_id', and returned. The `trades` is an array of transactions happened after submitting the order. So it includes the IDs of both buyer and seller who participate in transaction and, the price and the quantity of the transaction. There could also be multiple transactions as one crossing order can fill many orders with fewer quantities, or no transactions at all if the submitted order is not matched. The structure of the `trade` object is given in Table 6.4. The mechanism uses `process_order()` subroutine to process the incoming orders. It is given in Algorithm 4. The mechanism is continuously connected to the underlying market and updates the new asset prices every trading day. Because the underlying market is simulated using the mathematical models given in Chapter 3, the data arrival to the mechanism happens once every trading day before the traders start submitting their orders with the new asset prices. Hence all the trading algorithms readjust their models to the new

Trade
buyer: (order_id, trader_id)
seller: (order_id, trader_id)
price
quantity

Table 6.4: Trade Class

asset price through computing the new option price of the day. This option price represented by $\hat{p}$ is then used to create bid-ask spread around it. This would also require the mechanism to cancel all the existing bids and asks from previous day, as they are considered obsolete by all traders. However the traders are not reinitialised, and all their adapted parameters, portfolios and cash are preserved, apart from the option price. So every trading day start with empty LOB. Once the new asset price is announced, the traders

Algorithm 4 Processing Order

```
Require: order
  trades ← []
  if order is bid then
    while there is askList in asksTree do
      while there is ask in askList that match order do
        if order.quantity ≤ ask.quantity then
          decrease ask.quantity by order.quantity and remove ask if empty
        else
          decrease order.quantity by ask.quantity and remove ask
        end if
        trades.append(ask)
      end while
    end while
  else
    {similar to bid}
  end if
  return trades, order
```

are randomly solicited for an order. Of course, the first trader who submits the order has no information about the bids and asks of other traders, hence he cannot adjust his trading algorithm for the market. So the opening price usually exhibits the traders internally calculated option price processed by the initial settings of the trader to create reasonable bid-ask spread. Similarly, the closing order taken from the last trader summarises all the information coming from posted orders and reflects the trading day's final option price. This can be tricky when there are heterogeneous traders involved in the market, as their methods of reflecting the changes in the market are also different, so they produce different option prices. So for analysis purposes, we can also use the average of the day's highest and lowest prices. This should provide better approximation of an option price for a given day.

Every trading day, the trader submits his order only once when it is requested by the mechanism. Instead of traders pushing orders to the market through a Poisson process, I decided to shift this role to the mechanism due to its simplicity. Mechanism shuffles the order of traders to post their bids and asks everyday, in this way the agents at the bottom of the queue can observe the arrival of orders and consider it as random process. In other words, the lower is the agent in the queue, the greater is the number of new orders he observes before submitting his own. Although order arrivals are not correspond to Poisson process as it is usually implemented in stock markets, it still mimics the casual interest in particular type of option out of many variations of available options in option chain.

The simulation timeline is the same as the option's expiry time. I normally use option which expires in one year, so the number of trading days is equal to 365. On option's expiry date, the agents will be requested to exercise their options and evaluate the cash they have made. Those who hold the option receive the payoff from those who wrote the option. The agents can run into loss due to the liability they bear for the issued option. This definitely contradicts with the individual rationality of traders, as their utilities can end-up below zero from the uncertainty of future prices. However if we consider individual rationality from the standpoint of sole trading and disregarding the option payoff, then for each bid or ask submitted by the trader, he is not charged more than the submitted amount, so the trader is weakly better off participating in the market.

6.3 Design of Experiments

In section, I describe the experimental cases and provide their important parameters. The experiments mainly consist of simulating different option trading agents in online double auction and observing the option prices they generate. I provide the instances of traders that are used in these experiments, and determine their fixed parameters. I also emphasise the important aspects of the traders that will be observed throughout the simulation. Next, I talk about the simulation of the market involving the underlying market, and the main indicators of the market that are of an important concern. Finally, I describe how I analyse the option Greeks in current setup.

6.3.1 Traders

As I covered in Chapter 4, there are 4 trading algorithms that I use to simulate option traders. These are 2 dealers: Garman's portfolio-based dealer, and Copeland-Galai's information-based dealer. I have already adapted these algorithms to option pricing, so they can be used according to this implementation. I also use the ZIP and GD algorithms as the speculative option trading behaviour. Besides these proxy trading algorithms, I also need new auxiliary traders for specific cases. I will use -MC option pricer, and -RND quantity chooser for all traders. For example, according to the nomenclature declared in Table 4.1, if I refer to Garman trader, I shall say 'MC-RND-GAR'. This would mean that the trader uses Monte-Carlo option pricing method, random integer for choosing option quantity and Garman's algorithm for proxy trading in online double auction. However, because I use Monte-Carlo option pricing method and random quantity chooser, for shortness sake, I only use the last part of the traders name, which is 'GAR'. The quantities for each order are capped at $\bar{q} = 2000$, so the randomly picks from 1 to 2000 options to bid or to ask. The MC option pricing method uses the GBM asset pricing model with NASDAQ calibrated parameters such as $r = 0.175$, $\sigma = 0.0089$, $S_0 = 3563.57$, and simulates the asset prices till option's maturity date.

The experiments consider several important properties of traders during simulation. First, we observe the bid-asks spread of the trader, how it changes depending on the changes in underlying market, the option market and its internal state. Then I also present the inventory and cash dynamics of the traders changing as they participate in market transactions. This shows the current market position of the trader at given point of the simulation, and the money he made so far through participating in trades. Traders can profit from selling and buying options at different times, but the profit generated from the option payoff considerably greater than speculative profit. Therefore I also show the distribution of the final balance of options before execution, and then the profit made after execution.

Now let me first start with auxiliary traders, and then I go on to the instances of other traders used in my experiments.

Auxiliary Traders

I need two auxiliary traders for two different cases. First is the verification of the proposed online double auction to see if it really reflects the agent prices. And the second is to provide the population of informed traders to the market, so the belief of the Copeland-Galai trader about the informed traders in the market is true.

For the verification of the mechanism, I simply strip the trader from its proxy trading algorithm, and use whatever value it generates using Monte-Carlo option pricing. In this trader does not create bid-ask spread, but simply posts his private evaluation of the Monte-Carlo simulation. I shall name such trader as MC trader. In verification case, I populate the market with MC traders, and run the simulation. The resulted outcome should approximately replicate the Black-Scholes prices. Also the option's sensitivity to the other factors has to correspond with Black-Scholes analytical solutions. I use 100 MC traders to populate the online double auction.

The second type of auxiliary trader is an informed trader. This trader does not use any option pricing method at all. He simply knows the future asset price at the expiry of the option, and hence can compute the option's real payoff. If option is going to end up ITM, the informed trader is better off buying the option when it is underpriced, and selling the option when it is overpriced. If the option is going to expire in ATM or OTM, then informed trader is better off selling it any price greater than zero. So informed trader can take advantage of the information he knows about the future asset price and make rational decision. The informed trader uses -RND to pick the quantity of the option he wishes to sell or buy. I shall refer the informed trader as INF in this chapter.

Speculative Traders

As speculative traders I use ZIP and GD traders. Experiments with these traders are important because they are the most commonly used trading algorithms in CDAs. I shall test their performance in option markets where the prices are not only controlled by the orders of these traders, but they also emerge from the underlying market. Simulating the option market solely

Experimental Case	Traders	Population
ZIP Traders	ZIP	100
ZIP-GD Traders	ZIP GD	70 30

Table 6.5: Experiments with Speculative Traders

Parameter Name	Parameter Value
Learning Coefficient (β)	$U(0.1, 0.5)$
Momentum (γ)	$U(0, 1)$
Initial Bid Margin	$U(0.1, 0.5)$
Initial Ask Margin	$U(0.1, 0.5)$
Relative Downward Perturb.(R_-)	$U(0.95, 1.00)$
Relative Upward Perturb.(R_+)	$U(1.00, 1.05)$
Absolute Downward Perturb.(A_-)	$U(-0.05, 0)$
Absolute Upward Perturb.(A_+)	$U(0, 0.05)$

Table 6.6: ZIP Trader Parameters

populated with ZIP traders should realistically replicate the human trading behaviour, as it combines both the trend following and learning behaviour of agents. These parameters are set randomly for ZIP traders, so they are truly heterogeneous in their trading behaviour.

The GD traders, on the other hand, represent other category of speculative traders who take advantage of the current mood of the market. However GD traders cannot create the mood in the market by themselves as they simply rely on others' decisions in making their own. Therefore I cannot simulate a market solely populated with GD traders, as it will result in an erratic output with occasional traders happening due to mere chance. GD traders should populated with ZIP traders to provide liquidity and facilitate their decision making. I shall also mix both ZIP and GD traders to observe the option prices resulted, and to review the performance of each type of agent. In Table 6.5 the different trader scenarios are listed.

As it is known the ZIP traders are defined by a number of parameters such as β agent's learning coefficient, γ agent's momentum from previous orders, initial bid and ask margins, and τ agents target price perturbation parameters for decrease and increase (R_-, A_-) and (R_+, A_+) respectively. Below Table 6.6 lists how these parameters are populated throughout the simulation where $U(a, b)$ represents uniform distribution:

The GD traders use the full history of previous trades starting from the first trading day till the option's expiry date. Every GD trader keeps track of the rejected and accepted orders computing their private price ratio with risk-neutral price, and based on this information decide which margin to use for bids and asks. It has been shown that they maximise the probability of their orders being successful, and if the market mood is more likely to accept lower bids or higher asks, the GD trader posts such orders. Also as it was mentioned before that unlike original GD algorithm which computes the success probability for raw prices, my GD trader uses the ratio of his private option price $\hat{p}$ and the risk-neutral price to compute the probability of the posted price getting successfully filled. This as explained due to the changing nature of the option price. In this way, GD algorithm finds the the optimal margin for evaluated option price which maximises the probability of trade happening in given mechanism.

Dealers

The dealers are the special types of traders that submit both bid and ask for the option, and maintain bid-ask spread based on certain factors. Dealers are less interested in making profit, rather than making the market, so they cannot trade with each other in the absence of speculative traders. Therefore I use additional traders in the market to provide liquidity for the dealers. I populate GAR dealers with different portfolios of option and cash, and then observe the width of their bid-ask spread as they involve in trading. Although GAR dealer expects the intensity of the order arrivals depend on the bid and ask price, my simulation omits this assumption. Dealers cannot calibrate the intensity of orders from incoming orders due to the specific design of my CDA. This is because traders are solicited to submit orders at random time by the mechanism, and they can only submit a single order per trading day. In a real-life market, traders can submit multiple orders at any time during the day, so there is a possibility to learn about the intensity of orders directly from the market. As I mentioned earlier in Section 4.4, this approach would require the implementation of sophisticated algorithms for traders to decide when to enter the market, what orders (if multiple) represent traders preferences, how they are broken down into smaller chunks to minimise the market impact, and what is the best timing to submit them, and what is the best time to completely exit the market. These issues require more rigorous research and outside the scope of my thesis. Therefore, I have to assume that GAR dealer uses hypothetical

Dealer Name	Cash	Options
GAR1	$1K	100
GAR2	$500K	10K
GAR3	$1M	10K
GAR4	$500K	50K
GAR5	$1M	100K
GAR6	$1M	1M

Table 6.7: GAR Dealers and their inventories

linear functions of supply and demand as his model of the intensity of arrivals in the online double auction. Although this belief is detached from the mechanism itself, it is reasonable to expect such behaviour in real markets. In other words, the intensity of ask arrivals is low if the outstanding bid is low, and the intensity of bid arrivals is low if the outstanding ask is high. Therefore we use standard linear supply and demand functions as constraints for the GAR's objective function which minimises the probability of failure. I assign GAR dealer with different portfolios of cash and options, and use these instances throughout the experiments. Table 6.7 lists the GAR dealer instances and their corresponding portfolios used through the simulation.

It is worth mentioning that the quantities ordered in the online double auction are capped at 2000, and the average ITM (i.e. the most expensive one) option price throughout the timeline of the simulating fluctuates between $40 to $80. Based on these parameters I set up different extremal portfolios for GAR traders to observe the width of their bid-ask spread. For example, the GAR1 trader has very limited portfolio of $1000 cash, and 100 options, hence in ITM option market he can only buy at most 20 options, or barely satisfy any ask, as the quantity requested may exceed 100. In order to safeguard himself from possible asks, the GAR1 trader has to maintain lowest possible bid, but very high ask. At the other spectrum of the GAR trader is the GAR6 dealer with $1M in cash, and 1M options. Hence this dealer can safely submit narrower bid-ask spread, and engage in trader without fearing to fail. I shall observe the bid-ask spreads of the GAR trader in comparison to the their portfolio.

The second type of dealers are the information-based COP dealers who adjust their bid-ask spread to safeguard themselves from the informed traders. As I said I will use INF traders to simulate the informed trader behaviour. COP dealer has a special parameter θ indicating the proportion of informed traders in the market. In practice, this parameter θ is obtained through using Bayesian learning as it was given in Chapter 4, but for the sake of experiment and

Dealer Name	Informed Traders (θ)
COP1	0.01
COP2	0.05
COP3	0.1
COP4	0.5
COP5	0.8

Table 6.8: COP Dealers and their inventories

Experimental Case	Traders	Population
GAR Dealers	GAR1	1
	GAR2	1
	GAR3	1
	GAR4	1
	GAR5	1
	GAR6	1
	ZIP	90
COP Dealers	COP1	1
	COP2	1
	COP3	1
	COP4	1
	COP5	1
	INF	10
	ZIP	85

Table 6.9: Experiments with Dealers

simplicity, I will manually assign these parameters to COP dealers and observe their behaviour in choosing bid-ask spread. I listed the COP dealers that are used in the experiments in Table 6.8. As I did for GAR dealers, I also simulate different parameters including the extremal cases for COP dealers. For example, the COP1 trader with $\theta = 0.01$ thinks that there are only 1% of informed traders in the market, and hence it is less likely he is going to loose much money from them. So COP1 trader can use narrower bid-ask spread and freely engage in trading process. However the COP5 trader assumes that 80% of the market is populated with informed traders, and therefore he needs to set wider bid-ask spread. I shall observe the dynamics of the bid-ask spreads for COP traders based on their θ parameter.

Now once I clarified the dealers that are used in experiments, I list the two experimental cases for each of them. Table 6.9 shows the configuration of each experiment. It can be seen from the Table 6.9 that the 'COP Dealers' experiment include 10 INF traders, which constitutes the 10% of the trader population. Hence, the COP3 dealer has the right guess about θ.

Trader Name	Population
GAR1-GAR6	6
COP1-COP5	5
ZIP	59
GD	30

Table 6.10: Experiment with All Traders

Mixed Traders

In this experimental scenario, I use every developed trader and dealer to simulate the option prices and observe their trading behaviour. It includes ZIP and GD traders, and all GAR and COP dealers. The main reason to do so is to maximise the heterogeneity of the traders in the market, and see how the option prices are affected. It is also interesting to observe the performance of the traders in such scenario. Table 6.10 illustrates all traders involved in mixed case:

6.3.2 Market Simulation

In this section, I describe how both underlying and option markets are simulated in proposed experiments. I already simulated the asset prices using GBM in Chapter 5, and I use the same asset prices path for consistency. It calibrated to NASDAQ-100 indices and can be viewed in Figure 5.4. The option market is run using proposed online double auction. I create 100 traders at the beginning of the simulation and then continuously solicit orders from them. I also use the same set of ATM, OTM and ITM options that I used in Chapter 5. They are displayed in Table 5.6.

Mechanism simulates 365 trading days, going up to the point the option expires. For each trading day, I capture key outcomes of the mechanism for further analysis. This involves the opening, closing, high and low prices of the day, volumes traded and some other relevant indicators. The mechanism is analysed using following parameters for each trading day.

- Black-Scholes Price: This shows the current Black-Scholes price for comparison. In this way, we can see how the option prices differ from theoretical price.
- Opening Price: This shows the price resulted from the first trade on a trading day.

- Closing Price: This shows the price resulted from the last trade on a trading day.
- High Price: This shows the highest price resulted from the trade on a trading day.
- Low Price: This shows the lowest price resulted from the trade on a trading day.
- Average Price: This shows the average highest and lowest prices of a trading day. It is used for the analysis of option's Greek.
- Trade Volume: This shows how many options have been traded on a trading day.

From captured details, I plot corresponding charts and analyse the simulation results. The closing prices are plotted as line graph, indicating trading days on horizontal axis, and the prices on vertical axis. It could be presented as a candlestick graph like most of the price charts in real markets having ask and bid prices shown separately, but due to very narrow spread between cleared bid and ask prices, the candlestick graph looked less comprehensive. The highest and lowest prices are indicated with a shaded yellow area.

Besides the market prices, I also capture the bid-ask spread of each trader involved in the market. Below is the list of details that are saved for each trader for a given trading day.

- Bid Price: This shows the trader's bid for the option on a given trading day. It might not be the bid that is submitted to the market, as the trader may choose to sell an option, and submit ask instead. But every trader maintains his internal bid for the option on a given trading day.
- Ask Price: This shows the trader's ask for the option on a given trading day, not necessarily submitted one according to the case above.
- Cash: This shows the cash account of the trader on a given trading day. It can also be negative meaning that the trader borrowed cash, and this is considered as his obligation until the end of the simulation so he can exercise his options and get a payoff to compensate it.
- Options: This shows the number of options held by the trader on a given trading day. It can also be negative meaning that the trader wrote the option to someone else.

I need this information to observe how the cash and inventories change for different types of traders, and more importantly, how the portfolio affects the GAR dealer's bid-ask spread.

Once the market reaches the option's expiration date, the options are exercised and the payoffs are received. So I also summarise this information for each trader to see how much money in total is made, and what was the trader's final portfolio of options. In this way, we can evaluate the performance of each trader by the amount of cash he made. Also we can plot the distribution of cash and options for each trader in a histogram to see the overall performance of the market.

6.3.3 Greeks Simulation

In Greeks simulations, I analyse the sensitivity option prices generated by the mechanism with respect to changes in 3 relevant factors such as asset price, delta and time-to-maturity. Namely, they are the option's delta, gamma and theta. I have mentioned the importance of these indicators in option pricing in previous chapters.

I compute each Greeks parameters with respect to two parameters. In first case, I change the asset price linearly from \$3465 to \$3665 covering moneyness range of all three initialised options listed in Table 5.6. However, the way I do it is a bit different from the one described in Chapter 5. Instead of directly feeding these linear asset prices into the online mechanism, I use the data obtained from the simulation of the market to approximate the Greeks. The main reason for doing this is that the mechanism is continuous and the traders exhibit continuous interest in the market. The linearly increasing asset prices would simply adjust all traders to follow the trend, and does not diverge in their bids and asks preventing the possibility of any trade. Therefore, I compute the implied volatility from the closing prices of the market, and use its average to obtain analytical Black-Scholes Greeks of options. I also compute Greeks for linearly decreasing the time-to-maturity parameter by fixing the asset price at ATM $S_0 = K$. In this way, we can observe how option Greeks change according to the time from $T = 1$ to $T = 0$. I obtain the Greeks with respect to the time-to-maturity in the same way I did for the linear asset prices.

I present the results of the simulation of Greeks along with theoretical Black-Scholes Greeks for each type of option. This should enable us to see how these aspects of option pricing can be different from theoretically computed ones.

6.4 Verification of Proposed Mechanism

In this section, I present the first results from the simulation of both market and Greeks in order to verify if it gives expected results in a hypothetical case populated with MC traders. This is an important factor because it exhibits the several aspects of the proposed mechanism:

- Unanimity: The prices obtained by the mechanism must be the overall representation of submitted orders, and when all traders in the market are MC traders, the price should correspond with Black-Scholes price. This shows the unanimity of the mechanism.
- Consistency: The prices obtained by the mechanism must also be consistent with option's definition. This involves option's final payoff according to its strike and asset price at expiry date. For example, OTM option price should end up at zero at its maturity, because it generates no payoff.
- Robustness: The sensitivity of option prices to changes in asset prices, delta and time-to-maturity must be consistent when the Greeks are simulated. This exhibits how option's delta, gamma and theta correspond to Black-Scholes delta, gamma and theta when the mechanism is put to simulation of one parameters while fixing the others. In the context of this research, these control factors are the asset prices and time-to-maturity.

Figure 6.2 shows the results obtained from simulating MC traders in proposed online mechanism. It can be seen that it corresponds with Black-Scholes prices in all ATM, OTM and ITM option markets. We can see from the high and low prices in the yellow shaded area that the mechanism does not significantly vary from the risk-neutral prices. Also we can observe that the OTM ends up at zero at its maturity, which shows the consistency of the mechanism. ATM and ITM options expire in-the-money, hence their prices are positive and equal to their corresponding payoffs.

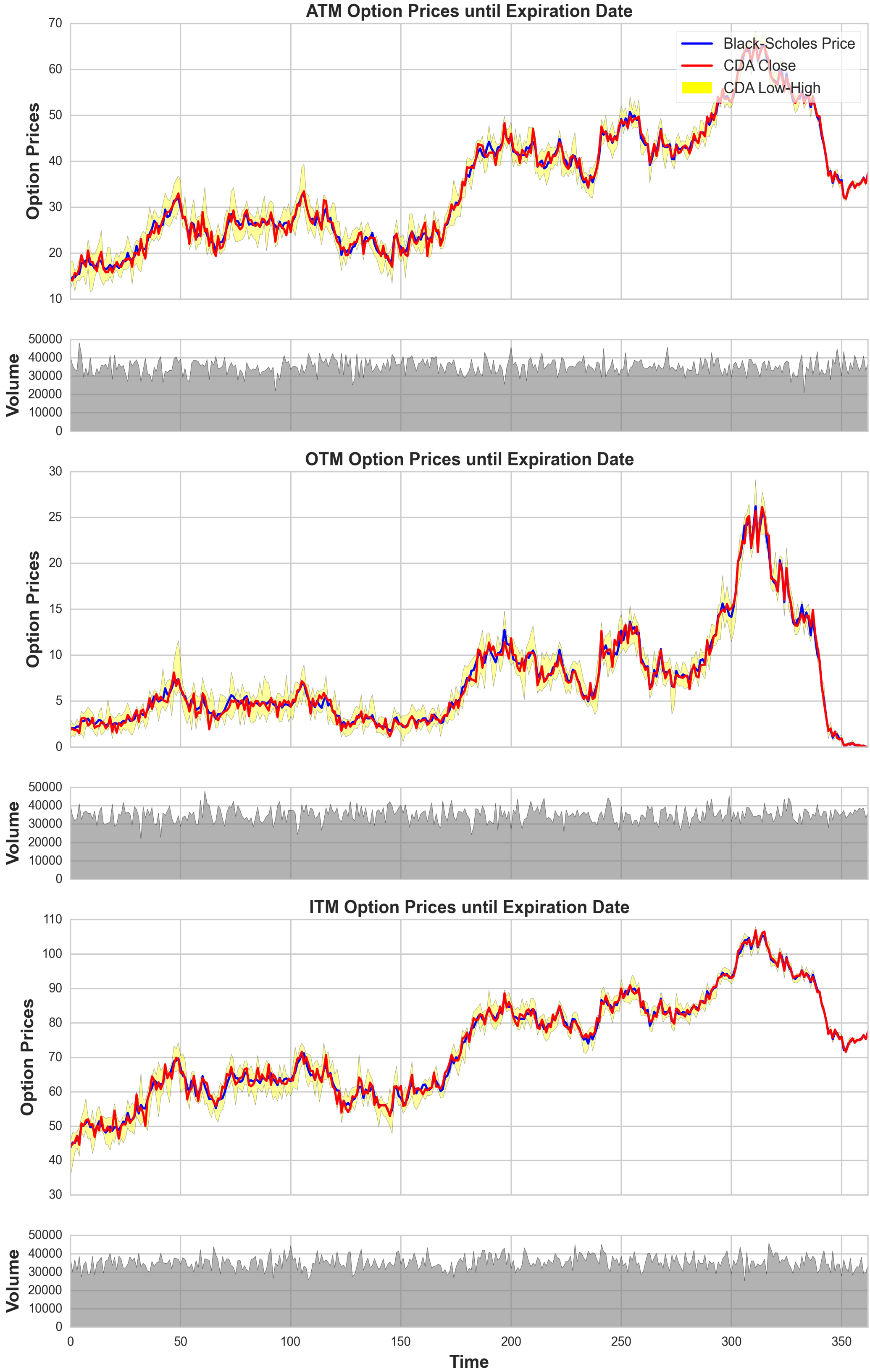


Figure 6.2: CDA prices of ATM, OTM and ITM options for MC traders

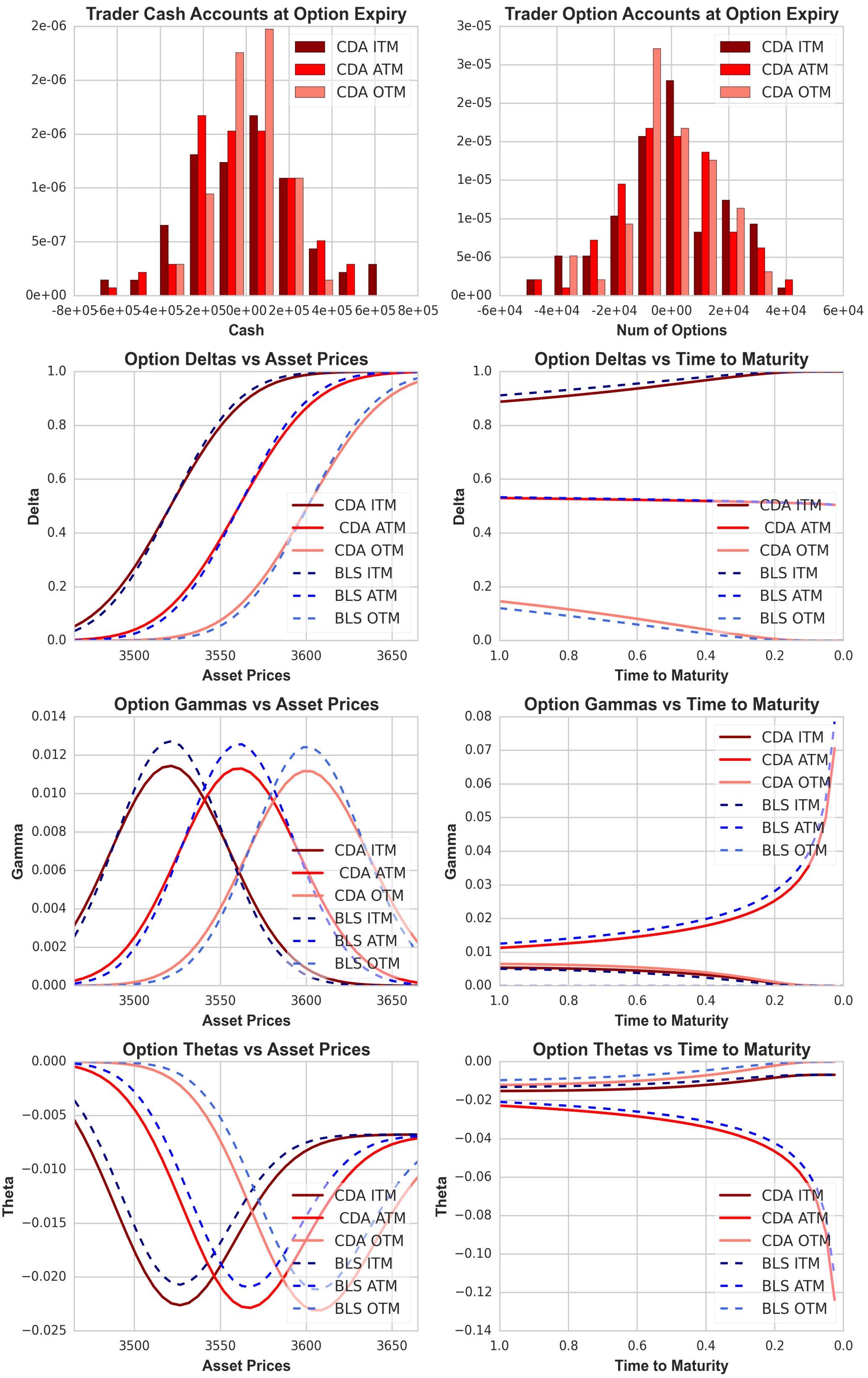


Figure 6.3: CDA inventory distribution and Greeks of ATM, OTM and ITM options for MC traders

The option Greeks for the MC traders given in Figure 6.3 show slight divergence in deltas and other parameters. This is due to a numeric error caused by the approximation of the implied volatility from market prices and using that in Black-Scholes analytic solution. If fact, this shows that the option prices are robust to changes in asset prices and time-to-maturity, as they do not diverge significantly from Black-Scholes analytical solutions.

6.5 Experimental Results

In this section, I present the important findings from simulation of different traders in proposed online DA. I enclosed all results of the simulation in Appendix E. I review the results from simulating ZIP, GD, GAR, COP and mixed traders and point out their important characteristics.

6.5.1 ZIP traders

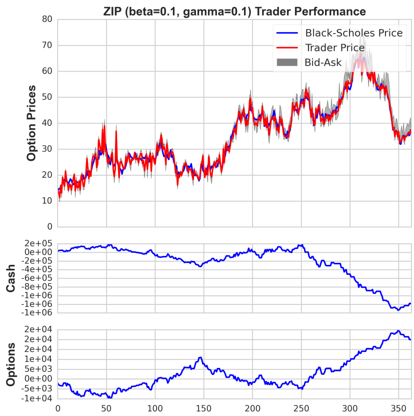
ZIP traders turned out to be great liquidity providers among other agents, as they were very likely to adjust quickly their bid-ask upon arrival of new information. Hence they adjusted their margins to the submitted bids and asks as soon as they see any success in their execution. Therefore I used ZIP traders in all other experiments to provide a heterogeneity for other traders.

Figure E.2 shows the simulation results for the market solely populated by ZIP traders. As it can be noticed, it is very close to Black-Scholes prices in all cases, although there is slight more volatility at the beginning. The important thing to notice is the very narrow difference between day's high and low prices. It is because ZIP traders continuously adjust their own bids and asks to the others' orders, and quickly reach an approximated equilibrium around Black-Scholes prices. Also we can see that the closing price summarises the day's option prices the best because all traders are ZIP traders, and the randomly chosen last trader is also ZIP trader who actually reflects the day's trader in his order.

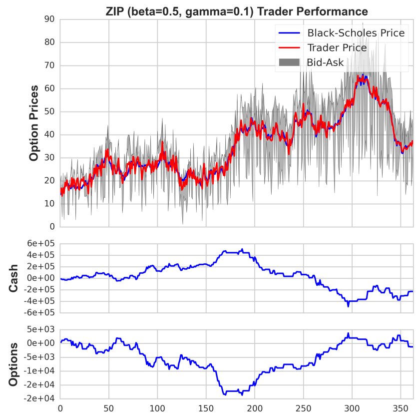
Another interesting point to observe is the volume goes down as the options approach their maturity. If we compare this with MC option pricing agents in direct double auctions (see Figure D.6) where the volumes stayed approximately the same throughout the year, in CDA with ZIP traders the volume declined by the time option reached maturity. This is because the option price converges to its Black-Scholes price as it approaches maturity. In other words, there is less uncertainty about option's payoff when it is near its maturity, so the prices the MC agents come up are nearly the same. In direct DA, they do not use any profit margins, and post the prices as they are. Therefore agents have more chances to match, and the prices change accordingly. However in CDA, ZIP traders maintain margin for MC computed prices. At the beginning of the option life, there is more uncertainty in pricing, so the bids and asks overlap making trade higher. But as option nears maturity, MC computed prices converge Black-Scholes, due to profit margin there is less chance for bids and ask to match.

Figure E.3 shows the distribution of cash and options among the other traders, and presents a snapshot on the option Greeks. As it can be seen, the final payoffs are normally distributed among ZIP traders. This is also true for the options held by trader at the end of simulation. The other Greek parameters are slightly diverged from Black-Scholes, but in general they follow the same pattern. I do not stress on the meaning of the Greeks, as I have explained what they represent in Chapter D.

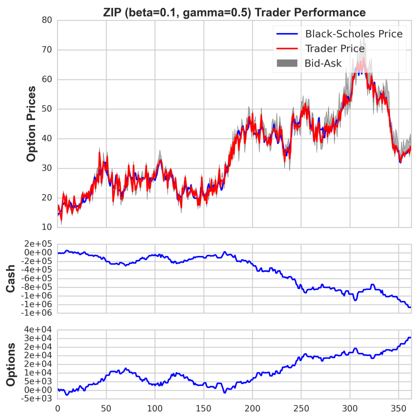
As we know, the ZIP traders are defined with two main parameters β and γ, and the formed one is drawn from $U(0.1, 0.5)$, the latter $U(0, 1)$. I shall consider the performance of the 4 traders with following configurations: $(\beta_1 = 0.1, \gamma_1 = 0.1), (\beta_2 = 0.5, \gamma_2 = 0.1), (\beta_3 = 0.1, \gamma_3 = 0.5)$ and $(\beta_4 = 0.5, \gamma_4 = 0.5)$. Figures 6.4a,6.4b, 6.4c and 6.4d illustrate the bid-ask spread, cash and options accounts of ZIP traders with above parameters. It is known that β parameter defines the trader's ability to reach his target price τ which is derived from previously submitted order. The lower is the parameter, the less likely the trader is going to adapt to other orders in the market. In other words, the trader is less likely to react to the change in underlying market or the orders submitted within the mechanism. These prices can be attractive to other ZIP traders with higher β because such orders might be underpriced, because they dismiss the newly arrived information. If we compare the Figures 6.4a and 6.4b, we can see that the trader with $\beta = 0.5$ participated less in trades, therefore he bought about 5000 options and spent about \$60K on them. However we can see that the first trader bought 20K options, 4 times more than the previous trader, but spent about \$1M for obtaining them, about 17 times more than the previous trader. This is because the trader with $\beta = 0.5$ was able to quickly adapt his quote to the latest orders and get better deals.



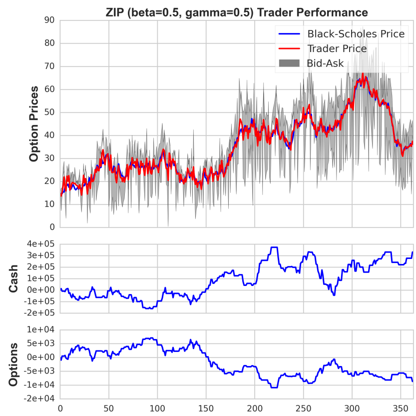
(a) ZIP($\beta_1 = 0.1, \gamma_1 = 0.1$) Performance



(b) ZIP($\beta_2 = 0.5, \gamma_2 = 0.1$) Performance



(c) ZIP($\beta_3 = 0.1, \gamma_3 = 0.5$) Performance



(d) ZIP($\beta_4 = 0.5, \gamma_4 = 0.5$) Performance

Figure 6.4: ZIP Trader Performances

The other factor is the momentum γ where the trade has the tendency to keep his profit margin less affected with the arrival of new orders. We can see in Figures 6.4a and 6.4c that the trader with $\gamma = 0.5$ spent the same amount of cash to obtain 10K more options than the one with $\gamma = 0.1$. This is because $\gamma = 0.5$ agent was more reluctant to change his profit margin, hence traded with better prices.

All in all, from results provided, we can see that ZIP algorithm can be used as a proxy trading algorithm for trading options and its performance depends on its 2 parameters β and γ. We have seen that the agents with lower β and higher are more likely to get less attractive deals due to their reluctance to adapt their price and profit margin quickly to ongoing changes. However, by controlling these parameters of ZIP traders, we can simulated trading agents with different behaviour in the market, and somehow replicate its heterogeneity. In next sections, I use ZIP traders as main liquidity providers in the market due to their ability to generate randomized bids or asks constrained by their learning algorithm.

6.5.2 GD Traders

In this section, I present the results of the simulation of GD traders along with ZIP traders. As said before, GD traders base their decisions from the performance of other traders, through Bayesian update of the posteriors on the probability of successful bid or ask. Figure E.4 illustrates the option prices obtained from this simulation. The option prices are aligned around Black-Scholes, and the range of orders is not very wide. The interesting results can be observed in Figure E.5 where the distribution of cash and options are somewhat segregated into different groups. This is because all GD traders are somewhat deterministic in their behaviour. There is no random component in GD trading strategy, apart from their option pricing method. Hence all GD traders using full history of trades in the market should exhibit almost the same bids and asks. The only difference is the time when each of the GD traders submit their bids/asks, as the ones who submit the latest order gets more complete information about the preferences of others. But from Figure E.4, we can see that the effect of this difference is not noticeable on the option prices results.

In my experiment, their bids were considered efficient and purchased by ZIP

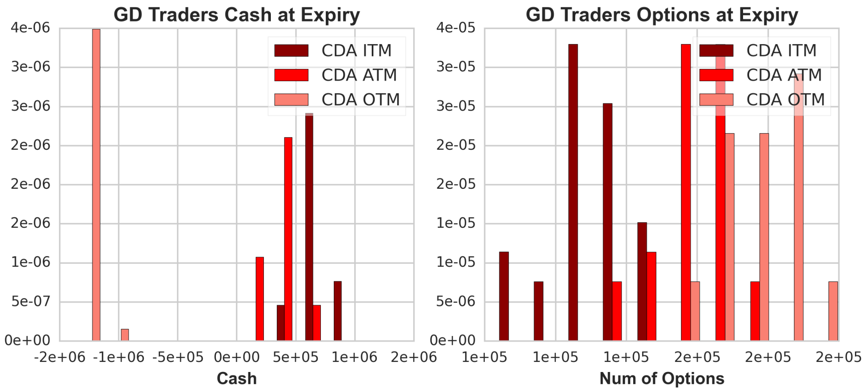
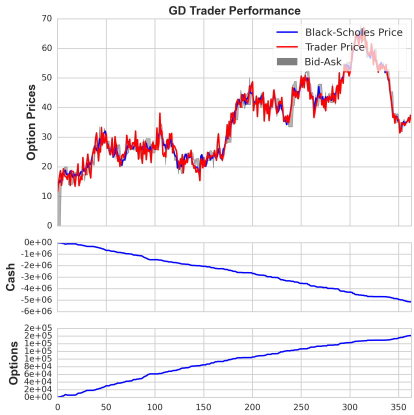
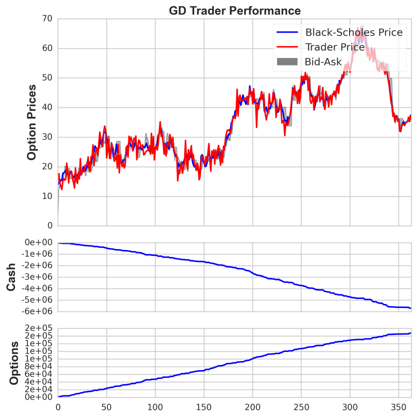


Figure 6.5: GD Trader's distribution of cash and options in ATM, OTM and ITM markets



(a) GD Trader 1 Performance



(b) GD Trader 2 Performance

Figure 6.6: GD Trader Performances

traders. So in all option markets, GD traders engaged in buying out options. Of course, in case of ATM and ITM options, this yielded them enormous profits, while in OTM they lost much money. The distributions of cash and options for only GD traders are given in Figure 6.5. We can see from the Figure 6.5 that traders clearly owned options in all cases, and that caused them enormous profit in ATM and ITM cases, and loss in OTM case.

In order to highlight the similar behaviour of all GD traders, and to view the dynamics of their bid-ask spreads, cash and options accounts. I picked two random GD traders for comparison. Figures 6.6a and 6.6b illustrate them. It can be seen that both of the GD traders are involved in buying options, and their bid-ask spread is not much different from their actual private option price. Apparently their bid price was attractive to ZIP traders, so they bought those options from GD traders.

To sum up, the key message we can deduce from the simulation is that the GD algorithm can be used as a proxy trading algorithms for trading options, although most of their trading strategy is influenced by the behaviour of other traders. In provided example, the prices were mostly driven by ZIP trader, and GD traders simply adapted to such these prices. GD traders with full historical record of orders submitted lower bids which was below the profit margin of other traders. This led to GD traders purchasing options all the time, and making profit in case if option ends up ITM.

6.5.3 Garman Dealers

In this section, I present the results of the simulation of 6 GAR dealers along with 90 ZIP traders. GAR dealers are the agents who make their trading decisions based on the portfolio they currently hold, therefore their bid-ask spread directly depend on what they currently have. I simulated extremal cases where GAR1 trader has incompatible portfolio of cash and options to trade in the market, and GAR6 has enough portfolio to make an attractive bid-ask spread.

Figure E.6 illustrates the overall option prices generated from the simulation of these agents. It can be seen that the closing prices are fluctuating around Black-Scholes prices with high and low prices being not far from them. The volume also tends to decrease as the option reaches its maturity for above stated reasons in ZIP simulation case (i.e. because option prices converge nearing maturity and traders maintain profit margin). In Figure E.7, we can see the distribution of cash and options among all traders. Note, that I had to subtract the initial endowment of the GAR traders to compute their net performance. It can be seen that the final payoffs are centralised around zero, which is caused by the majority of ZIP traders. However there is an exceptional case in OTM market. These are the GAR trader who bought the OTM options and end up not exercising them. Hence they made a loss for the money they spent for purchasing the options. The final balances of all 6 GAR traders displayed in Figure 6.7 shows how they made profit in ATM, ITM markets, and made loss in OTM markets. In Figure 6.7 we can see that all GAR dealers incurred loss in OTM market, and profit in ATM and ITM market because of only buying options. The GAR1 with least portfolio did not even participate in trade due to its large bid-ask spread. However, in OTM market, when the option prices approached zero, GAR1 dealer had a chance to buy some options,

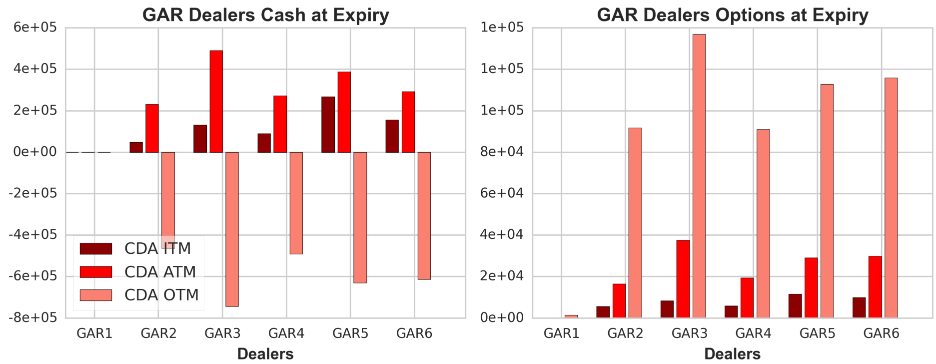
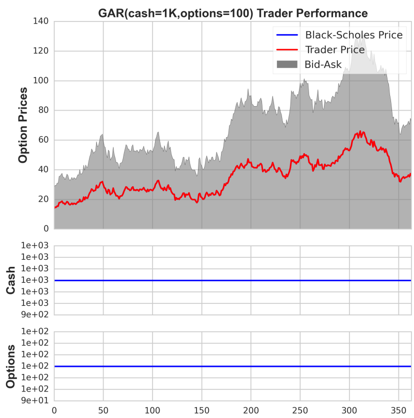


Figure 6.7: GAR Dealer Inventories At Expiry

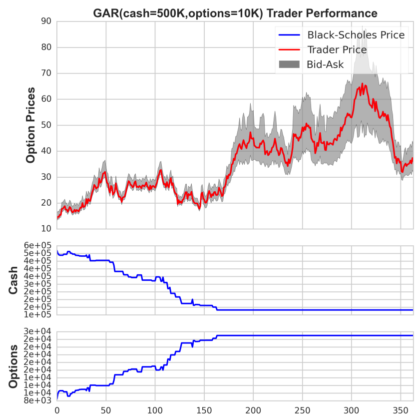
as they were falling into GAR1's bid-ask range.

I shall review the dynamics of 4 GAR dealers, namely GAR1, GAR2, GAR3 and GAR5, in setting bid-ask spread, and maintaining portfolio during the simulation of ATM market. As expected in Figure 6.8a, we can see the GAR1 dealer's unrealistic bid-ask which bids almost 0 for the option, and asks more than twice his private price. Naturally, it can be seen from the cash and option accounts, GAR1 dealer does not trade in ATM market at all, so his balance is constant. However GAR2 dealer has a realistic portfolio, hence it begins with moderate bid-ask spread, and starts buying out options, until he realises that he is becoming low in cash. This forces the GAR2 dealer to widen his bid-ask spread which makes his bid less competitive. We can also see the changes in the cash and option account stop changing, once the bid-ask spread is increased. Another interesting situation is the GAR3 dealers result. Although GAR3 had twice more cash endowment than GAR2, his bid-ask spread becomes even wider than GAR2 dealer's spread. At the beginning GAR3 has smaller bid-ask spread, but with the increase flow of orders, GAR3's cash falls faster than GAR2, thus he becomes much lower in cash, and will be forced to make a wider bid-ask spread. This eventually stops the incoming orders for GAR3. The most affluent GAR5 is observed to maintain smaller bid-ask spread and it gets slightly increased until it stops the flow of orders. As predicted, all GAR dealer correctly reacted to the changes in the portfolio.

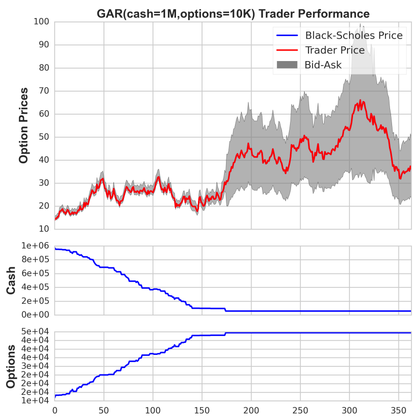
We can deduce important characteristics from above simulation. First GAR proxy trading algorithm can be used for trading options for dealers who have budget constraints. Due to this fact, it generates conservative bid-ask spread which gets continuously adjusted to the trades current inventories. We saw that it stops selling options as soon as it reaches its budget limit by signifi-



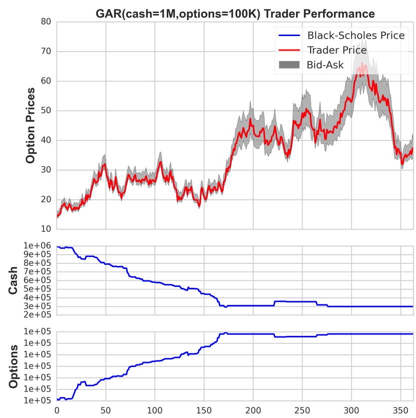
(a) GAR1 Dealer Performance



(b) GAR2 Dealer Performance

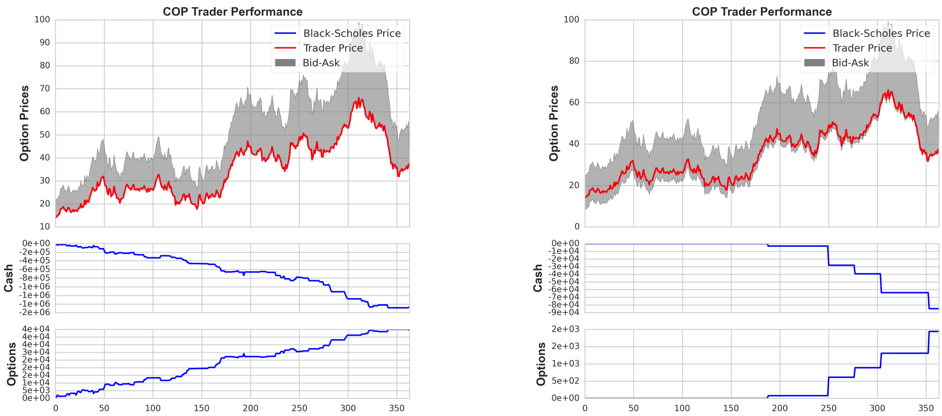


(c) GAR3 Dealer Performance



(d) GAR5 Dealer Performance

Figure 6.8: GAR Dealer Performances



(a) COP1 Dealer Performance

(b) COP3 Dealer Performance

Figure 6.9: COP Dealer Performances

cantly increasing his bid and decreasing his ask. We can also observe that the evaluated option price is always in the middle of the bid-ask spread.

6.5.4 Copeland-Galai Dealers

In this section, I review the results of the COP dealers simulated together with INF and ZIP traders. COP dealers set their bid-ask spreads based on their belief in the proportion of informed traders θ in the market. I set this parameter to different extremes to see how COP dealers handle the situation. Figure E.8 illustrates the option prices resulted from the simulation of above traders. We can see they align with Black-Scholes prices and the high-low prices are not significantly different from the closing price.

In Figure E.9, we can see the distribution of trader inventories and Greeks from the market simulation. The results are pretty much the same as in ZIP trader, but the Greeks are more skewed due to higher implied volatility. Let me concentrate the attention on 2 COP dealers: COP1 who thinks that there are only 1% informed dealers in the market, and COP3 who correctly believes that informed dealers are the 10% of the overall population. Figure 6.9 illustrates these two dealers. We can see that the COP1 dealer just maintains the fixed bid-ask spread, setting his current private option price $\hat{p}$ equal to his bid, and about twice more for the ask. The width of the bid-ask spread is controlled by the linear model for expected number of orders to arrive at given price. Changing the slope of the linear function, or replacing it with less aggressive order flow model, or even using exponential distribution or Poisson distribution changes the width of the bid-ask spread. If we look at the COP3 dealer, we can see that he shifts his fixed bid-ask spread downward to lower his bid in order to discourage INF traders selling to him. As it can be seen from the cash and inventory of the COP3 trader, this significantly deters the trades until the COP3 trader pushes back his bid to a near $\hat{p}$ price. The reason for changing the bid-ask spread upwards, similar to COP1 dealer is that as the option approaches its maturity, the European Compound option price goes down. We know from the definition of COP dealers that they use straddle portfolio made using European Compound options to evaluate the expected loss from the informed traders. Hence their price goes down, the expected loss from the informed traders also goes down, replicating the situation as if the INF traders were in minority.

6.5.5 Mixed Traders

I have simulated all the traders in proposed online double auction to see what option prices they generate, and what distribution of cash an options the traders will have. Figure E.10 illustrates the results for the simulation of ATM, OTM and ITM markets. We can see some spikes caused by GD traders due to the numerical issue described above. In general, the option prices are around Black-Scholes. The high and low of the trading day are more volatile due to the heterogeneity of the traders quoting wider range of prices. The Figure E.11 illustrates the distribution of trader inventories and the Greeks. It can be see that the both cash and options are unequally distributed causing some agents to loose more from buying OTM options, and some to gain more from selling them.

6.6 Summary and Contribution

This chapter provided a design of a LOB based online double auction. It used self-balancing binary trees to organise bids and asks. Then we used this mechanism to simulate various trading algorithms proposed in Chapter 4 and analysed the option prices along with the performance of the each trader. Below are the main messages of the chapter:

- *Design and Implementation of Online DA:* I proposed the design of an online double auction which uses the LOB to match bids and asks. The mechanism uses the fast RB-trees to order the bids and asks in corresponding OrderLists. Therefore mechanism spends $O(\log n)$ for the insertion of new orders into LOB, and on average $O(1)$ for matching them. I verified the mechanism with MC traders to see if they generate Black-Scholes prices. The mechanism is then used for the simulation of trading agents.

- *Option Trading Agents:* I used various option trading methods for pricing option in online double auction environment and provided detailed analysis of their results. I set the option pricing method for all traders as Monte-Carlo, and used random integers to pick the quantity. Below is my key finding from simulating each trading agent:

  - *ZIP traders:* ZIP traders exhibited the trend following behaviour, and provided a liquidity in the option market. I analyse how their bids and asks change based on their learning coefficient β and momentum γ, and has shown that the traders with greater β are capable of quickly adapting their bids and asks to the changing option values and other submitted orders. Therefore the final payoffs for the ZIP trader with higher β were greater than for the one with the lower β. The option prices generated by ZIP traders were very close to Black-Scholes prices, as their inventory distribution at the end of the simulation was centralised at zero.

  - *GD traders:* GD traders were considered as traders whose orders depend on the behaviour of the other traders, because they used submitted bids and asks to evaluate the probability of their own successful orders. All GD traders exhibited a uniform behaviour by buying out options to make profit in ATM and ITM markets, while to incur loss in OTM market. The overall option prices generated by GD traders along with ZIP traders corresponded the Black-Scholes prices.

  - *Garman dealers:* GAR dealers were good at maintaining the inventories, as they directly derived their bid and ask prices from their inventory levels. GAR dealers with impractically low inventories did not participate in trade due to their big bid-ask spread. However GAR dealers with affluent inventories started with narrow bid-ask spreads until their inventory in cash run low, so they quickly adjusted their bid-ask spread to deter the flow of asks lowering their cash. GAR dealers made profit from buying ATM and ITM options, but loss for the same behaviour in OTM market. We have seen that the GAR dealers yielded the similar to Black-Scholes prices.

  - *Copeland-Galai dealers:* COP dealers were simulated using INF traders with different beliefs about their proportions in the market. Although in practice the θ indicating this proportion is obtained through Bayesian learning method, we fixed these parameters to see the differences in their behaviour. We have seen that the COP dealer with lowest belief in INF traders posted his private option price as his bid, and end up in only buying options from all traders. However the COP correctly believing that there are 10% INF in the market, lowered his bid below his private option price until the expected loss from the informed traders became insignificant. This is caused by the drop in value of the European Compound option as its underlying option approaches maturity.

From above results, let me point out the key contributions of the chapter over previous researches in this area.

- There have been number of CDA designs proposed in literature starting from simple market design [150] to recently proposed self-evolving CDAs [105, 104]. Although the design of my CDA is somewhat trivial, it is driven by the an exogenous source in the form of an underlying market, it uses efficient RB trees to manage LOB and puts the mechanism in charge of soliciting orders. It is also relevant to option's market as it admits the option price is dependent on its underlying price, and clears all of its active orders once new information about the asset prices arrive. Also mechanism is responsible for exercising option upon their maturity, and compute necessary statistics about the sensitivity of option prices and the distribution of traders wealth.

- I have used the ZIP algorithm initially proposed by Cliff [29] as a proxy trading algorithm that uses pre-computed option price as its private value. This algorithm showed good performance in terms of generating wealth by trading options, and played a key role in simulating liquidity traders in option market. I also illustrated that its key parameters such as β and γ can be tuned to the specifics of the underlying market to increase its profitability.

- I have adapted the GD algorithm [69] to options by replacing the price based probability estimation to ratio-based probability estimation due to the changing nature of option prices. Because GD algorithm uses the orders posted by other traders to generate his own orders, it can be useful algorithm in option markets with heterogeneous traders, as it aggregates their diverse beliefs and takes advantage of those trades which are acceptable with his own private valuation.
- Inventory-based Garman's dealer [63] has been re-purposed for trading options to model a dealer who could generate bid-ask spread around pre-computed option prices subject to the limitations in the inventory, namely in cash and options accounts. This algorithm can be used for trading options where budget constraints are prevalent, so the trader significantly changes his bid and ask as soon as his inventory runs low.
- Information-based Copeland-Galai's dealer [33] has been re-purposed for trading options using Geske's formulas [66] and Straddle option portfolio. This algorithm can be used for trading options where there is a sentiment that there are informed traders in the option market, hence making it not efficient. After inferring the percentage of informed traders in the market as shown in Section 4.2, this algorithm can be used to mitigate the loss from informed traders.

There are couple of limitations and pitfalls pointed out in the mechanism and trading algorithms used. First, the mechanism did not simulate the order arrivals as a random process such as Poisson or exponential process. It simply reshuffled the order of traders to submit their orders consecutively for every trading day. It would be more realistic if traders were modelled with intelligent entry and exit algorithms from the signals in the market. At least the submission of orders by the traders could be modelled using above mentioned stochastic models with an intensity being the function of the prices quotes. In this way, dealers like Garman or Copeland-Galai could correctly calibrate the arrivals and use them in setting their bid-ask spreads. In this way, the width of their bid-ask spread would be adapted to the market environment. We have seen that the bid-ask spread remained almost unchanged for Copeland-Galai dealer due to the deterministic linear model of order arrivals. Second, the traders were all submitting only limit orders, and there were no market orders that could add extra volatility to the market. This is because our assumption of each trader as a price-setter due to the option pricing method in its core. The trading algorithms could be enhanced further to submit market orders once attractive prices are detected in the market. Third, the traders were using random integers to select the quantities of the options they needed, where more sophisticated ways of splitting the quantities into smaller chunks to reduce the market impact could be applied.

Chapter 7

Perspectives on Combinatorial Exchanges

7.1 Introduction

In this chapter, I present potential perspectives that can be implemented in contemporary financial exchanges, particularly in option market, to advance the trading of compound contracts such as option portfolios. I use Parkes *et al.* [130] Iterative Combinatorial Exchange (ICE) as a reference model for the design of combinatorial exchange for option portfolios. I apply this mechanism to the context of option portfolio trading and use its Tree-based Bidding Language (TBBL) [25] to enable the structured bidding of option portfolios. I describe the process of bidding and allocating the options to the winners through solving the Winner Determination Problem (WDP). The payment rule is defined as a VCG alike scheme named *threshold rule* which was described in Chapter2. I do not provide numerical results for the proposed combinatorial exchange due to the lack of the implementation of agents capable of trading option portfolios.

I have reviewed the combinatorial auctions and the combinatorial exchanges in earlier chapters, and emphasised their use in trading packaged goods. Most of the financial instruments traded in the markets like MBSs and CDOs are constructed as compound contracts, and were mostly responsible for the crash of 2008. In the context of options, this would mean the use of option portfolios. The standard option portfolios such as bullish spread, bearish spread, butterfly spread, etc can be considered as a separate financial instrument and can be traded as a single contract in the market. However in standard finance these type of contracts are priced from the perspective of arbitrage-free assumption. The price of an option portfolio should not be less or more than the combination of financial instruments that could replicate its cash flows. For example, the price of an option itself is derived from the replicated portfolio of cash and stock, so the same principles work here.

What if the option prices are derived not only from the perspective of arbitrage-free pricing, but also from the perspective of simulated markets with heterogeneous traders? In other words, what if we assume that the option price is mostly influenced by the price of its replicating portfolio, but there are also other factors such as market forces which could distort its value to some extent? These forces could be the frictions of the market, or the diversity of trader valuations in the market. In Chapters 5 and 6, I proposed how different option pricing methods and trading algorithms can be used to price the individual option in a simulated market environment. If we start holding to this view, where the option price cannot be simply derived from its replicating portfolio's price, then this is also true for the option portfolio which is made from one's market positions on different options. In other words, we cannot derive the option portfolio's price from its constituent option prices. This would involve its pricing in a combinatorial exchange environment where the options could be bundled and priced together.

The ICE presented by Parkes *et al.* is an expressive and a complete mechanism stemming from earlier works accomplished in combinatorial auctions [36] involving such concepts as price discovery and activity rules. Its bidding language TBBL generalises the expressiveness and completeness of multiple bidding languages such as `OR`, `XOR`, etc proposed earlier for combinatorial auctions [123]. Its efficiency can be asymptotically approximated to the number of goods or traders participating in the mechanism. Its payment rule is budget-balanced, and most importantly DSIC and individual rational. However similar to combinatorial auctions, its WDP is NP-hard as it can be directly transformed into set packaging problem.

This chapter is organised as follows. Section 7.2 describe the options from the new perspective of their substitutability and complementarity for the traders, and emphasise the need for the combinatorial exchange. In Section 7.3, I present the design of a combinatorial exchange for option portfolios based on reference model given by Parkes *et al.* . I provide the real application scenarios for trading option portfolio with the use of TBBL. In Section 7.4 I extend the existing TBBL to more general perspective where the concepts of valuation and preferences are separated in the bidding language, and some generic cases can be reused by the traders and mechanisms to express not only the bids and asks for particular types of options, but the trader beliefs in a succinct and complete way. Section 7.5 draws the summary of the chapter, lists the contributions made and discusses the future works in this field.

7.2 Substitutability and Complementarity of Options

Normally, the substitutability and complementarity of options are not defined in financial literature. In a risk-neutral and frictionless world, option is a contract which mimics the payoff function of a delta-hedged portfolio. Therefore its value is nothing but the value of that replicating portfolio which consists of cash and assets. As long as the cash and assets are identical and anonymously priced, the options on top of them can be priced according to the same parameters which are invariant. This would mean that every option has uniquely defined price which is risk-neutral compared to the conditions of the underlying market. This would also mean that these prices have nothing to do with the supply and the demand in the option market itself. Once the supply and the demand are not involved in pricing the options, the economic concepts such as cross-demand (cross-supply) elasticity XDE (XSE) which define substitutability (i.e XDE is positive) and complementarity (i.e. XDE is negative) of goods are also disregarded in option market. However as I introduced the market component into the option pricing methodology, and determined the option prices using the supply and the demand for options, then the question of XDE and XSE may naturally arise in studying the substitutability and complementarity relationships between different options.

For example, let us consider a case where options can be viewed as substitutes. There is bearish sentiment in underlying market and majority of option traders wish to take bearish spread which consists of buying one OTM call and selling one ITM call. However traders have multiple choices for OTM calls with different strikes. If over-demand is assumed to be the main reason for the rise of OTM call prices, then for some OTM if the price remains the same, the traders would be willing to buy it at cheaper price compared to other OTMs. This, as a result, would increase the demand for this option. Hence OTM options are substitutes in bearish spread.

I have presented several commonly used option portfolios in Table 4.12, and above example is one case where the use of these option portfolios may result in options being considered as substitutes. Another aspect of goods as substitutes is the unwillingness of the traders buying two substitute items simultaneously. Using our previous example, trader willing to take bearish spread does not want to buy two OTM options, having sold only one ITM. This would distort his option portfolio and change its payoff function. In given example, trader can have a valuation as $v(\{OTM1, OTM2\}) = v(\{OTM1\}) \oplus v(\{OTM2\})$, because he wants to buy only one OTM, so he pays only for one option allocated to him and takes the second one for free. Let us assume that out of multiple OTMs allocated to the trader, it is for the cheapest OTM he pays. So his valuation function is, indeed, $v(\{OTM1, OTM2\}) = \min[v(\{OTM1\}), v(\{OTM2\})]$. This is definitely smaller than their combined valuation, so according to the definition of substitutability below (7.1), these two options OTM1 and OTM2 are substitutable for the trader who wants to take a bear spread.

$$v(\{OTM1, OTM2\}) \leq v(\{OTM1\}) + v(\{OTM2\}) \tag{7.1}$$

In order to understand the complementarity of options, let us consider the trader who wants to take bull spread. In this case, the trader needs to buy one ITM call, and sell one OTM call. We know that the ITM costs more than OTM, so the trader's linear price of this contract is $v(\{ITM, -OTM\}) = \hat{v}(\{ITM\}) - \hat{v}(\{OTM\}) \geq 0$[1]. However the trader is determined to take bullish spread, and considers no other choice. So this would mean that if the trader gets allocated with either one of the options, but he is refused for the other one, his bid for ITM is zero, and ask for OTM is infinity. In other words, both $v(\{ITM\}) = 0$ and $v(\{-OTM\}) = \infty$ is true. Hence we know that $v(\{ITM\}) - v(\{-OTM\}) < 0$, we can derive an inequality given below which defines the strong complementarity of goods.

$$v(\{ITM, -OTM\}) > v(\{ITM\}) - v(\{-OTM\}) \tag{7.2}$$

[1]Minus sign means short position. $\hat{v}(\cdot)$ is the intrinsic value of the option for the trader, not the reported one.

To sum up, the traders engaging in taking option portfolios may exhibit a valuation behaviour which makes OTM options (and ITM options by symmetry) with different strikes mutually substitutable. Also we have seen that ITM and OTM options could be complements to each other when they are wanted as an integral part of an option portfolio. Once we can establish such relationships between options, there is an immediate necessity for a combinatorial exchange which can accept such preferences from traders and efficiently allocate them to winning traders. Below in this chapter, I propose a design of such combinatorial exchange that can deal with the substitutability and complementarity of options inside option portfolios.

7.3 Design of Combinatorial Exchange

There are two ways of looking at option market for traders using option portfolios. First way is looking at the market as a multi-unit multi-item double auction where traders simply submit their orders for each option separately and then once their orders satisfied, they hold a certain option portfolio. We can also consider this case as simultaneous multi-unit double auctions run in parallel for different types of options. Cramton [36] describes an issue for simultaneously ascending auction where traders had an incentive to snipe in an auction which bid the least price. In order to prevent bidders engaging in sniping, the auction introduced set of activity rules into its protocol. One of them was not to allow bidders increase their volumes as the price goes up, as it contradicts to the law of demand. Not imposing such activity rule would allow bidders to put a bid with an insignificant volume to stay active in multiple simultaneous auctions until the last moment when the trader put all his required volume into the cheapest auction and wins the lot. Once the activity rules are applied, the mechanism can produce surplus maximised outcome and hence be efficient.

However, the second way of dealing with option portfolios is matching them as in a combinatorial exchange. In this way, traders may not reveal their individual valuations of the options to the mechanism. Also traders are not required to coordinate their bids and asks in multiple auctions to make sure that their option portfolio is compiled. In more general perspective, combinatorial exchange allows the traders to express much more information other than simple quotes on options or option portfolios they want. In fact, in combinatorial exchange traders can reveal their whole strategy, or bearish or bullish beliefs, to allow the mechanism to decide which option portfolio one needs in order to maximise his utility. Although combinatorial exchange so many flexibilities and advantages to traders, it simply shifts these responsibilities to the mechanism, and makes his problem of finding best allocation result and DSIC payment rule NP-hard classed problem. As regards the allocation rule, the simultaneous multi-unit double auctions can provide surplus maximising allocation for combinatorial exchanges when the goods are substitutes. In other words, when the individual sums of the valuations of goods are not less than the combined valuation of the goods, then the surplus maximisation of simultaneous multi-unit double auctions will always be higher than the one found in combinatorial exchange for substitutable goods. Roughgarden also mentions such relationship between combinatorial auctions and simultaneous ascending auctions in one of his remarkable lectures [141].

However the combinatorial exchange allocation becomes NP-hard when the goods are complements, and the simulation of parallel multi-unit double auctions cannot produce better surplus maximisation. I mentioned that the options can be both substitutes and complements depending on the trader's strategy in option market. Hence we have to consider case of a combinatorial exchange which resolve the issue of allocating options through the use of a bidding language and an appropriate WDP.

In this section, I propose a design of a combinatorial exchange for trading option portfolios. I use Parkes *et al.* [130] ICE as a reference model and show how it can be used to implement a marketplace for option portfolios. I present the TBBL bidding language, and use it to specify the preferences of option traders. Then I formulate the WDP and payment rules. Let us specify the notation I used for denoting traders, options, allocations etc:

$N = \{1, \ldots, n\}$ is the set of traders

$G = \{1, \ldots, m\}$ is the option types listed in the option chain

$x^0 = (x^0_1, \ldots, x^0_n)$ is the the limits for traders in selling options. $x^0_i = (x^0_{i1}, \ldots, x^0_{im})$ where $x^0_{ij} \in \mathbb{Z}_+$ is the limit for each trader to sell particular type of option. This limit can be imposed by the margin account of the trader, or by the company's policy, or as we did in previous chapters, by the mechanism itself. Hence these limits are the quantity cap for the maximum number of options a trader can sell.

$\lambda = (\lambda_1, \ldots, \lambda_n)$ denotes the change in the allocation of options (i.e. trade), while each $\lambda_i = (\lambda_{i1}, \ldots, \lambda_{im})$ and $\lambda_{ij} \in \mathbb{Z}$. So λ_{ij} is the change in agent i's jth option account, negative number meaning the sales, and positive the acquisition.

$M = \sum_{i \in N} \sum_{j \in G} x_{ij}^0$ is the maximum number of options that could be traded in the combinatorial exchange.

7.3.1 Efficient Trades

For each possible trade for the agent i, we can define valuation function $v_i(\lambda_i) \in \mathbb{R}$ which denote how much the trader is willing to pay or receive for given set of trades in λ_i. Using our previous example, if the trader wants bear spread only, then his valuation for OTM options should be $v_i(\lambda_{OTM}) = \inf_{\lambda_{ij}>0, j \in \lambda_{OTM}}(v_i(\lambda_{ij}))$, as he considers the OTM options as substitutes. Let us denote the final position for the trader i as $x_i^0 + \lambda_i \geq 0$ which would mean that the agent's capacity decreases as he sells more options, and it should not exceed the given cap x_i^0.

I use the *free disposal* assumption so $v_i(\lambda_i') \geq v_i(\lambda_i) \rightarrow \lambda_i' \geq \lambda_i, \forall j \quad \lambda_{ij}' \geq \lambda_{ij}$ is true. Also let us denote the overall surplus from the trade as $v(\lambda) = \sum_i v_i(\lambda_i)$. Traders use quasi-linear utility to evaluate each trader: $u_i(\lambda_i, p) = v_i(\lambda_i) - p$ where p is the price paid for the trade λ_i. We know from Chapter 2 that the quasi-linearity of utilities guarantee that any Pareto improvement to the allocation maximises the social surplus. So for the given profile of valuations and caps (v, x^0), we can define a Pareto improvement, or an *efficient trade* as λ^* such that

$$\lambda^* = \arg\max_{\lambda} \sum_i v_i(\lambda_i) \tag{7.3}$$

$$\text{s.t. } \lambda_{ij} + x_{ij}^0 \geq 0, \quad \forall i, \forall j \tag{7.4}$$

$$\sum_i \lambda_{ij} = 0, \quad \forall j \tag{7.5}$$

$$\lambda_{ij} \in \mathbb{Z}$$

Constraint (7.4) is used to enforce the cap of the mechanism in the volume of options traded, and in (7.5) the strict budget-balance is enforced through the free-disposal assumption. This would mean that the unwanted options could freely allocated to traders. For example, if the bear spread taker considers

OTMs as substitutes, and pays only for the cheapest one, in case if the mechanism allocates the trader 2 OTM options, it is acceptable that the trader pays for only one option, and gets the other one for free.

It can be seen from the formulation of the efficient trade that the mechanism is aware of the caps for each trader. This is plausible because the traders participating in derivatives market maintain margin accounts which are continuously marked to the market by the broker as the market changes. Hence the mechanism knows the capacity of each trader to issue options. Let us denote any feasible set of allocations for given x^0 as $\mathcal{F}(x^0)$, and let the any feasible set of allocations for trader i as $\mathcal{F}_i(x^0)$.

In a competitive equilibrium, there is no Pareto improvement possible in given option allocation λ. Let us define the linear prices $\pi = (\pi_1, \ldots, \pi_m)$ where π_j is the price defined for option $j \in G$. These prices are anonymous for every trader. For the given profile of linear prices π, and the allocation for the trader λ_i, the total amount the trader has to pay or receive from the mechanism must be equal to $p^\pi = \lambda_i^\intercal \pi$. Then we can determine the competitive equilibrium prices for the market as follows:

$$v_i(\lambda_i) - p^\pi(\lambda_i) \geq v_i(\lambda_i') - p^\pi(\lambda_i'), \quad \forall \lambda_i' \in \mathcal{F}_i(x^0) \tag{7.6}$$

where $\lambda \in \mathcal{F}(x^0)$ is a feasible allocation which maximises the overall utility of the market. It is also said that the trade λ is supported by prices π.

The π prices are not always formed from the pool of fully efficient trades, but they can also use any other feasible allocation to approximate the efficient trades. Parkes *et al.* defines the δ-approximate competitive equilibrium as show below:

$$v_i(\lambda_i) - p^\pi(\lambda_i) + \delta \geq v_i(\lambda_i') - p^\pi(\lambda_i'), \quad \forall \lambda_i' \in \mathcal{F}_i(x^0) \tag{7.7}$$

Theorem 2 in Parkes *et al.* [130] states that any feasible trade λ supported by δ-approximate equilibrium prices π is a $2\min(M, n/2)\delta$-approximate efficient trade. Hence by finding such δ for any given feasible trade λ and the δ-approximate equilibrium prices π, we can find the exact approximation of the efficient trades in the given mechanism.

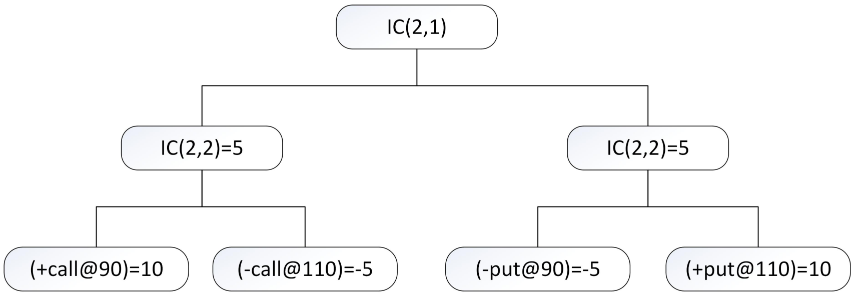


Figure 7.1: Bullish spread expressed using TBBL

7.3.2 Bidding Language

TBBL is specifically designed for combinatorial exchanges by Parkes *et al.* [25, 130] and it can be used to express option portfolios as combinatorial bids to the mechanism. This language is fully expressive, and designed to be as concise and structured as possible. Because it is specifically designed for combinatorial exchange, it allows bidders to submit bids and asks simultaneously. The tree structure is used for expressing bids and asks connected through series of generalised logical connectives, such as 'interval-choose' (IC_x^y) operator. It specifies at least x and at most y of its child nodes must be satisfied. Hence, all intermediate nodes in TBBL are IC_x^y nodes with corresponding price bounds. IC_x^y can replicate `OR` as IC_1^n, `XOR` as IC_1^1 and `AND` as IC_n^n. The leaf nodes of the TBBL are the actual bids or asks on options.

For example, trader is taking bull spread, so he can submit following bid which generates positive cash flow if price goes up. Figure 7.1 shows how it can be represented in tree format, where each node has its own value, and plus sign implies bid, and minus ask. This shows that the trader is willing to buy call at strike \$90 for \$10, and (i.e. $IC(2,2)$) sell call at strike \$110 for \$5. We implicitly assume that the current asset price is $S_0 = \$100$, hence first call is ITM, and the second call is OTM. Similarly, the trader is also indifferent to buy the same bull spread with puts. Trader states that he wants to sell OTM put at strike \$90 for \$5 and buy ITM put at strike \$110 for \$10. Both spreads are evaluated at the price of \$5 for the trader.

The important aspect of an above bidding structure displayed in Figure 7.1 is that the trader can express the bull spread both in terms of calls or puts. This gives him the flexibility in choosing equivalent option portfolios among possible allocations, and fully express his preferences to the mechanism. This is a great advantage of a combinatorial exchange over a double auction, because in this example bull spread taker can fully reveal his indifference to trading various sets of options, as long as they make equivalent portfolios and generate the same payoff. In other words, instead of expressing his quotes to individual options as it was normally done in double auctions, in combinatorial exchange the trader is now capable of expressing his entire strategy to the mechanism. It is important to note that the combinatorial exchange is DSIC [2], and the traders are always better off revealing their true strategy to the mechanism. Complete revelation of trader's strategies consisting of option portfolios also provides more allocation choices to the mechanism.

Now let us understand the main components of TBBL. Every trader i can submit bid T_i. Let $\theta \in T_i$ denote any node in the tree, and $v_i(\theta) \in \mathbb{R}$ is the value of this node for bidder i. A function $Leaf(T_i) \subset T_i$ returns all leaves of bid T_i, and $Child(\theta) \subset T_i$ returns all child nodes inside node θ. Any node θ is said to be satisfied by $IC_x^y(\theta)$ if:

- R1: Node θ with $IC_x^y(\theta)$ may be *satisfied* if only at least x and at most y of its children are satisfied.
- R2: If some node θ is not satisfied, then none of its children may be satisfied.

Let $G \in \{1, \ldots, m\}$ represents the options listed in the option chain. Let $\lambda_i \in \mathbb{Z}$ be the vector representing which option to take and which option to give for bidder i, or in other words a trade, or an allocation. Then the value v_i of the allocation λ_i for bid T_i is equal to the sum of all satisfied nodes. In order to represent satisfaction, let us define $sat_i(\theta) \in \{0, 1\}$ function which represents if $\theta \in T_i$ is satisfied. Valid set of solutions for T_i can be derived through applying R1 and R2 to all internal nodes of T_i, such that $\theta \in \{T_i \backslash Leaf(T_i)\}$. Hence for $IC_x^y(\theta)$ following condition should hold:

$$x sat_i(\theta) \leq \sum_{\theta' \in Child(\theta)} sat_i(\theta') \leq y sat_i(\theta) \tag{7.8}$$

Secondly, we also do not want for any given trade λ, the number of options supplied is less than demanded. So we can write this constraint as follows:

$$\sum_{\theta \in Leaf(T_i)} sat_i(\theta) \leq \lambda_{ij}, \forall j \tag{7.9}$$

[2] See Parkes *et al.* [130] for proof.

So the rules R1 (7.8) and R2 (7.9) form the validity function $valid(T_i, \lambda_i)$ for a given bid tree T_i and allocation λ_i. This validity function returns the mapping for the satisfiability of each node θ in T_i under given λ_i. Hence we can right the satisfiability function as $sat_i \in valid(T_i, \lambda_i)$.

The valuation of the given tree T_i holds free disposal rule where unsold options can be freely allocated to any bidder. With given constraints we can formulate a valuation function for given bid T_i,

$$v_i(T_i, \lambda_i) = \max_{sat_i} \sum_{\theta \in T_i} v_i(\theta) sat_i(\theta) \tag{7.10}$$

$$\text{s.t. } (7.8)\ (7.9) \tag{7.11}$$

7.3.3 Winner Determination

I have covered the WDP for combinatorial auctions and combinatorial exchanges in Chapter 2 and formulated them in the form of a Integer Linear Programme (ILP) which can be reduced to NP-hard set packaging problem. In this section, I define the WDP using the TBBL for a combinatorial exchange. This finds the efficient set of allocations λ for the given capacity x^0 in the combinatorial exchange. Let us define $T = (T_1, \ldots, T_n)$ as the TBBL bids submitted to the mechanism. Also let us denote the tree node $\theta \in \lambda_i$ if $\theta \in T_i$ and it is satisfied by trade λ_i written as $sat_i(\theta) = 1$. Then we can formulate the WDP for the option exchange as shown below:

$$WD(T, x^0) = \lambda^* = \arg\max_{\lambda, sat} \sum_i \sum_{\theta \in T_i} v_i(\theta) sat_i(\theta) \tag{7.12}$$

$$\text{s.t. } (7.4),\ (7.5)$$

$$sat_i \in valid(T_i, \lambda_i), \forall i \tag{7.13}$$

$$sat_i(\theta) \in \{0, 1\}, \lambda_{ij} \in \mathbb{Z} \tag{7.14}$$

The solution of the above WDP should give the matrix of allocations λ that maximise the surplus for posted options. The mechanism has to choose the valid mappings for sat_i for the nodes of submitted tree bids T and at the same time maximise the valuations of these nodes.

7.3.4 Threshold Payments

I have covered the threshold payment rule in Chapter 2, and formulated it as an optimisation problem (2.67) using the minimisation of the worst difference in VCG payments and threshold payments. The same payment rule is used here to make the payments budget-balanced and conform with δ-approximate equilibrium prices. Let me define the threshold payment rule for the current case, the equation (7.15) shows its formulation. Also for determining the VCG discounts let λ^*_{-i} be the combinatorial exchange's allocation of options where trader i did not participate.

$$\rho_{vcg,i} = v_i(\lambda^*_i) - \Delta_{vcg,i} \tag{7.15}$$

$$\Delta_{vcg,i} = \left(\sum_j v_j(\lambda^*_j) - \sum_{j \neq i} v_j(\lambda^*_{-i,j}) \right) \tag{7.16}$$

where $\rho_{vcg,i}$ is the VCG payment for the trader i, and $\Delta_{vcg,i}$ is called as the VCG discount for the trader i. Also note that the trader produces a scalar valuation for the given allocation vector λ^*_i, as the trader values the bundle as a whole. Then we can find such $\Delta_{thresh,i}$ which solves the minimisation of the worst difference between VCG discount and the threshold discount.

$$\boldsymbol{\Delta}^*_{thresh} = \underset{\boldsymbol{\Delta}_{thresh}}{\arg\min}\, \epsilon \tag{7.17}$$

$$\text{s.t. } \Delta_{vcg,i} - \Delta_{thresh,i} \leq \epsilon \quad \forall i \tag{7.18}$$

$$\Delta_{vcg,i} - \Delta_{thresh,i} \geq 0 \quad \forall i \tag{7.19}$$

$$\sum_i \Delta_{thresh,i} \leq \sum_i v_i(\lambda^*_i) \tag{7.20}$$

The solution $\boldsymbol{\Delta}^*_{thresh}$ can be used to compute the budget-balanced payments for traders given formula below:

$$\rho_{thresh,i} = v_i(\lambda^*_i) - \Delta_{thresh,i} \tag{7.21}$$

To illustrate the combinatorial exchange in option market, consider following example given in Figure 7.2. There are two traders who submitted their bids to the combinatorial exchange to take a corresponding position in the option market. First trader is bullish trader because he is willing to buy a bullish spread. Although he specified which ITM call he wants, he is indifferent for the OTM call he wants to sell. So he can sell either call at strike \$105 for \$4, or

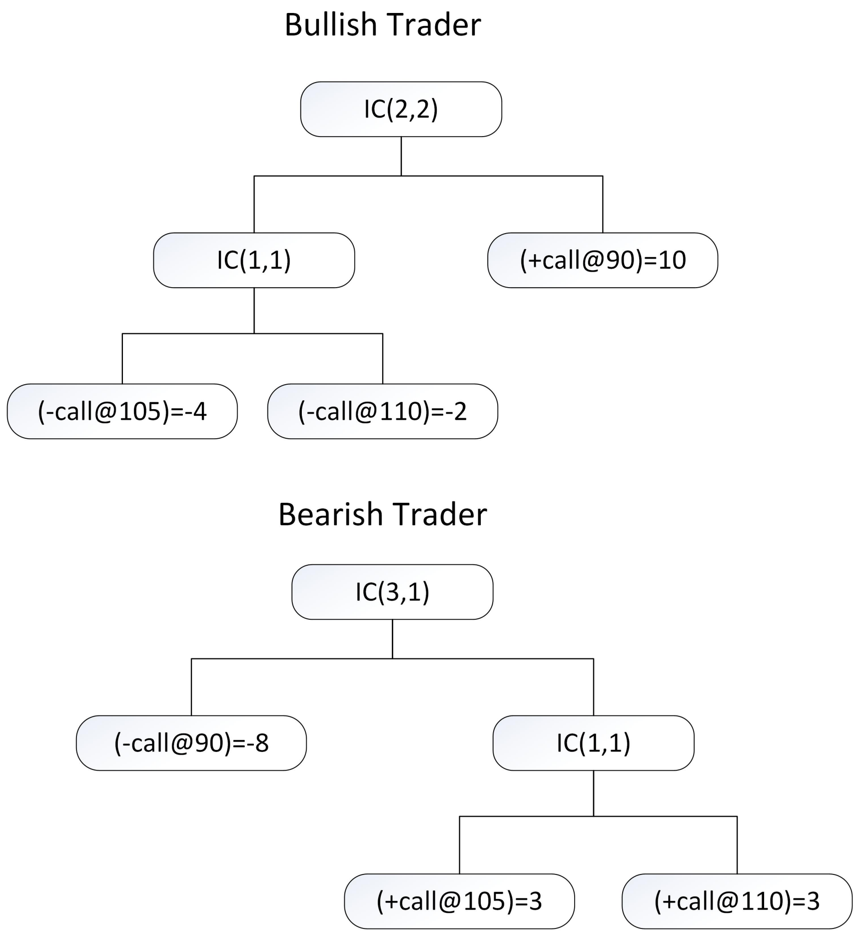


Figure 7.2: Bullish and Bearish traders' bids using TBBL

call at strike \$110 for \$2. Note that here bullish trader is regarding the OTM calls as substitutes. The second trader is bearish in his belief on asset prices, so he expects a asset prices to go down, and wants to take a bearish spread. So he submits one ask for the ITM call, and he is indifferent in buying either of OTM calls. There are 2 potential matches in given example. In the first match, bullish trader is going to buy ITM call from bearish trader creating a surplus of \$2, and sell OTM call at strike \$105 for \$4 to the bearish trader creating a deficit of \$1. In the second case, bullish trader does the same with the ITM call, but sells OTM call at strike \$110 for \$2 creating a surplus of \$1. The surplus maximising allocation would choose the second matching, as it maximises the overall surplus to \$3.

Now let us consider the payments paid and received by both traders using threshold rule. Considering it using VCG scheme, we understand that the removal of any trader would cause null trade. Hence the VCG discount of both traders is equal to the surplus made, which is \$3. In this way, the bullish trader's total payment to the mechanism is $\$10 - \$2 - \$3 = \$5$. However the bearish trader's total payment to the mechanism is $-\$8+\$3-\$3 = -\$8$, so he should receive \$8 from the mechanism which runs into deficit of $\$5-\$8 = -\$3$. But now using the threshold rule, we can find the minimised deviance of the discounts from the VCG discounts. In this example, the threshold discount is \$1.5. So applying the threshold discount instead of VCG discounts, we get a payment from bearish trader $\$10 - \$2 - \$1.5 = \$6.5$ and get a payment from the bullish trader $-\$8 + \$3 - \$1.5 = -\$6.5$. As it can be seen mechanism balances the deficit, through decreasing the amount of the VCG discount for both traders.

7.4 More on TBBL

In this section, I introduce a new version of TBBL where the concepts of valuation and preferences are separate. Although it is common for existing bidding languages to combine both concepts into single atomic bid, in option portfolio market it becomes redundant to provide valuations for the identical goods appearing in different portfolios. It enforces the consistency of option prices throughout multiple portfolios. The mechanism can also easily check if the individual option prices are within their legal bounds. Moreover there is possibility for the trader just to disclose his belief and the number of options in his portfolio, and the mechanism can automatically pick the right option portfolio for him.

The trader sends to the mechanism two pieces of information. First is the list of his private valuations for the options listed in option chain. The second is his preference over these options expressed through TBBL. While referencing the options in the TBBL, the trader does not indicate his valuation, but just indicate the option type and the strike he wishes to buy or sell. Of course, this way of bidding the options reveals the individual valuations of options to the market maker which might not be a desired property of a combinatorial exchange in general. However this requirement is necessary in option market because the mechanism should be able to check the consistency of option prices. Let me list these checks that the mechanism should confirm in order to accept the traders bid:

1. Option prices have lower and upper bounds given in (2.4) and (2.4), so they should fall into these ranges.

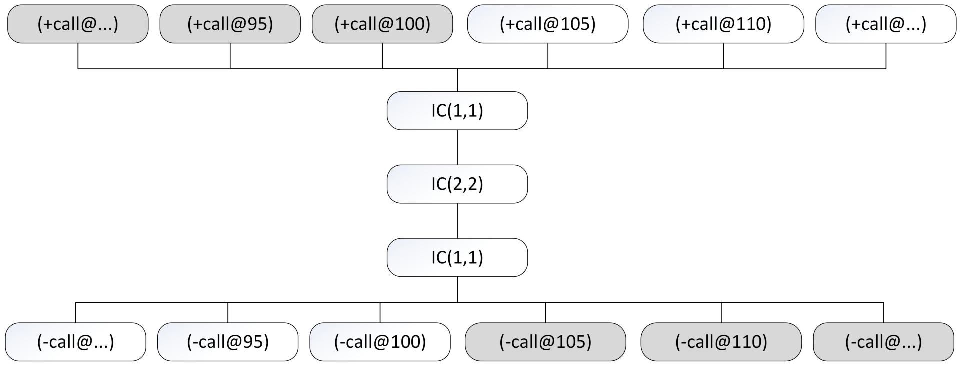


Figure 7.3: Generic 2 option spread using TBBL

2. Put and call prices with the same strike are connected through put-call parity relationship (2.6), so the trader prices must be consistent with each other.

3. The deeper the option goes into moneyness, its price should never decrease. In other words, if there are two calls with $K_1 < K_2$, then the value of $v_i(K_1) \geq v_i(K_2)$ as the first option is more deeper in moneyness than the second one.

4. If mechanism can establishing the arbitrage-free bound for option prices taking into account the frictions of the market, the potential magnitudes of the asset price rises and falls, then mechanism can also enforce the arbitrage-free prices of the options on traders.

For above reasons the traders are required to declare their valuations of individual options to the mechanism.

On the other hand, the traders can be freed from the burden of building the TBBL to express their preferences. For instance, consider a bullish trader who wants to take a bullish spread, but he has not clarified which concrete ITM call and OTM call he wants to buy and sell. He can submit to the mechanism following generic TBBL shown in Figure 7.3 along with his option valuations for each option in the option chain. Hence the trader declares all of his valuations in one list, and then interprets his generic bullish preference to the mechanism. The options that the trader would like to use are highlighted with shaded background in Figure 7.3. Note that the trader did not give his valuations inside TBBL, but sent them in a separate list.

As we can see from above Figure 7.3, it is the job of the mechanism to decide which ITM and OTM options to use in order to satisfy trader's bullish preference. The mechanism will choose the options that maximise the allocative efficiency in the market, and therefore the utility of the trader. The option trader does not need to employ any algorithm for finding the correct portfolio, as he only needs to disclose his prediction, or just a belief in the direction of the asset price future movement, and provide his corresponding valuations for each option. The mechanism will then find the best allocation for him given the bids and asks of others.

This generic and declarative approach can be used for other types of options portfolios which contain more than 2 options. For instance, the neutral option portfolios such as butterfly, long ladder, etc contain more than 2 options, and can also be combined into generic format in the same way it was done with spreads. So the mechanism can store all these generic TBBLs as master templates and have nomenclature for their naming. The traders in their turn can provide only the name of the master template which matches with their preference structure and send it to the mechanism. This definitely simplifies the communication between the trader and the mechanism, and helps the trader to express more about his belief through very limited traffic.

7.5 Summary and Contribution

In this chapter, I reviewed the potential perspectives for the option market using the combinatorial exchanges and pointed out their role in pricing compound products. Once option is priced not only using the parameters of its underlying market, but also the forces in the option market itself, then the back my argument. I also provided couple of scenarios where the options can be considered as substitutes and compliments for the traders. I can list the contributions of the chapter below:

- *Substitutability and Complementarity of Options:* I described the possible cases where the substitutability and complementarity relationship between different options may emerge. I have shown that in spread-based option portfolios, the OTMs (also ITMs) are substitutes among each other, as the trader can choose any of the OTM to form, for example, a bullish spread. Also I have shown that OTM is the complement for the ITM in a spread-based option portfolio, as the trader is only interested in having both of his orders on ITM and OTM satisfied simultaneously, so he does not accept the case where only one of them is satisfied by the mechanism.

- *Design of Combinatorial Exchange:* I used the Parkes *et al.* 's ICE design as the main reference model, and have shown its potential application to the option market through several examples. I also gave insight on representing different option portfolios using TBBL, and formulated WDP and budget-balanced payment rule for their pricing. The proposed combinatorial exchange also holds an approximate efficiency measured by $2\min(M, n/2)\delta$ error, individual rationality and most importantly DSIC properties.

- *Declarative TBBL:* I also presented the further improvements that can be done to the proposed mechanism by changing the structure of TBBL. In particular, I suggested to separate valuations from the preferences, and pointed out its important in option market case. I also introduced a generic class of TBBLs that could be used to represent any kind of option portfolio with no specification on the strike prices of OTMs and ITMs. These classes of generic TBBLs can be predefined and repetitively used by the mechanism to communicate the trader's preferences in a succinct and complete way.

Beyond the made contributions, there are still many aspects of the research that are at its early stages. To my current knowledge, there are only couple of solid combinatorial exchange designs have been proposed so far, and none of them are continuous. Online models of the combinatorial exchanges can emerge from the online double auctions holding BIC that have been recently developed. Also, there is no trading algorithm that has been developed for a continuous trading in a combinatorial exchange. In the context of option trading, this would involve the different ways of selecting the option portfolios based on the conditions of the option market. The traders would have to evaluate not only the option price and the quantity, but also the future predictions of asset prices, and their use in compiling the option portfolios. The traders would have to target the outstanding combinatorial bids in online exchange to learn more about the preferences of other traders and construct their own to meet the demands and make profit. Indeed, bidding languages provide lot more information about the particular trader's preferences, and this can be harnessed to build more advanced market mechanisms in future.

Chapter 8

Conclusion and Future Works

In this chapter, I summarise the main findings of the research. Then I summarise the contributions by providing the answers to the research questions posed at the beginning of the thesis. I also list the current limitations and drawbacks of the work. I shall briefly discuss about the potential application domains of the research. Then I propose my future works aimed at tackling the presented issues, improving the current results and extending this research to new frontiers.

8.1 Research Findings

In this work, I introduced a new simulated market component into the existing option pricing methodology. The title of this thesis suggests that the options are priced 'via' double auctions, hence the main concern of the research was not abandoning the existing methodologies, but putting them into a market environment in order to obtain competitive option prices. We have seen different option pricing methods starting from classic Black-Scholes methodology, to numerical Monte-Carlo pricing. I have used the indifference option pricing methods to reflect the risk-averseness of traders to potential loss. Also I used cutting-edge LMSR option pricing methodology and adopted it for pricing the options from the perspective of an option portfolio holder. These diverse option pricing methodologies are then channelled into a direct double auction to obtain the corresponding option prices and to test their sensitivity to changes in the different parameters of the underlying market. In the process, I designed the direct double auction which enables the satisfaction of multi-unit and atomic orders at the cost of budget-balance. This proposed mechanism holds the desired properties such as efficiency, individual rationality and DSIC. Most of the obtained option prices were close to the Black-Scholes theoretical price, but we learned more about the important factors that form these prices in the market. Few of these influencing factors were the range of accepted orders, the proportions of traders, the volume of trades, the efficiency and the budget-balance of the mechanism as well as the parameters of the underlying market and options themselves. I can summarise the results for simulation of different trading agents in the list below:

- *Risk-Neutral Traders:* The risk-neutral traders included volatility-based traders and Monte-Carlo traders. Their results were very similar to the analytical prices generated using Black-Scholes given that their beliefs aligned with the parameters of set in Black-Scholes formula. The option pricing results were similar for both random and linear methods of choosing supply and demand, although for the former case the mechanism's relative efficiency error was skewed to %2, while the latter case exhibited two peaks one in %0, and the other is in %3.5. The budget-balance was centralised around 0 in both cases. Another important finding was that the range of accepted orders decreased as the option approached its maturity date. This was explained by the option's payoff becoming less uncertain as it neared its maturity date. We have also seen that the option price sensitivity was similar to its theoretic values. For risk-neutral traders, the only exception was the Monte-Carlo trader simulating the asset prices according to jump-diffusion process (MON*). In this case, MON* traders slightly overpriced the ATM and OTM options due to potential upward jumps they expected. This difference was not significant for ITM options because the expectation of both MON* and the other risk-neutral traders was the same that the given ITM option ends up in-the-money. MON* prices also converged with Black-Scholes prices as the option approached its maturity. One finding for such traders was that the mechanism was almost fully efficient due to wider range of orders submitted. We have seen that the option prices were less sensitive to the changes in the underlying market for MON*. Mixing all risk-neutral traders together resulted in prices close to analytical ones in ATM and ITM markets, however the OTM options were slightly overpriced due to the higher accepted bids submitted by MON* traders.
- *Risk-Averse Traders:* In risk-averse traders, I have simulated traders

with an exponential utility who exhibited risk-averseness in selling and buying options. This would mean that they deliberately overprice asks, and underprice bids. The market fully populated with risk-averse traders would not result any trade as depending on their risk-averseness factor the chances that they match their bids asks were exponentially decreasing. I found out that the threshold risk-averseness parameter is $a = 0.03$, as for the lower risk-averseness, the orders match, and for the higher not. Simulating the risk-averse traders with equal number of risk-neutral ones would not generate any information about risk-averse traders, because they could not beat the bids and asks posted by risk-neutral traders, and therefore do not trade. In order to provide liquidity, I simulated the risk-averse traders with zero-intelligence traders who could post random price for the option with in its valid range. This resulted option prices aggregated around Black-Scholes option prices, but the range of accepted order varied from 0 to even 3 times more than the theoretical price. The volatility was higher for ITM options. The option's Greek analysis has shown that the ATM and the OTM options were more sensitive to the changes in the asset prices, compared to the ITM option. I have simulated the mixed market of risk-averse, risk-neutral and zero-intelligence traders, where the risk-neutral traders were made deliberately less to give an opportunity for trader to risk-averse traders. The results were more smoothed and less volatile compared to the only risk-averse traders case. However the sensitivity of option prices to the changes in the asset price were similar.

- *Portfolio Holders:* The portfolio holders were simulated using LMSR option pricing method where the traders had to price options based on their current portfolios. I have included LMSR traders with neutral, non-neutral, bullish and bearish portfolios. Due to the determinism in computing the option payoffs among LMSR traders, the resulted range of prices were not big. Actually, for each LMSR trader with distinct portfolio, it gave one bid and one ask price of the option. For example, simulating 2 neutral and 2 non-neutral LMSR traders resulted the overall of 8 distinct quotes from which the mechanism had to choose the winning prices. Few number of diverse quotes caused discrete jumps in option prices, although they were fluctuating around theoretical prices. I also found out that when the liquidity parameter of LMSR traders is set to $b = 100$, the traders produce reasonable range of bids and asks that match. Let me point out the findings from each scenario simulated for LMSR traders:

- *Neutral and Non-Neutral Portfolio Holders:* Initially, all options were underpriced, and as the option approached its maturity date, they crossed the theoretical price. The range of fluctuations did not diminish even if the option approached its maturity date. This was especially vivid for OTM options, where the neutral portfolio holders sold underpriced options to non-neutral portfolio holders, and when the OTM became ATM it was the non-neutral traders selling to neutral until the option's maturity. I found out that as the OTM option approaches maturity, the differences in pricing this option increase, hence there is more volatility involved. These are the differences in prices between neutral trader who view OTMs less profitable, and the non-neutrals who view them potentially profitable. From Greeks analysis, we have seen that the option prices are very sensitive to the changes in asset prices.
- *All Portfolio Holders:* Similar to above case, the prices were initially underpriced, and as the option approached its maturity, the prices were approximated along theoretical price. However, in contrast to the previous case, the volatility started to decrease when option neared its maturity. This is because there were more prices available to the mechanism to choose, and consequently the range of accepted orders was bigger too. I found out that the OTM prices were still overpriced after they turn ITM. Greeks analysis has shown similar results as it was for the previous case.
- *More Bullish Portfolio Holders:* The behaviour of the bullish portfolio holders was similar to previous simulations. Although the range of accepted orders was slightly smaller, there was less volatility in the prices. This would suggest that the prices of other traders were close to the ones submitted by bullish portfolio holders. Greeks analysis was similar to the previous results.
- *More Bearish Portfolio Holders:* The bearish portfolio holders underpriced all options because when the expectation is the decrease in asset prices, even the ITM calls appear worth less from bearish perspective. However the prices converge to the theoretical price in both ATM and ITM markets as the option approaches maturity,

while the OTM options are exhibited the similar trajectory as in other experiments ending up overpriced near maturity. Also the options were more sensitive to the changes in asset prices compared to other results. For example, the gamma of the ATM option has peaked at 0.28, while in previous results it was around 0.22.

- *Mixed All Traders:* In this experiment, I ran all developed option pricing methods in a direct double auction. The results were similar to theoretical prices, although the range of accepted order were wide enough even before the option's maturity. The prices were more volatile for the OTM options compared to ATM and ITM options. The option's sensitivity to the changes in the asset prices was asymmetric. The option prices were less sensitive when they are in an OTM range, and the sensitivity became higher when the option went into the ITM range.
- *Zero Intelligence Traders and Their Aggregated Forecasts:* In this series of experiments, I ran the ZI traders with different forecasts in a revealed mechanism. I controlled the quantities of options traded by ZI traders, and observed how they affect their aggregated forecasts. The experiment has shown that the aggregated forecasts were higher than the current asset prices when there was more demand for the call option. The opposite was true for when there was more supply. The asset prices and the forecasts aligned over the same trajectory when there was balance between supply and demand. I also used the forecasts of ZI traders to select option portfolios and traded them in a simultaneously run revealed mechanisms. It turned out that when the bullish portfolios are selected more, the aggregate forecasts were higher than the current asset prices, and vice versa when the majority of selected portfolios were bearish. When all option portfolios selected equally, the forecasted prices were similar to the asset prices.

Besides aggregating the different option pricing methodologies in a direct double auction, I also tested the proxy trading algorithms based on the Monte-Carlo option price in an online double auction. The other option pricing methodologies could also be used along with proxy trading algorithms, but because of the space limitation of this work, I dropped other pricing methodologies from the experiments. For the online double auction, I developed a LOB data structure in Python using the self-balancing binary trees. Then I funnelled the proxy trading algorithms into the mechanism, and obtained the option prices emerging from the continuous interaction of traders in the market. In this case, I used novel adaptations of Garman's and Copeland-Galai's dealership models which based their bid-ask spreads based on inventory and information respectively. I also adapted commonly known trading algorithms such as GD and ZIP to trade as a proxy algorithm for the option pricing methodology, and tested them for their performances. Below are my findings for each case:

- *ZIP Traders:* The ZIP traders provided a great liquidity to the market and exhibited very flexible behaviour. The closing option prices were volatile but approximated the theoretical prices. The range of day's high and low prices were very narrow. The payoffs as well as the number of options held by ZIP traders were normally distributed around zero. I simulated ZIP traders with different learning coefficients β and momentum γ. When $\beta = 0.1$, trader quotes were less flexible to the changes in option's value, and thus they ended up buying options at higher, and selling at lower prices. For $\gamma = 0.1$ the bid and ask margins remained almost the same as it was set initially, as they were slow to adjust them to the changing prices. These facts together also affected the ZIP trader's final payoff to be lower compared to other traders with higher β and γ. For ZIP traders with higher β and γ, the margins were flexible to the changes in markets and thus the bid-ask spreads were wider. This gave them an advantage in taking better deals from the mechanism, hence they bought at cheaper prices, and sold at higher prices. This, of course, increased their final payoff.
- *GD Traders:* GD traders were built to optimise the probability of successful order submitted to the mechanism and their prior is determined by the set of all historical orders submitted to the mechanism. Hence GD traders do not form their own prices, but learn from the experience of other traders and submit corresponding orders. For this reason, GD traders could not be simulated alone, and had to be mixed with ZIP traders to facilitate the trading process. The final results were the approximation of a theoretical prices, however there were numerical artefacts caused by the optimiser. The range of accepted prices were also very tight. However the interesting finding was that the distribution of payoffs and options were polarised with ZIP traders mostly taking short position and GD trader taking long position in the market. It happened because GD bid prices were attractive to ZIP traders. This resulted in

an enormous profit from buying out ATM and ITM calls for GD traders, but incurred loss in OTM market.

- *Garman Dealers:* In this model, I endowed the GAR dealers with different amounts of cash and options, and then observed how their bid-ask spreads change based on what their inventory level. I simulated the GAR dealers along with ZIP traders. GAR dealer gave impractical bid-ask spreads when their portfolios were incapable of satisfying majority of potential orders in the market, so it made near zero price for the bid, and almost 3 times more than the market price for the ask. This definitely kept out the poorest GAR dealer from any trade in the market whatsoever. GAR dealer with enough portfolio gave very attractive bid-ask spreads and continuously participated in trade until either his cash or option inventories run low. In that case, GAR dealer chose to increase his bid-ask spread and stopped the aggressive flow of unbalanced orders. In one situation, GAR dealer with more cash made a wider bid-ask spread compared to the one with less cash after spending some time in the market. This was caused by tight bid-ask spread the affluent GAR dealer posted at the beginning of the simulation to attract more orders and cause imbalance in inventories faster than the one who started with moderate bid-ask spread. In my simulation, the affluent GAR trader engaged in buying options, thus he quickly became low in cash, and was forced to take wider bid-ask spread. As the consequence of only buying behaviour GAR traders made profit in ITM and ATM markets, but incurred loss in OTM market.

- *Copeland-Galai Dealers:* The COP dealers used the expected loss from the informed traders as the main characteristic to determine their bids and asks. I simulated the COP dealers along with informed traders and ZIP traders, and the obtained option prices which corresponded with theoretical price. Also the final payoffs were normally distributed. The interesting finding about COP dealers was that they kept the width of the bid-ask spread constant throughout the simulation timeline. This was because of the linear order arrivals model used to predict the order arrival rates per given bid or ask. I reviewed two COP dealer performances one with belief of only %1 informed traders, and the other one correctly believed that they were %10 of market population. The former trader declared his private option valuation as his bid, and actively bought out options for such an attractive quote. The latter dealer feared more from potential loss incurred by informed traders, so he lowered his bid, while at the same time resulting to lower his ask proportionally. The bid price started to rise back to the private valuation of the trader as the option approached its maturity, and the expected loss from the informed traders became less.

- *Mixed Traders:* The overall results from the simulation mixed traders approximated the theoretical option prices well, although the range of accepted orders were wider due to heterogeneity of all traders. It also exhibited the numerical error spikes caused by GD traders. The distribution of payoffs and options were unequal, and scattered over different ranges, proving that some type of traders made more profit compared to other traders. It was noticed that the ones who made the most profit were those who kept either buying or selling an option with no change in strategy.

8.2 Addressing the Research Questions

In this section, I provide my conclusions to the questions posed in Chapter 1 based on the accomplished research. By answering these questions, I also emphasise the contributions made. Below are the questions and my conclusions:

1. *Can European options be priced via double auctions?*

   From the accomplished research, we have seen that the simulated option prices were very close to the theoretical Black-Scholes prices when the traders were using similar option pricing methodologies. But at the same time, this gave us an important insight into the market conditions and the traders that resulted these option prices. After trying multiple option pricing methodologies and proxy trading algorithms in direct and online double auctions respectively, we have observed option prices with different volatilities, slightly underpriced or overpriced values, with smaller or greater ranges of accepted orders, with certain sensitivity to the changes in the underlying market, with corresponding volume of trades, etc. I have explained when and how particular cases emerge while presenting my results. All of this information would not be available if the options were priced analytically. Therefore it would be correct to propose that pricing options via double auctions can not only simulate the Black-Scholes prices, but also may produce slightly different results with the different traders, and more importantly, it can provide more detailed description of the market state that formed these prices. Hence financial analysts and researcher can learn more about the evolution of option prices by incorporating the double auction mechanism into the option pricing methodology, which is, of course, course cannot be done using traditional mathematical approaches. Also it is impossible to represent the complex behaviour and the heterogeneity of this behaviour using standard approaches, other than modelling them with agents and simulating them in a virtual option market.

2. *What are the important components involved in pricing European option via double auctions?*

   I have reviewed the existing option pricing methodologies, and determined the key components that are required to model the pricing of options via double auctions. I classified them into three groups: underlying market, option traders and option markets. In first underlying market component, I implemented different asset pricing and interest-rate pricing models and calibrated them to the real underlying market parameters. These models included GBM and jump-diffusion model for asset prices, and Vasicek and Vasicek-Jump model for interest-rates. The second set of components were the option traders, which by themselves consisted of multiple layers. I have specified the role of each layer, and provided a conceptual design of trading agents supporting modularity of algorithms for pricing options, choosing quantities and trading options. The final component was the market mechanism itself, which collected the heterogeneous orders from the traders and cleared them to obtain useful information about market conditions. I have provided the simulation flow of such mechanisms and constructed their conceptual design in UML.

3. *What types of trading agents involved in option market and how are they designed for double auctions?*

   The trading agents for the double auction were designed using IIKB architecture where the important tasks were allocated to multiple layers. The Information layer was responsible to collect historical data from both underlying market and the mechanism. The Information layer was responsible for storing the inventory of the trader at given time. In

Knowledge layer, traders computed the aggregated knowledge from the available information. This also included learning from new information about the informed traders, and continuously calibrating the indicators such as implied volatility from the option prices in the market. The behavioural layer of the agent determined the agent's type, and it decided how to price the option, how to pick the quantities and what proxy trading algorithm to use. The traders were classified by the option pricing methodologies they used into ZI, Risk-Neutral, Risk-Averse and Portfolio Holding traders, and by the proxy trading algorithm they use into 2 groups: Dealers and Speculative Traders.

4. *How is the direct double auction designed for trading options, and what is learned from its simulation?*

   I took McAfee's direct double auction as the basis for designing the direct multi-unit double auction with atomic orders. The mechanism is built using Grove's argument on efficient mechanisms, Myerson's argument on monotonicity of an allocation rule the Vickery's argument on DSIC payments. It uses a surplus maximising allocation rule, and rejects the partially satisfied multi-unit bids or asks. The mechanism uses the rejected bids and asks to determine the clearing price of the mechanism. Any budget-deficit or the exposed asks or bids resulted from the rejection of efficient trades are covered by the mechanism. I have mentioned the key findings from the simulation of this mechanism with different agents in Section 8.1 of this chapter.

5. *How is the online double auction designed for trading options, and what is learned from its simulation?*

   I used LOB implementation of the online double auction and developed it using RB binary trees. It was linked to the underlying market and at the beginning of every trading day, the participants were informed with the new asset prices, and the previously submitted orders are truncated. Instead of enabling traders with sophisticated entry or exit strategies to trigger the submission of orders, I took more simplified approach by enabling the mechanism randomly solicit the traders for their orders. The mechanism accepted multi-unit orders and cleared them as soon as they match the existing orders in LOB. The online double auction produced opening, closing, high and low option prices per trading day, of which the closing price gave the good summary of the day's option price. I listed the key finding learned from the simulation of online double auction with different proxy trading algorithms in Section 8.1 of this chapter.

6. *What are the benefits of using combinatorial exchanges for pricing option portfolios?*

   The main benefit of using combinatorial exchanges is that the traders can fully reveal their preferences on bundles of goods to the mechanism which holds DSIC property in more structured and comprehensive way using bidding languages. This enables pricing option portfolios and other compound financial products also involving the substitutability and complementarity relationship between its constituent components. I have shown that these relationships exist among options when the traders engage in taking option portfolios. I also proposed a combinatorial exchange design for trading option portfolios, and illustrated a generic class of 2-item spreads in TBBL, to simplify the communication between the traders and the mechanism. The traders can only submit their full valuations and express what type of portfolio they want, and the mechanism using its pre-defined class of generic TBBLs can formulate a structured bid and clear his order. Although it is computationally hard mechanism to run, this is a fundamentally different and more compound approach at pricing bundles of products, particularly, the option portfolios.

8.3 Current Limitations

In this section, I provide the limitations and the drawbacks of my work. First of all, some of the limitations basically arise from the time and scope restrictions of my research, and can be resolved given enough time and effort. The other limitations were proven to be practically impossible to handle, and one of them is the impossibility of having efficient, budget-balanced and individual rational mechanism along with BIC property. Another one is the complexity of a combinatorial exchange allocation problem which belongs to the NP-hard class. Below is the limitations that can be improved in future:

- *Budget-Deficiency:* The proposed direct double auction is not budget-balanced at the expense of the atomicity of orders. The weakly budget-balanced double auction would spread the excess demand or supply evenly among traders forcing them to decrease the number of options they bid or ask for. However we said that in the case of traders taking option portfolios from multiple double auctions run in parallel, this could result in distortion of the portfolios. I suggested to fix the quantities to certain size, and request traders to split their orders into smaller chunks. This would reduce to the McAfee's single-unit double auction with all its properties. Another approach suggested is to coordinate the fulfilment of portfolios among multiple traders in such way that the mechanism can counterbalance the excess of supplies and demands in different markets by substituting one with the others. Of course, in this approach all OTMs (ITMs) are considered as substitutes.

- *Collusion:* The direct double auction is susceptible to collusion, and there are cases where it can be exploited to buy options at zero price, and sell them at maximum price. In order to resolve this issue, the double auction must adopt reserve prices for both bids and asks, hence setting the acceptable bid-ask spread for future bids and asks. This acceptable bid-ask spread may be computed using the distribution of historical valuations of traders, and through maximising the expected revenue for given reserve price. In theory, this is usually referred as optimal auctions, and these concepts apply here while determining the reserve prices.
- *No Online Incentive Compatibility:* The proposed online double auction is not DSIC, and not even BIC. There are multiple suggestions that guarantee the incentive compatibility in online double auctions. Sandholm *et al.* [19] and Parkes *et al.* [131] provide interesting methods of allocating goods in online double auctions where the orders are cleared within a specified time frame, and the traders make bounded-rational decisions within this time frame.
- *No Algorithmic Or Probabilistic Entry/Exit:* The submission of orders by traders is not controlled by any algorithm or probabilistic process, but explicitly solicited by the mechanism in random order. The submission of orders can be approximated into a corresponding Poisson process with intensity as function of last quoted price. Indeed, both Garman and Copeland-Galai dealers expect such behaviour from the mechanism. However I have modelled the order arrival intensity as a linear function for both dealers. In practice, this intensity function should be calibrated to the market accordingly, and the orders should be submitted

according to a Possion process. Another perspective in improving this behaviour is the submission of orders according to some algorithm. One such algorithm is trend-following algorithm which initiates trade when the short-term moving average crosses the long-term moving average.

- *Limited Option Pricing, Quantity Models and Proxy Trading Algorithms:* Although I included the key option pricing methodologies into the scope of this research, these are not an exhaustive list of the methodologies available. For example, there are other risk-neutral models such as the Black-Scholes-Merton model which prices options from the perspective of discontinuous asset prices, or the Heston model which simulates the asset prices with a stochastic volatility model. The indifference pricing methodologies involve various utility functions other than exponential, some involve the stochastic volatility, risk-measure and market frictions. The quantity models provided can also be extended into a more sophisticated ways of picking the right amount to trade depending on the given strategy. There are also many other trading algorithms such as Kaplan strategy, Roth-Erev, Time-Weighted Moving Average, Volume-Weighted Moving Average, and so forth.
- *Only European Options:* Along with American options, there are many other exotic types of options such as Asian, Bermudan, Binary, Barrier, Basket, Swaption, etc that are not included into the scope of this research. Basically, one can engineer any sort of a financial derivative similar to exotic options, and price them using Monte-Carlo simulation. The results can be further enhanced and enriched if heterogeneous pricing methodologies are channelled through a double auction.

8.4 Application Domains

The outcome of my research will enable financial analysts and researchers to better understand the formation of option prices in a simulated market environment, to undertake complex analysis of option prices by running various scenarios in underlying markets, to evaluate the performance of different trading algorithms, to observe the impact of different market clearing rules on option prices and to better understand the role of option portfolios in determining option prices. This research can also find its applications in pricing compound financial products such as MBSs, CDOs and other portfolios in a combinatorial exchange environment. For example, traders using the Capital Asset Pricing Model (CAPM) to compile a diversified portfolio of assets can post their orders on them as bundle to the exchange, and the mechanism can find the DSIC prices for give portfolio. The compilation of MBSs and CDOs contracts would require more expressive bidding languages to express the structure of the payable tranches and the combinations of constituent assets.

Auctions are also used beyond their direct and natural applications. For example, a good application for the proposed double auction can be the organisation of a prediction market where information about some uncertain event can be solicited from experts for a certain reward in the future [26, 135]. Experts can post their predictions in the form of options, and the mechanism can collect the aggregate forecast for the event. This can be applied to sports betting, presidential elections, or even the possible outcomes of wars.

Auctions have also practically proven applications in ranking search words [141], supply chain and network flow formation, optimisation and computation of a price of anarchy [162], grid resource allocation or sensor networks [56]. One example of using double auctions for allocating ad slots can be the double auction between content providers and the advertisers freely selling and buying the ad locations (or search positions) in a truthful mechanism.

8.5 Future Work

In the future prospects of the research, I shall put my efforts at tackling the limitations stated above. In particular, I would like to concentrate my further research in studying the combinatorial exchanges and their applications into financial markets. I have listed the topics that I will be working in future:

- *Bidding Languages:* The design and implementation of bidding languages for expressing the trader preferences on portfolios of assets based on their key indicators such as beta, alpha, Shapley ratio, etc. Also enabling the interpretation of compound financial derivatives through the use of bidding languages which are sufficient and succinct to reveal the trader's true intentions and methodology of constructing these financial instruments in a DSIC environment. I will develop specific bidding languages for certain financial instruments and formulate a corresponding WDP for them.

- *Combinatorial Trading Algorithms:* The design, implementation and simulation of adaptive trading algorithms not only for single asset, but for multiple assets also involving their complementarity and substitutability aspects. Although the CAPM is the great way of constructing diversified portfolios, it does take into consideration the trader's private preferences in building such portfolios. The trading algorithms should be developed from the perspective of combinatorial exchanges, so the traders can provide their own pricing results for any given bundle of assets. In case of financial derivatives, not only the traders use their private estimations of risk for the given contract, but also determine the optimal combinations of such contracts that can hedge their risk on one hand, and also yield desired profit on the other hand. Instead of using standardised compound derivatives that are commonly traded in exchanges, the combinatorial trading algorithms should be able to construct such contracts on the fly and trade them truthfully in the combinatorial exchange. The development and analysis of such algorithms would be one of the key areas of my future research.

- *Applications of Truthful Mechanisms in Finance:* The design, implementation and simulation of truthful mechanisms for determining the prices of commonly traded financial securities and derivatives. This frees the traders from strategising their trading behaviour, and consequently simplifies their interaction with the mechanism. Using such mechanisms can produce honest prices of the assets and financial derivatives practically achieving the same efficiency as in currently established market, and thus prevent the emergence of potential bubbles in the market, and glitches in trading algorithms. I will extensively research such mechanisms in future, not only with the use of existing trading algorithms, but also with human traders to compare the results and to find similarities in their behaviour.

- *Applications of Combinatorial Exchanges in Finance:* Although combinatorial exchange problem belongs to the NP-Hard class and it is impractical to implement such mechanism in a high-frequency financial exchanges, fast and specialised approximation algorithms can be found to clear the combinatorial bids and asks in real time. The approximation of mechanisms has been extensively studied by Hartline [79, 78], and I hope I can apply some of these approximation results to concrete cases in finance in my future work.

Appendices

Appendix A

Historical Market Data

Table A.1: Various US T-Bill daily returns for the period from 2014-01-02 to 2014-12-31

Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-01-02	0.07	0.13	0.39	1.72	3.00	3.92
2014-01-03	0.07	0.13	0.41	1.73	3.01	3.93
2014-01-06	0.05	0.12	0.40	1.70	2.98	3.90
2014-01-07	0.04	0.13	0.40	1.69	2.96	3.88
2014-01-08	0.05	0.13	0.43	1.77	3.01	3.90
2014-01-09	0.04	0.13	0.44	1.75	2.97	3.88
2014-01-10	0.05	0.12	0.39	1.64	2.88	3.80
2014-01-13	0.03	0.11	0.39	1.60	2.84	3.77
2014-01-14	0.04	0.11	0.39	1.65	2.88	3.80
2014-01-15	0.04	0.13	0.41	1.68	2.90	3.81
2014-01-16	0.04	0.11	0.41	1.66	2.86	3.77
2014-01-17	0.05	0.11	0.40	1.64	2.84	3.75
2014-01-21	0.04	0.12	0.40	1.67	2.85	3.74
2014-01-22	0.04	0.11	0.44	1.72	2.87	3.75
2014-01-23	0.04	0.11	0.39	1.62	2.79	3.68
2014-01-24	0.04	0.11	0.37	1.58	2.75	3.64
2014-01-27	0.05	0.11	0.37	1.61	2.78	3.67
2014-01-28	0.05	0.11	0.38	1.59	2.77	3.68
2014-01-29	0.04	0.11	0.36	1.52	2.69	3.62
2014-01-30	0.02	0.10	0.36	1.55	2.72	3.65
2014-01-31	0.02	0.10	0.34	1.49	2.67	3.61

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-02-03	0.05	0.11	0.30	1.44	2.61	3.55
2014-02-04	0.06	0.12	0.31	1.46	2.64	3.59
2014-02-05	0.07	0.12	0.32	1.50	2.70	3.66
2014-02-06	0.07	0.13	0.33	1.52	2.73	3.67
2014-02-07	0.08	0.12	0.30	1.47	2.71	3.67
2014-02-10	0.07	0.12	0.32	1.48	2.70	3.66
2014-02-11	0.05	0.12	0.35	1.54	2.75	3.69
2014-02-12	0.05	0.12	0.35	1.59	2.80	3.72
2014-02-13	0.03	0.12	0.32	1.51	2.73	3.70
2014-02-14	0.02	0.11	0.32	1.53	2.75	3.69
2014-02-18	0.05	0.12	0.31	1.50	2.71	3.68
2014-02-19	0.06	0.11	0.33	1.53	2.73	3.71
2014-02-20	0.05	0.12	0.34	1.57	2.76	3.73
2014-02-21	0.05	0.12	0.33	1.56	2.73	3.69
2014-02-24	0.05	0.11	0.35	1.57	2.75	3.70
2014-02-25	0.05	0.11	0.34	1.53	2.70	3.66
2014-02-26	0.05	0.11	0.33	1.50	2.67	3.63
2014-02-27	0.04	0.11	0.33	1.49	2.65	3.60
2014-02-28	0.05	0.12	0.33	1.51	2.66	3.59
2014-03-03	0.05	0.12	0.32	1.46	2.60	3.55
2014-03-04	0.05	0.12	0.33	1.54	2.70	3.64
2014-03-05	0.06	0.13	0.33	1.54	2.70	3.64
2014-03-06	0.05	0.12	0.37	1.57	2.74	3.68
2014-03-07	0.06	0.13	0.38	1.65	2.80	3.72
2014-03-10	0.05	0.12	0.37	1.64	2.79	3.73
2014-03-11	0.05	0.13	0.37	1.62	2.77	3.70
2014-03-12	0.05	0.12	0.37	1.59	2.73	3.66
2014-03-13	0.05	0.12	0.34	1.53	2.66	3.60
2014-03-14	0.05	0.12	0.36	1.55	2.65	3.59
2014-03-17	0.06	0.13	0.38	1.58	2.70	3.63
2014-03-18	0.05	0.13	0.36	1.56	2.68	3.62
2014-03-19	0.06	0.15	0.47	1.75	2.78	3.66
2014-03-20	0.06	0.14	0.45	1.73	2.79	3.67
2014-03-21	0.06	0.14	0.45	1.73	2.75	3.61
2014-03-24	0.06	0.14	0.47	1.76	2.74	3.57

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-03-25	0.05	0.13	0.47	1.76	2.75	3.59
2014-03-26	0.05	0.12	0.45	1.70	2.71	3.55
2014-03-27	0.04	0.12	0.45	1.70	2.69	3.52
2014-03-28	0.04	0.13	0.45	1.74	2.73	3.55
2014-03-31	0.05	0.13	0.44	1.73	2.73	3.56
2014-04-01	0.04	0.13	0.44	1.74	2.77	3.60
2014-04-02	0.02	0.12	0.47	1.80	2.82	3.65
2014-04-03	0.02	0.11	0.46	1.79	2.80	3.62
2014-04-04	0.03	0.11	0.43	1.71	2.74	3.59
2014-04-07	0.03	0.11	0.41	1.68	2.71	3.56
2014-04-08	0.03	0.11	0.40	1.67	2.69	3.54
2014-04-09	0.04	0.10	0.37	1.65	2.71	3.57
2014-04-10	0.04	0.09	0.37	1.59	2.65	3.52
2014-04-11	0.04	0.09	0.37	1.58	2.63	3.48
2014-04-14	0.04	0.10	0.37	1.61	2.65	3.48
2014-04-15	0.04	0.11	0.39	1.63	2.64	3.46
2014-04-16	0.04	0.11	0.39	1.67	2.65	3.45
2014-04-17	0.03	0.11	0.43	1.75	2.73	3.52
2014-04-21	0.04	0.11	0.42	1.74	2.73	3.52
2014-04-22	0.03	0.11	0.45	1.76	2.73	3.50
2014-04-23	0.02	0.11	0.43	1.73	2.70	3.47
2014-04-24	0.01	0.10	0.43	1.74	2.70	3.46
2014-04-25	0.03	0.11	0.43	1.72	2.68	3.45
2014-04-28	0.03	0.10	0.44	1.73	2.70	3.47
2014-04-29	0.02	0.11	0.44	1.74	2.71	3.49
2014-04-30	0.03	0.11	0.42	1.69	2.67	3.47
2014-05-01	0.03	0.10	0.41	1.66	2.63	3.41
2014-05-02	0.02	0.10	0.42	1.67	2.60	3.37
2014-05-05	0.03	0.11	0.43	1.68	2.63	3.41
2014-05-06	0.03	0.10	0.43	1.68	2.61	3.38
2014-05-07	0.03	0.10	0.41	1.65	2.62	3.40
2014-05-08	0.03	0.10	0.40	1.63	2.61	3.45
2014-05-09	0.03	0.10	0.40	1.63	2.62	3.47
2014-05-12	0.03	0.09	0.41	1.67	2.66	3.49
2014-05-13	0.03	0.10	0.39	1.62	2.61	3.45

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-05-14	0.03	0.10	0.39	1.57	2.54	3.37
2014-05-15	0.03	0.09	0.38	1.55	2.50	3.33
2014-05-16	0.03	0.09	0.38	1.56	2.52	3.34
2014-05-19	0.03	0.09	0.36	1.56	2.54	3.39
2014-05-20	0.03	0.09	0.35	1.53	2.52	3.38
2014-05-21	0.04	0.09	0.37	1.55	2.54	3.42
2014-05-22	0.03	0.09	0.37	1.57	2.56	3.43
2014-05-23	0.04	0.10	0.37	1.55	2.54	3.40
2014-05-27	0.04	0.09	0.39	1.56	2.52	3.37
2014-05-28	0.04	0.10	0.37	1.50	2.44	3.29
2014-05-29	0.04	0.10	0.37	1.52	2.45	3.31
2014-05-30	0.04	0.10	0.37	1.54	2.48	3.33
2014-06-02	0.04	0.10	0.39	1.60	2.54	3.38
2014-06-03	0.04	0.10	0.41	1.65	2.60	3.43
2014-06-04	0.04	0.10	0.41	1.65	2.61	3.45
2014-06-05	0.04	0.10	0.40	1.63	2.59	3.44
2014-06-06	0.04	0.11	0.41	1.66	2.60	3.44
2014-06-09	0.04	0.11	0.43	1.69	2.62	3.45
2014-06-10	0.04	0.11	0.45	1.71	2.64	3.47
2014-06-11	0.04	0.11	0.44	1.70	2.65	3.47
2014-06-12	0.04	0.10	0.42	1.66	2.58	3.41
2014-06-13	0.04	0.11	0.45	1.70	2.60	3.41
2014-06-16	0.04	0.11	0.49	1.71	2.61	3.40
2014-06-17	0.04	0.11	0.51	1.77	2.66	3.44
2014-06-18	0.03	0.10	0.48	1.71	2.61	3.43
2014-06-19	0.02	0.09	0.48	1.71	2.64	3.47
2014-06-20	0.02	0.09	0.50	1.71	2.63	3.44
2014-06-23	0.03	0.10	0.48	1.72	2.63	3.45
2014-06-24	0.03	0.12	0.49	1.70	2.59	3.41
2014-06-25	0.03	0.11	0.48	1.68	2.57	3.38
2014-06-26	0.04	0.11	0.46	1.64	2.53	3.35
2014-06-27	0.03	0.10	0.45	1.64	2.54	3.36
2014-06-30	0.04	0.11	0.47	1.62	2.53	3.34
2014-07-01	0.02	0.11	0.47	1.66	2.58	3.40
2014-07-02	0.02	0.12	0.49	1.71	2.64	3.46

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-07-03	0.01	0.11	0.52	1.74	2.65	3.47
2014-07-07	0.04	0.12	0.52	1.74	2.63	3.44
2014-07-08	0.03	0.11	0.51	1.70	2.58	3.38
2014-07-09	0.03	0.11	0.51	1.68	2.57	3.37
2014-07-10	0.02	0.10	0.46	1.66	2.55	3.38
2014-07-11	0.02	0.11	0.48	1.65	2.53	3.34
2014-07-14	0.03	0.11	0.48	1.68	2.55	3.36
2014-07-15	0.02	0.11	0.49	1.70	2.56	3.37
2014-07-16	0.02	0.11	0.50	1.71	2.55	3.35
2014-07-17	0.02	0.10	0.47	1.65	2.47	3.27
2014-07-18	0.02	0.10	0.51	1.69	2.50	3.29
2014-07-21	0.03	0.11	0.51	1.70	2.49	3.26
2014-07-22	0.03	0.11	0.49	1.68	2.48	3.25
2014-07-23	0.03	0.11	0.50	1.67	2.48	3.26
2014-07-24	0.03	0.11	0.53	1.72	2.52	3.30
2014-07-25	0.03	0.11	0.53	1.69	2.48	3.24
2014-07-28	0.04	0.11	0.54	1.73	2.50	3.26
2014-07-29	0.02	0.12	0.54	1.70	2.47	3.22
2014-07-30	0.04	0.13	0.56	1.77	2.57	3.31
2014-07-31	0.03	0.12	0.53	1.76	2.58	3.32
2014-08-01	0.03	0.13	0.47	1.67	2.52	3.29
2014-08-04	0.04	0.12	0.47	1.66	2.51	3.30
2014-08-05	0.03	0.12	0.47	1.66	2.49	3.28
2014-08-06	0.03	0.11	0.48	1.66	2.49	3.27
2014-08-07	0.03	0.11	0.44	1.60	2.43	3.23
2014-08-08	0.03	0.10	0.45	1.62	2.44	3.23
2014-08-11	0.04	0.10	0.47	1.62	2.44	3.24
2014-08-12	0.03	0.10	0.45	1.63	2.46	3.27
2014-08-13	0.04	0.10	0.43	1.59	2.43	3.24
2014-08-14	0.04	0.10	0.42	1.58	2.40	3.20
2014-08-15	0.03	0.09	0.42	1.55	2.34	3.13
2014-08-18	0.03	0.10	0.44	1.58	2.39	3.20
2014-08-19	0.03	0.11	0.46	1.59	2.40	3.21
2014-08-20	0.04	0.12	0.49	1.65	2.43	3.22
2014-08-21	0.02	0.10	0.49	1.64	2.41	3.19

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-08-22	0.03	0.10	0.53	1.68	2.40	3.16
2014-08-25	0.04	0.11	0.53	1.69	2.39	3.13
2014-08-26	0.03	0.12	0.52	1.68	2.39	3.15
2014-08-27	0.04	0.11	0.51	1.65	2.37	3.11
2014-08-28	0.03	0.11	0.50	1.63	2.34	3.08
2014-08-29	0.03	0.09	0.48	1.63	2.35	3.09
2014-09-02	0.03	0.10	0.53	1.69	2.42	3.17
2014-09-03	0.03	0.11	0.52	1.69	2.41	3.15
2014-09-04	0.03	0.10	0.54	1.71	2.45	3.21
2014-09-05	0.03	0.10	0.52	1.69	2.46	3.23
2014-09-08	0.02	0.10	0.54	1.72	2.48	3.23
2014-09-09	0.02	0.11	0.56	1.76	2.50	3.23
2014-09-10	0.02	0.11	0.58	1.79	2.54	3.26
2014-09-11	0.02	0.11	0.58	1.79	2.54	3.27
2014-09-12	0.02	0.11	0.58	1.83	2.62	3.35
2014-09-15	0.02	0.11	0.58	1.80	2.60	3.34
2014-09-16	0.02	0.13	0.55	1.78	2.60	3.36
2014-09-17	0.02	0.12	0.59	1.82	2.62	3.37
2014-09-18	0.02	0.12	0.59	1.85	2.63	3.36
2014-09-19	0.02	0.11	0.59	1.83	2.59	3.29
2014-09-22	0.01	0.10	0.58	1.80	2.57	3.28
2014-09-23	0.01	0.10	0.57	1.78	2.54	3.25
2014-09-24	0.02	0.11	0.59	1.82	2.57	3.28
2014-09-25	0.01	0.10	0.56	1.75	2.52	3.22
2014-09-26	0.01	0.11	0.59	1.80	2.54	3.22
2014-09-29	0.02	0.11	0.58	1.77	2.50	3.18
2014-09-30	0.02	0.13	0.58	1.78	2.52	3.21
2014-10-01	0.02	0.10	0.53	1.69	2.42	3.12
2014-10-02	0.01	0.10	0.53	1.70	2.44	3.15
2014-10-03	0.01	0.11	0.57	1.73	2.45	3.13
2014-10-06	0.02	0.11	0.54	1.70	2.43	3.12
2014-10-07	0.02	0.10	0.52	1.64	2.36	3.06
2014-10-08	0.01	0.10	0.46	1.57	2.35	3.07
2014-10-09	0.01	0.10	0.46	1.58	2.34	3.07
2014-10-10	0.01	0.10	0.45	1.55	2.31	3.03

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-10-14	0.02	0.09	0.39	1.45	2.21	2.95
2014-10-15	0.02	0.10	0.34	1.37	2.15	2.92
2014-10-16	0.03	0.10	0.35	1.39	2.17	2.94
2014-10-17	0.02	0.11	0.39	1.44	2.22	2.98
2014-10-20	0.02	0.10	0.37	1.41	2.20	2.96
2014-10-21	0.02	0.10	0.38	1.44	2.23	3.00
2014-10-22	0.02	0.11	0.41	1.46	2.25	3.01
2014-10-23	0.01	0.11	0.41	1.52	2.29	3.05
2014-10-24	0.01	0.11	0.41	1.52	2.29	3.05
2014-10-27	0.02	0.11	0.41	1.51	2.27	3.04
2014-10-28	0.02	0.11	0.42	1.53	2.30	3.06
2014-10-29	0.03	0.11	0.48	1.61	2.34	3.06
2014-10-30	0.01	0.11	0.48	1.58	2.32	3.04
2014-10-31	0.01	0.11	0.50	1.62	2.35	3.07
2014-11-03	0.02	0.12	0.52	1.63	2.36	3.07
2014-11-04	0.03	0.11	0.52	1.63	2.35	3.05
2014-11-05	0.03	0.11	0.52	1.63	2.36	3.06
2014-11-06	0.03	0.12	0.54	1.67	2.39	3.09
2014-11-07	0.03	0.12	0.51	1.60	2.32	3.04
2014-11-10	0.02	0.13	0.55	1.65	2.38	3.09
2014-11-12	0.02	0.14	0.55	1.65	2.37	3.09
2014-11-13	0.02	0.15	0.53	1.64	2.35	3.08
2014-11-14	0.02	0.15	0.54	1.62	2.32	3.04
2014-11-17	0.03	0.15	0.54	1.64	2.34	3.06
2014-11-18	0.02	0.14	0.53	1.63	2.32	3.05
2014-11-19	0.01	0.15	0.54	1.66	2.36	3.08
2014-11-20	0.02	0.14	0.53	1.64	2.34	3.05
2014-11-21	0.01	0.14	0.53	1.63	2.31	3.02
2014-11-24	0.02	0.14	0.53	1.62	2.30	3.01
2014-11-25	0.02	0.14	0.51	1.58	2.27	2.97
2014-11-26	0.02	0.14	0.53	1.56	2.24	2.95
2014-11-28	0.02	0.13	0.47	1.49	2.18	2.89
2014-12-01	0.03	0.13	0.49	1.52	2.22	2.95
2014-12-02	0.03	0.14	0.55	1.59	2.28	3.00
2014-12-03	0.01	0.15	0.57	1.61	2.29	2.99

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Date	3 Mo	1 Yr	2 Yr	5 Yr	10 Yr	30 Yr
2014-12-04	0.02	0.14	0.55	1.59	2.25	2.94
2014-12-05	0.02	0.18	0.65	1.69	2.31	2.97
2014-12-08	0.03	0.18	0.64	1.67	2.26	2.90
2014-12-09	0.04	0.23	0.64	1.63	2.22	2.87
2014-12-10	0.03	0.21	0.59	1.58	2.18	2.83
2014-12-11	0.03	0.21	0.62	1.62	2.19	2.84
2014-12-12	0.02	0.19	0.56	1.53	2.10	2.75
2014-12-15	0.04	0.22	0.60	1.58	2.12	2.74
2014-12-16	0.03	0.21	0.58	1.53	2.07	2.69
2014-12-17	0.03	0.23	0.62	1.61	2.14	2.74
2014-12-18	0.04	0.25	0.67	1.68	2.22	2.82
2014-12-19	0.04	0.26	0.67	1.66	2.17	2.77
2014-12-22	0.05	0.28	0.71	1.67	2.17	2.75
2014-12-23	0.03	0.26	0.73	1.76	2.26	2.85
2014-12-24	0.01	0.26	0.73	1.76	2.27	2.83
2014-12-26	0.01	0.26	0.73	1.75	2.25	2.81
2014-12-29	0.03	0.25	0.72	1.72	2.22	2.78
2014-12-30	0.03	0.23	0.69	1.68	2.20	2.76

Table A.2: NASDAQ-100 indices for the period from 02-01-2014 to 31-12-2014

Date	Open	High	Low	Close	Volume	Adj. Close
2014-01-02	3575.60	3577.03	3553.65	3563.57	1,738,820,000	3563.57
2014-01-03	3564.94	3567.51	3537.61	3538.73	1,667,480,000	3538.73
2014-01-06	3539.02	3542.52	3512.45	3526.96	2,292,840,000	3526.96
2014-01-07	3539.29	3562.99	3535.50	3557.85	2,278,220,000	3557.85
2014-01-08	3558.30	3575.15	3551.12	3567.54	2,345,220,000	3567.54
2014-01-09	3576.33	3579.40	3541.81	3552.58	2,214,770,000	3552.58
2014-01-10	3565.68	3568.47	3536.45	3565.08	2,143,070,000	3565.08
2014-01-13	3559.39	3572.40	3499.37	3512.80	2,322,240,000	3512.80
2014-01-14	3526.20	3581.60	3525.47	3580.65	2,034,180,000	3580.65
2014-01-15	3593.81	3615.70	3592.62	3609.84	2,101,870,000	3609.84
2014-01-16	3605.43	3614.66	3599.87	3611.29	2,005,850,000	3611.29
2014-01-17	3597.41	3608.72	3580.39	3591.25	2,150,370,000	3591.25
2014-01-21	3611.63	3617.70	3586.05	3617.70	2,034,030,000	3617.70
2014-01-22	3623.31	3634.65	3615.48	3627.72	2,026,910,000	3627.72
2014-01-23	3612.53	3613.88	3588.83	3613.76	2,191,980,000	3613.76
2014-01-24	3596.93	3601.00	3541.38	3541.48	2,489,470,000	3541.48
2014-01-27	3543.50	3548.48	3482.89	3509.02	2,398,280,000	3509.02
2014-01-28	3484.39	3509.80	3483.71	3505.72	2,091,180,000	3505.72
2014-01-29	3472.42	3500.82	3461.64	3467.82	2,231,850,000	3467.82
2014-01-30	3514.25	3544.21	3508.01	3532.41	2,168,410,000	3532.41
2014-01-31	3488.37	3539.44	3488.04	3521.92	2,300,570,000	3521.92
2014-02-03	3524.23	3533.48	3433.64	3440.50	2,617,030,000	3440.50
2014-02-04	3460.13	3480.74	3449.19	3470.20	2,173,360,000	3470.20
2014-02-05	3457.04	3469.35	3418.88	3454.90	2,168,360,000	3454.90
2014-02-06	3465.83	3502.52	3465.42	3497.60	1,942,700,000	3497.60
2014-02-07	3522.33	3563.14	3508.83	3561.91	2,055,850,000	3561.91
2014-02-10	3558.59	3582.84	3557.78	3582.11	1,811,970,000	3582.11
2014-02-11	3587.58	3628.59	3585.08	3621.72	1,993,950,000	3621.72
2014-02-12	3626.14	3635.92	3618.06	3627.36	2,035,890,000	3627.36
2014-02-13	3601.75	3659.56	3600.86	3659.56	2,249,990,000	3659.56
2014-02-14	3652.88	3671.04	3643.79	3663.88	1,881,510,000	3663.88
2014-02-18	3668.99	3685.18	3658.64	3679.43	1,886,210,000	3679.43
2014-02-19	3668.72	3679.69	3646.13	3652.85	1,956,720,000	3652.85
2014-02-20	3654.85	3676.89	3640.63	3671.93	1,992,780,000	3671.93

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-02-21	3683.42	3687.16	3661.46	3662.60	2,138,250,000	3662.60
2014-02-24	3670.07	3701.27	3668.34	3685.81	2,161,300,000	3685.81
2014-02-25	3689.68	3697.60	3669.53	3679.14	2,137,150,000	3679.14
2014-02-26	3691.84	3702.09	3663.75	3676.60	2,108,270,000	3676.60
2014-02-27	3678.31	3704.65	3672.64	3699.80	2,049,160,000	3699.80
2014-02-28	3703.48	3722.38	3664.83	3696.10	2,617,730,000	3696.10
2014-03-03	3657.32	3678.38	3637.67	3668.37	2,077,500,000	3668.37
2014-03-04	3710.19	3723.00	3706.82	3719.93	2,477,850,000	3719.93
2014-03-05	3721.53	3732.98	3715.38	3727.19	2,215,980,000	3727.19
2014-03-06	3735.85	3738.32	3709.41	3720.93	2,136,260,000	3720.93
2014-03-07	3736.93	3737.36	3686.94	3703.40	2,175,560,000	3703.40
2014-03-10	3702.14	3709.78	3683.97	3706.34	2,111,610,000	3706.34
2014-03-11	3715.89	3725.71	3681.37	3691.50	2,477,780,000	3691.50
2014-03-12	3678.36	3707.20	3661.87	3707.06	2,131,880,000	3707.06
2014-03-13	3719.36	3720.62	3636.79	3651.49	2,383,600,000	3651.49
2014-03-14	3643.21	3656.94	3626.86	3627.87	2,196,890,000	3627.87
2014-03-17	3653.19	3678.32	3651.14	3662.51	1,810,410,000	3662.51
2014-03-18	3667.40	3708.26	3664.79	3706.62	1,962,890,000	3706.62
2014-03-19	3705.51	3709.01	3661.06	3682.74	1,992,750,000	3682.74
2014-03-20	3675.62	3705.31	3665.83	3693.97	1,847,270,000	3693.97
2014-03-21	3711.47	3717.36	3643.35	3653.07	3,245,740,000	3653.07
2014-03-24	3665.38	3665.58	3585.15	3617.39	2,434,650,000	3617.39
2014-03-25	3640.26	3659.59	3600.65	3629.73	2,270,760,000	3629.73
2014-03-26	3645.73	3654.69	3582.84	3582.89	2,455,460,000	3582.89
2014-03-27	3578.54	3594.08	3543.07	3563.13	2,270,650,000	3563.13
2014-03-28	3576.18	3609.08	3560.58	3571.49	2,029,840,000	3571.49
2014-03-31	3598.08	3618.42	3593.70	3595.74	2,090,850,000	3595.74
2014-04-01	3615.91	3659.16	3614.16	3658.40	2,153,130,000	3658.40
2014-04-02	3670.33	3676.36	3649.79	3665.99	2,187,100,000	3665.99
2014-04-03	3670.43	3675.70	3617.41	3637.58	2,067,370,000	3637.58
2014-04-04	3659.92	3663.78	3532.28	3539.38	2,621,270,000	3539.38
2014-04-07	3522.21	3551.21	3482.75	3507.75	2,554,680,000	3507.75
2014-04-08	3511.82	3543.98	3499.00	3538.23	2,198,900,000	3538.23
2014-04-09	3552.07	3602.35	3544.09	3600.44	1,957,560,000	3600.44
2014-04-10	3597.58	3599.64	3479.15	3487.76	2,421,210,000	3487.76

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-04-11	3453.24	3502.33	3440.55	3446.85	2,264,480,000	3446.85
2014-04-14	3479.66	3493.17	3443.26	3474.63	1,890,480,000	3474.63
2014-04-15	3484.29	3505.67	3414.11	3487.85	2,413,110,000	3487.85
2014-04-16	3517.50	3533.09	3489.29	3533.09	1,863,110,000	3533.09
2014-04-17	3526.57	3551.02	3511.27	3534.53	1,954,720,000	3534.53
2014-04-21	3543.65	3559.95	3525.59	3559.95	1,544,440,000	3559.95
2014-04-22	3570.51	3596.92	3566.87	3588.81	1,875,930,000	3588.81
2014-04-23	3587.65	3588.05	3555.19	3557.04	1,794,760,000	3557.04
2014-04-24	3609.83	3613.20	3552.93	3591.03	2,130,870,000	3591.03
2014-04-25	3569.92	3573.95	3525.60	3533.10	2,087,140,000	3533.10
2014-04-28	3546.96	3570.11	3489.60	3545.03	2,348,320,000	3545.03
2014-04-29	3557.77	3579.03	3538.24	3573.99	1,911,240,000	3573.99
2014-04-30	3556.00	3585.50	3548.11	3582.02	2,151,400,000	3582.02
2014-05-01	3589.66	3613.03	3580.02	3594.36	2,077,040,000	3594.36
2014-05-02	3608.84	3611.56	3578.53	3587.64	1,844,790,000	3587.64
2014-05-05	3566.26	3605.26	3556.40	3605.09	1,561,170,000	3605.09
2014-05-06	3598.27	3599.65	3556.10	3556.51	1,850,610,000	3556.51
2014-05-07	3561.47	3567.55	3506.35	3546.47	2,486,030,000	3546.47
2014-05-08	3533.39	3583.64	3524.85	3540.42	2,411,940,000	3540.42
2014-05-09	3537.68	3556.35	3516.24	3555.70	1,976,160,000	3555.70
2014-05-12	3574.24	3613.37	3572.69	3612.73	1,880,020,000	3612.73
2014-05-13	3614.91	3625.77	3605.78	3611.13	1,923,480,000	3611.13
2014-05-14	3603.58	3614.63	3585.68	3593.25	1,764,430,000	3593.25
2014-05-15	3594.59	3598.34	3542.63	3565.17	2,083,030,000	3565.17
2014-05-16	3568.20	3590.41	3545.63	3587.20	1,741,070,000	3587.20
2014-05-19	3577.85	3618.96	3572.28	3615.62	1,601,400,000	3615.62
2014-05-20	3612.86	3624.60	3585.88	3600.31	1,797,910,000	3600.31
2014-05-21	3605.79	3636.47	3605.51	3635.61	1,703,500,000	3635.61
2014-05-22	3639.49	3660.59	3633.85	3650.86	1,835,810,000	3650.86
2014-05-23	3656.27	3678.17	3646.23	3677.33	1,536,610,000	3677.33
2014-05-27	3693.18	3723.07	3691.09	3723.07	1,812,330,000	3723.07
2014-05-28	3720.89	3726.72	3708.65	3712.26	1,785,140,000	3712.26
2014-05-29	3723.27	3735.72	3717.66	3735.72	1,714,000,000	3735.72
2014-05-30	3738.71	3741.57	3716.09	3736.82	1,903,660,000	3736.82
2014-06-02	3740.71	3740.71	3707.89	3732.96	1,631,310,000	3732.96

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-06-03	3719.17	3735.97	3715.46	3730.07	1,718,640,000	3730.07
2014-06-04	3720.00	3750.01	3715.24	3743.59	1,610,090,000	3743.59
2014-06-05	3748.36	3782.00	3735.19	3776.95	1,926,750,000	3776.95
2014-06-06	3792.42	3795.14	3782.12	3794.57	1,616,650,000	3794.57
2014-06-09	3795.85	3804.04	3783.90	3795.74	1,783,060,000	3795.74
2014-06-10	3790.48	3802.46	3786.56	3800.86	1,787,120,000	3800.86
2014-06-11	3789.41	3803.95	3783.61	3797.85	1,778,460,000	3797.85
2014-06-12	3790.59	3794.99	3751.69	3763.80	1,908,190,000	3763.80
2014-06-13	3782.84	3784.84	3759.87	3775.56	1,754,560,000	3775.56
2014-06-16	3768.37	3789.93	3761.42	3779.93	1,675,120,000	3779.93
2014-06-17	3775.73	3790.92	3769.94	3781.32	1,814,360,000	3781.32
2014-06-18	3786.19	3807.49	3764.53	3804.61	1,860,660,000	3804.61
2014-06-19	3808.76	3811.24	3783.30	3800.80	1,845,460,000	3800.80
2014-06-20	3803.10	3807.92	3794.51	3802.64	2,721,380,000	3802.64
2014-06-23	3802.25	3806.19	3794.85	3805.31	1,712,930,000	3805.31
2014-06-24	3807.49	3837.17	3791.12	3799.53	2,014,700,000	3799.53
2014-06-25	3794.84	3831.85	3792.31	3827.33	1,722,820,000	3827.33
2014-06-26	3826.70	3827.70	3798.63	3826.91	1,554,070,000	3826.91
2014-06-27	3825.02	3844.89	3823.21	3844.44	3,964,930,000	3844.44
2014-06-30	3844.45	3860.65	3841.70	3849.48	1,848,110,000	3849.48
2014-07-01	3864.14	3902.93	3863.81	3894.33	1,942,550,000	3894.33
2014-07-02	3895.18	3904.35	3890.77	3899.27	1,599,480,000	3899.27
2014-07-03	3911.11	3923.15	3901.33	3923.01	1,001,730,000	3923.01
2014-07-07	3917.89	3923.92	3904.67	3910.71	1,691,390,000	3910.71
2014-07-08	3903.37	3906.22	3848.18	3864.07	2,221,820,000	3864.07
2014-07-09	3873.17	3896.38	3863.20	3892.91	1,736,960,000	3892.91
2014-07-10	3837.29	3895.72	3837.16	3880.04	1,682,920,000	3880.04
2014-07-11	3888.76	3905.40	3879.40	3904.58	1,511,250,000	3904.58
2014-07-14	3923.60	3937.73	3916.53	3929.46	1,579,660,000	3929.46
2014-07-15	3934.79	3940.55	3887.50	3914.46	1,772,030,000	3914.46
2014-07-16	3943.60	3947.49	3927.01	3932.33	2,059,340,000	3932.33
2014-07-17	3921.87	3933.88	3866.22	3878.01	2,055,240,000	3878.01
2014-07-18	3899.38	3942.15	3894.56	3939.89	1,823,580,000	3939.89
2014-07-21	3933.11	3941.93	3917.74	3934.14	1,557,820,000	3934.14
2014-07-22	3950.83	3966.10	3949.09	3961.62	1,724,440,000	3961.62

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-07-23	3974.66	3990.64	3967.96	3986.19	1,909,810,000	3986.19
2014-07-24	3994.18	3997.50	3975.66	3983.19	1,935,090,000	3983.19
2014-07-25	3959.35	3970.44	3945.05	3965.17	1,711,430,000	3965.17
2014-07-28	3966.31	3977.00	3936.72	3967.24	1,783,250,000	3967.24
2014-07-29	3974.06	3983.42	3957.22	3959.03	2,090,810,000	3959.03
2014-07-30	3982.10	3989.00	3960.29	3976.07	1,872,430,000	3976.07
2014-07-31	3938.31	3948.90	3889.10	3892.50	2,273,380,000	3892.50
2014-08-01	3886.10	3908.21	3855.35	3879.67	2,050,340,000	3879.67
2014-08-04	3890.63	3922.25	3875.10	3908.77	1,677,400,000	3908.77
2014-08-05	3891.13	3901.17	3858.65	3874.94	1,916,180,000	3874.94
2014-08-06	3851.44	3896.65	3849.45	3874.27	1,819,310,000	3874.27
2014-08-07	3889.69	3895.74	3845.20	3857.94	1,866,330,000	3857.94
2014-08-08	3864.09	3890.63	3848.49	3888.09	1,759,060,000	3888.09
2014-08-11	3899.98	3921.19	3899.37	3910.46	1,537,880,000	3910.46
2014-08-12	3904.45	3916.73	3887.58	3905.23	1,560,220,000	3905.23
2014-08-13	3924.47	3949.69	3921.44	3949.20	1,611,690,000	3949.20
2014-08-14	3953.27	3969.24	3949.08	3969.11	1,549,820,000	3969.11
2014-08-15	3993.00	3997.08	3951.25	3987.51	1,799,460,000	3987.51
2014-08-18	4007.19	4022.48	4003.46	4020.50	1,571,100,000	4020.50
2014-08-19	4025.51	4041.50	4024.17	4040.13	1,556,560,000	4040.13
2014-08-20	4032.91	4046.97	4032.20	4040.71	1,502,030,000	4040.71
2014-08-21	4041.00	4048.95	4035.75	4047.03	1,421,730,000	4047.03
2014-08-22	4052.14	4060.90	4040.22	4052.75	1,311,810,000	4052.75
2014-08-25	4073.48	4079.52	4058.60	4067.48	1,384,620,000	4067.48
2014-08-26	4073.05	4076.54	4062.73	4071.67	1,469,620,000	4071.67
2014-08-27	4073.40	4078.19	4064.07	4073.18	1,389,470,000	4073.18
2014-08-28	4059.37	4072.29	4054.61	4066.27	1,309,580,000	4066.27
2014-08-29	4081.02	4082.97	4062.18	4082.56	1,352,830,000	4082.56
2014-09-02	4093.12	4095.87	4078.19	4095.81	1,859,080,000	4095.81
2014-09-03	4103.97	4104.43	4063.49	4070.96	1,897,450,000	4070.96
2014-09-04	4078.82	4100.78	4056.53	4066.13	1,728,700,000	4066.13
2014-09-05	4067.37	4089.98	4051.75	4089.92	1,641,830,000	4089.92
2014-09-08	4086.52	4105.50	4076.63	4095.47	1,670,210,000	4095.47
2014-09-09	4094.02	4110.86	4053.11	4061.88	1,956,550,000	4061.88
2014-09-10	4063.85	4095.91	4054.61	4094.97	1,808,650,000	4094.97

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-09-11	4077.69	4092.79	4060.92	4092.65	1,704,850,000	4092.65
2014-09-12	4087.17	4089.62	4057.53	4069.23	1,784,180,000	4069.23
2014-09-15	4070.20	4070.82	4018.30	4029.89	1,940,520,000	4029.89
2014-09-16	4012.53	4073.14	4009.98	4067.27	1,879,350,000	4067.27
2014-09-17	4066.76	4090.95	4048.58	4073.57	1,796,710,000	4073.57
2014-09-18	4084.51	4103.42	4080.82	4103.08	1,774,840,000	4103.08
2014-09-19	4113.02	4118.91	4084.45	4100.09	3,178,490,000	4100.09
2014-09-22	4091.87	4092.14	4044.99	4061.23	1,881,520,000	4061.23
2014-09-23	4045.67	4069.21	4043.26	4051.57	1,847,730,000	4051.57
2014-09-24	4055.42	4096.27	4041.72	4094.31	1,765,260,000	4094.31
2014-09-25	4079.37	4087.19	4007.63	4007.82	1,939,610,000	4007.82
2014-09-26	4016.97	4057.47	4015.02	4053.72	1,637,480,000	4053.72
2014-09-29	4009.66	4056.43	4008.58	4047.18	1,737,750,000	4047.18
2014-09-30	4054.86	4070.44	4031.70	4049.45	2,200,380,000	4049.45
2014-10-01	4043.33	4043.44	3972.91	3984.74	2,312,630,000	3984.74
2014-10-02	3984.80	3996.75	3934.94	3985.87	2,165,500,000	3985.87
2014-10-03	4004.75	4040.56	3995.81	4027.31	2,765,750,000	4027.31
2014-10-06	4041.40	4046.71	4004.21	4016.27	2,467,830,000	4016.27
2014-10-07	3999.25	4007.94	3958.44	3958.59	2,111,360,000	3958.59
2014-10-08	3960.79	4048.79	3938.16	4041.12	2,451,630,000	4041.12
2014-10-09	4031.62	4040.54	3965.37	3969.32	2,264,220,000	3969.32
2014-10-10	3946.32	3969.48	3870.86	3870.86	2,765,750,000	3870.86
2014-10-13	3866.89	3893.66	3807.89	3808.00	2,467,830,000	3808.00
2014-10-14	3837.22	3860.04	3801.16	3810.45	2,496,120,000	3810.45
2014-10-15	3738.72	3801.42	3700.23	3785.97	3,058,740,000	3785.97
2014-10-16	3706.70	3795.08	3704.83	3765.28	2,591,940,000	3765.28
2014-10-17	3815.72	3848.98	3791.16	3815.47	2,260,070,000	3815.47
2014-10-20	3813.60	3872.36	3804.97	3870.08	1,717,370,000	3870.08
2014-10-21	3916.65	3971.40	3908.83	3971.39	1,997,580,000	3971.39
2014-10-22	3980.49	3988.40	3947.98	3949.59	1,967,020,000	3949.59
2014-10-23	3989.86	4032.22	3984.15	4012.27	1,952,380,000	4012.27
2014-10-24	4019.84	4045.19	4003.43	4042.02	1,754,300,000	4042.02
2014-10-27	4031.88	4052.50	4019.23	4046.02	1,585,580,000	4046.02
2014-10-28	4063.07	4107.19	4062.21	4106.63	1,966,920,000	4106.63
2014-10-29	4091.62	4104.06	4062.49	4090.55	2,184,050,000	4090.55

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-10-30	4074.26	4109.06	4064.45	4100.64	2,034,960,000	4100.64
2014-10-31	4168.50	4170.87	4143.20	4158.21	2,424,360,000	4158.21
2014-11-03	4160.60	4179.36	4156.79	4169.28	2,033,000,000	4169.28
2014-11-04	4154.88	4165.05	4126.72	4156.23	1,939,320,000	4156.23
2014-11-05	4179.23	4179.93	4140.21	4153.27	2,001,870,000	4153.27
2014-11-06	4147.31	4165.77	4133.98	4164.08	1,986,820,000	4164.08
2014-11-07	4170.46	4171.13	4139.46	4160.50	1,978,830,000	4160.50
2014-11-10	4162.59	4179.98	4151.48	4175.95	1,830,010,000	4175.95
2014-11-11	4173.61	4187.31	4165.25	4187.16	1,663,690,000	4187.16
2014-11-12	4174.47	4199.19	4173.02	4195.40	1,773,060,000	4195.40
2014-11-13	4202.77	4228.68	4194.33	4213.48	1,857,580,000	4213.48
2014-11-14	4214.97	4225.18	4198.59	4224.99	1,739,950,000	4224.99
2014-11-17	4215.29	4225.48	4194.18	4213.79	1,695,070,000	4213.79
2014-11-18	4215.18	4248.19	4214.32	4242.19	1,655,760,000	4242.19
2014-11-19	4233.71	4235.84	4204.16	4222.66	1,641,560,000	4222.66
2014-11-20	4204.54	4245.17	4202.36	4242.09	1,667,330,000	4242.09
2014-11-21	4284.41	4285.27	4237.38	4251.32	1,854,340,000	4251.32
2014-11-24	4262.09	4285.00	4258.41	4284.32	1,568,130,000	4284.32
2014-11-25	4290.68	4302.44	4279.90	4288.23	1,720,140,000	4288.23
2014-11-26	4291.87	4319.41	4289.51	4317.99	1,362,930,000	4317.99
2014-11-28	4330.96	4347.09	4326.21	4337.79	998,600,000	4337.79
2014-12-01	4323.95	4332.55	4274.68	4287.81	1,893,600,000	4287.81
2014-12-02	4291.33	4312.92	4283.89	4305.96	1,839,170,000	4305.96
2014-12-03	4312.17	4319.02	4287.04	4312.93	1,734,510,000	4312.93
2014-12-04	4309.48	4327.55	4293.94	4311.93	1,724,090,000	4311.93
2014-12-05	4318.28	4324.42	4302.04	4311.57	1,767,100,000	4311.57
2014-12-08	4301.27	4320.08	4259.01	4278.34	1,966,770,000	4278.34
2014-12-09	4228.21	4297.73	4216.87	4294.67	1,950,330,000	4294.67
2014-12-10	4284.50	4297.00	4218.85	4224.87	1,850,810,000	4224.87
2014-12-11	4241.69	4296.81	4238.29	4246.48	1,873,050,000	4246.48
2014-12-12	4207.61	4254.43	4199.28	4199.28	1,888,870,000	4199.28
2014-12-15	4224.17	4236.06	4145.01	4157.41	2,143,610,000	4157.41
2014-12-16	4122.23	4190.42	4089.19	4089.60	2,231,670,000	4089.60
2014-12-17	4095.75	4175.97	4089.09	4165.10	2,279,930,000	4165.10
2014-12-18	4230.35	4267.77	4213.21	4267.77	2,172,260,000	4267.77

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Date	Open	High	Low	Close	Volume	Adj. Close
2014-12-19	4272.75	4300.40	4258.33	4281.78	3,287,920,000	4281.78
2014-12-22	4271.87	4295.27	4270.54	4293.67	1,720,070,000	4293.67
2014-12-23	4308.95	4309.03	4274.51	4279.30	1,590,820,000	4279.30
2014-12-24	4283.19	4298.87	4281.80	4283.10	729,750,000	4283.10
2014-12-26	4295.72	4322.45	4294.80	4314.09	930,220,000	4314.09
2014-12-29	4309.70	4321.20	4306.67	4312.64	1,227,740,000	4312.64
2014-12-30	4301.14	4309.13	4277.75	4282.35	1,269,200,000	4282.35
2014-12-31	4294.88	4307.01	4233.71	4236.28	1,515,600,000	4236.28

Table A.3: Bids, asks and Black-Scholes prices of `'NDX-161216C04320000'` option contract with strike \$4320 and expiration date 2016-12-16, during the period from 2014-01-02 to 2014-12-31. Bids, asks and other greeks are inferred from the historical volatility surface.

Date	**Bid**	**Ask**	**BS price**	σ	Δ	ρ	θ	ν	T
2014-01-02	201.00	212.70	206.47	0.18	0.34	29.16	-0.20	22.46	2.96
2014-01-03	183.80	199.70	186.30	0.18	0.32	27.75	-0.19	21.84	2.95
2014-01-06	175.30	190.40	177.56	0.18	0.31	27.02	-0.18	21.51	2.95
2014-01-07	184.90	196.90	185.42	0.18	0.32	27.99	-0.18	21.95	2.94
2014-01-08	187.70	199.40	188.02	0.18	0.32	28.28	-0.19	22.09	2.94
2014-01-10	178.50	192.80	181.00	0.17	0.32	27.74	-0.18	21.84	2.93
2014-01-13	162.20	176.50	165.03	0.17	0.30	25.82	-0.17	20.95	2.93
2014-01-14	181.10	195.30	183.51	0.17	0.32	28.07	-0.18	22.00	2.92
2014-01-15	192.70	206.50	194.74	0.17	0.33	29.18	-0.19	22.49	2.92
2014-01-16	195.20	200.20	192.85	0.17	0.33	29.10	-0.19	22.45	2.92
2014-01-17	184.10	198.50	186.57	0.17	0.32	28.35	-0.18	22.13	2.92
2014-01-21	191.50	207.50	194.65	0.17	0.33	29.17	-0.19	22.49	2.90
2014-01-22	196.80	212.30	207.24	0.18	0.34	29.89	-0.20	22.80	2.90
2014-01-23	190.80	206.20	193.68	0.17	0.33	28.99	-0.19	22.41	2.90
2014-01-24	179.70	191.90	181.32	0.18	0.32	26.91	-0.18	21.48	2.90
2014-01-27	167.40	181.40	170.15	0.18	0.30	25.66	-0.18	20.90	2.89
2014-01-28	166.40	180.20	169.08	0.18	0.30	25.52	-0.18	20.83	2.88
2014-01-29	158.00	170.90	160.41	0.18	0.29	24.36	-0.18	20.26	2.88
2014-01-30	171.40	184.80	179.28	0.18	0.29	24.36	-0.18	20.26	2.88
2014-01-31	164.70	179.80	168.00	0.17	0.30	25.69	-0.18	20.92	2.88
2014-02-03	145.60	160.30	149.12	0.18	0.28	23.15	-0.17	19.65	2.87
2014-02-04	148.00	163.30	151.73	0.17	0.28	23.78	-0.17	19.98	2.87
2014-02-05	147.40	159.90	149.81	0.18	0.28	23.38	-0.17	19.78	2.86
2014-02-06	155.20	170.30	158.72	0.17	0.29	24.60	-0.17	20.40	2.86
2014-02-07	170.50	184.00	172.91	0.17	0.31	26.53	-0.18	21.33	2.86
2014-02-10	174.70	187.70	176.78	0.17	0.32	27.04	-0.18	21.57	2.85
2014-02-11	184.80	200.10	187.83	0.17	0.33	28.35	-0.19	22.16	2.85
2014-02-12	190.10	204.70	193.07	0.17	0.33	28.64	-0.19	22.29	2.84
2014-02-13	198.50	214.50	208.17	0.17	0.35	29.88	-0.20	22.81	2.84
2014-02-14	200.00	215.90	208.17	0.17	0.35	29.94	-0.20	22.84	2.84
2014-02-18	203.40	223.50	208.15	0.17	0.35	30.18	-0.20	22.94	2.83

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-02-19	197.30	212.90	208.25	0.18	0.34	29.55	-0.20	22.68	2.82
2014-02-20	200.80	220.70	207.37	0.17	0.35	30.02	-0.20	22.88	2.82
2014-02-21	201.20	220.70	207.40	0.17	0.35	29.78	-0.20	22.78	2.82
2014-02-24	203.30	223.40	207.35	0.17	0.35	30.21	-0.20	22.96	2.81
2014-02-25	202.10	222.00	207.37	0.17	0.35	30.03	-0.20	22.88	2.81
2014-02-26	200.10	220.70	207.38	0.17	0.35	29.94	-0.20	22.85	2.81
2014-02-27	205.10	226.00	207.30	0.17	0.35	30.45	-0.20	23.05	2.80
2014-02-28	208.00	228.00	207.32	0.17	0.35	30.33	-0.20	23.01	2.80
2014-03-03	200.00	216.00	207.44	0.17	0.35	29.61	-0.20	22.72	2.79
2014-03-04	215.20	234.60	219.92	0.17	0.36	31.23	-0.20	23.37	2.79
2014-03-05	218.80	238.60	223.68	0.17	0.37	31.51	-0.20	23.47	2.79
2014-03-06	216.10	236.50	221.34	0.17	0.36	31.24	-0.20	23.37	2.78
2014-03-07	209.70	227.80	207.41	0.17	0.35	30.28	-0.20	22.99	2.78
2014-03-10	212.00	230.50	207.43	0.17	0.35	30.26	-0.20	22.99	2.77
2014-03-11	208.10	227.90	207.49	0.17	0.35	29.89	-0.20	22.84	2.77
2014-03-12	211.90	231.60	207.43	0.17	0.35	30.22	-0.20	22.97	2.77
2014-03-13	200.00	215.10	206.83	0.18	0.35	29.00	-0.20	22.47	2.76
2014-03-14	195.30	206.30	195.61	0.18	0.34	28.02	-0.20	22.06	2.76
2014-03-17	200.90	214.10	206.84	0.18	0.35	29.13	-0.20	22.53	2.75
2014-03-18	208.10	227.00	206.66	0.17	0.35	30.09	-0.20	22.92	2.75
2014-03-19	202.80	223.30	206.79	0.17	0.35	29.51	-0.20	22.69	2.75
2014-03-20	205.90	225.70	206.76	0.17	0.35	29.74	-0.20	22.78	2.75
2014-03-21	192.30	208.10	195.01	0.17	0.34	28.33	-0.20	22.20	2.74
2014-03-24	180.50	192.50	181.58	0.17	0.32	26.91	-0.19	21.58	2.73
2014-03-25	178.70	194.10	181.45	0.17	0.33	27.13	-0.19	21.68	2.73
2014-03-26	162.50	178.30	165.76	0.17	0.31	25.41	-0.18	20.88	2.73
2014-03-27	158.00	173.30	160.46	0.17	0.30	24.83	-0.18	20.61	2.73
2014-03-28	157.80	171.60	159.51	0.17	0.30	24.91	-0.18	20.65	2.72
2014-03-31	161.30	175.30	163.01	0.17	0.31	25.48	-0.18	20.92	2.72
2014-04-01	177.80	192.10	179.25	0.17	0.33	27.53	-0.19	21.86	2.71
2014-04-02	178.90	193.20	180.33	0.16	0.33	27.71	-0.19	21.94	2.71
2014-04-03	170.20	182.70	171.66	0.16	0.32	26.58	-0.18	21.44	2.71
2014-04-04	147.00	158.80	148.67	0.17	0.29	23.46	-0.17	19.94	2.70
2014-04-07	138.10	149.40	139.73	0.17	0.28	22.33	-0.17	19.35	2.70
2014-04-08	144.10	156.50	146.13	0.17	0.29	23.20	-0.17	19.81	2.69

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-04-09	159.10	169.70	159.90	0.17	0.30	25.08	-0.18	20.75	2.69
2014-04-10	130.00	142.10	132.19	0.17	0.27	21.47	-0.16	18.90	2.69
2014-04-11	119.10	131.90	121.88	0.17	0.25	20.15	-0.16	18.16	2.68
2014-04-14	124.90	136.80	127.12	0.17	0.26	20.85	-0.16	18.57	2.68
2014-04-15	130.70	144.00	133.51	0.17	0.27	21.44	-0.17	18.89	2.67
2014-04-16	134.30	149.40	137.84	0.16	0.28	22.43	-0.17	19.43	2.67
2014-04-17	139.00	153.60	141.07	0.17	0.28	22.79	-0.17	19.62	2.67
2014-04-21	143.50	158.10	145.46	0.16	0.29	23.39	-0.17	19.94	2.66
2014-04-22	150.80	165.60	152.65	0.16	0.30	24.30	-0.18	20.39	2.65
2014-04-23	141.10	152.40	141.48	0.16	0.29	23.06	-0.17	19.77	2.65
2014-04-24	148.60	161.70	149.64	0.16	0.30	24.13	-0.18	20.31	2.65
2014-04-25	131.30	145.50	133.36	0.16	0.28	22.11	-0.17	19.28	2.65
2014-04-28	134.20	148.20	136.12	0.16	0.28	22.41	-0.17	19.44	2.64
2014-04-29	141.30	153.90	142.33	0.16	0.29	23.26	-0.17	19.89	2.64
2014-04-30	142.50	157.10	144.47	0.16	0.29	23.51	-0.17	20.01	2.63
2014-05-01	143.50	154.30	143.53	0.16	0.29	23.66	-0.17	20.09	2.63
2014-05-02	142.80	155.10	143.61	0.16	0.29	23.51	-0.17	20.02	2.63
2014-05-05	145.20	155.90	145.16	0.16	0.29	23.85	-0.17	20.19	2.62
2014-05-06	135.00	147.50	136.19	0.16	0.28	22.41	-0.17	19.46	2.62
2014-05-07	132.10	143.20	132.69	0.16	0.28	22.00	-0.17	19.25	2.61
2014-05-08	129.20	140.10	130.33	0.16	0.27	21.64	-0.16	19.06	2.61
2014-05-09	128.00	138.80	129.05	0.16	0.27	21.80	-0.16	19.14	2.61
2014-05-12	144.90	155.00	145.24	0.16	0.29	23.73	-0.17	20.14	2.60
2014-05-13	146.90	161.70	149.55	0.16	0.30	23.91	-0.18	20.24	2.60
2014-05-14	137.80	148.80	138.74	0.16	0.29	22.94	-0.17	19.75	2.59
2014-05-15	131.40	143.20	132.91	0.16	0.28	22.06	-0.17	19.30	2.59
2014-05-16	135.70	146.80	136.75	0.16	0.28	22.66	-0.17	19.61	2.59
2014-05-19	138.20	152.40	140.69	0.16	0.29	23.34	-0.17	19.96	2.58
2014-05-20	131.60	143.50	133.09	0.16	0.28	22.57	-0.17	19.57	2.58
2014-05-21	135.70	149.00	137.72	0.15	0.29	23.49	-0.17	20.04	2.58
2014-05-22	140.10	153.10	141.88	0.15	0.30	24.01	-0.17	20.30	2.57
2014-05-23	145.70	158.20	147.07	0.15	0.30	24.81	-0.17	20.69	2.57
2014-05-27	159.90	173.60	161.57	0.15	0.32	26.44	-0.18	21.44	2.56
2014-05-28	156.80	171.00	158.78	0.15	0.32	26.03	-0.18	21.26	2.56
2014-05-29	161.00	175.30	162.91	0.15	0.33	26.73	-0.18	21.57	2.55

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-05-30	161.50	176.40	163.71	0.15	0.33	26.77	-0.18	21.59	2.55
2014-06-02	163.20	176.80	164.82	0.15	0.33	26.64	-0.18	21.54	2.54
2014-06-03	161.90	177.60	164.61	0.15	0.33	26.53	-0.18	21.49	2.54
2014-06-04	166.50	182.00	169.03	0.15	0.33	27.02	-0.19	21.70	2.54
2014-06-05	177.30	190.20	177.60	0.15	0.34	28.25	-0.19	22.22	2.53
2014-06-06	182.70	196.50	183.31	0.15	0.35	28.89	-0.19	22.48	2.53
2014-06-09	183.00	199.00	184.74	0.15	0.35	28.88	-0.19	22.47	2.52
2014-06-10	184.30	199.70	185.73	0.15	0.35	29.01	-0.19	22.53	2.52
2014-06-11	184.10	199.40	185.51	0.15	0.35	28.90	-0.19	22.48	2.52
2014-06-12	175.10	191.10	177.80	0.15	0.34	27.63	-0.19	21.97	2.52
2014-06-13	176.90	192.90	179.54	0.15	0.34	27.95	-0.19	22.10	2.51
2014-06-16	176.80	192.40	179.27	0.15	0.34	27.95	-0.19	22.10	2.50
2014-06-17	176.00	192.50	178.93	0.15	0.34	27.93	-0.19	22.10	2.50
2014-06-18	185.50	198.90	186.72	0.15	0.35	28.79	-0.19	22.44	2.50
2014-06-19	181.00	195.20	181.99	0.15	0.35	28.56	-0.19	22.35	2.50
2014-06-20	183.10	197.10	183.97	0.15	0.35	28.65	-0.19	22.39	2.49
2014-06-23	179.80	195.80	181.73	0.15	0.35	28.53	-0.19	22.34	2.48
2014-06-24	178.50	194.50	180.47	0.15	0.35	28.30	-0.19	22.25	2.48
2014-06-25	187.00	203.00	188.76	0.15	0.36	29.30	-0.20	22.64	2.48
2014-06-26	187.30	200.70	187.77	0.15	0.36	29.21	-0.20	22.61	2.48
2014-06-27	192.90	206.90	193.55	0.15	0.37	29.85	-0.20	22.85	2.47
2014-06-30	195.50	209.60	195.35	0.15	0.37	29.95	-0.20	22.88	2.47
2014-07-01	208.00	226.80	210.71	0.15	0.39	31.66	-0.20	23.48	2.46
2014-07-02	207.50	226.40	210.27	0.15	0.39	31.74	-0.20	23.51	2.46
2014-07-03	218.80	237.30	221.17	0.15	0.40	32.72	-0.21	23.83	2.46
2014-07-07	213.70	233.50	216.88	0.15	0.39	32.08	-0.21	23.62	2.45
2014-07-08	201.00	222.00	195.40	0.15	0.37	30.05	-0.20	22.91	2.44
2014-07-09	209.50	229.60	212.97	0.15	0.39	31.39	-0.21	23.38	2.44
2014-07-10	204.90	225.90	209.68	0.15	0.38	30.80	-0.21	23.18	2.44
2014-07-11	213.00	233.60	217.41	0.15	0.39	31.68	-0.21	23.48	2.44
2014-07-14	222.80	242.70	226.73	0.15	0.40	32.54	-0.21	23.76	2.43
2014-07-15	218.80	238.00	222.50	0.15	0.40	31.96	-0.21	23.57	2.42
2014-07-16	229.50	248.90	233.16	0.16	0.40	32.71	-0.22	23.81	2.42
2014-07-17	214.40	233.70	218.36	0.16	0.39	30.76	-0.21	23.16	2.42
2014-07-18	229.80	249.90	233.78	0.15	0.41	32.87	-0.22	23.85	2.42

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-07-21	229.60	248.80	233.20	0.16	0.40	32.57	-0.22	23.75	2.41
2014-07-22	236.30	256.90	240.43	0.15	0.41	33.52	-0.22	24.04	2.41
2014-07-23	255.60	272.20	257.54	0.16	0.43	34.59	-0.23	24.35	2.40
2014-07-24	256.10	276.10	259.78	0.16	0.43	34.50	-0.23	24.32	2.40
2014-07-25	252.50	272.00	256.04	0.16	0.42	33.84	-0.23	24.13	2.40
2014-07-28	256.90	277.00	260.77	0.16	0.42	33.87	-0.23	24.13	2.39
2014-07-29	256.40	276.50	260.31	0.16	0.42	33.58	-0.23	24.04	2.39
2014-07-30	262.00	282.30	265.95	0.16	0.43	34.14	-0.24	24.20	2.38
2014-07-31	238.60	259.50	243.38	0.17	0.40	31.28	-0.23	23.32	2.38
2014-08-01	230.70	250.60	235.08	0.17	0.40	30.71	-0.23	23.12	2.38
2014-08-04	239.90	258.90	243.70	0.17	0.41	31.58	-0.23	23.41	2.37
2014-08-05	234.60	254.10	238.84	0.17	0.40	30.53	-0.23	23.06	2.37
2014-08-06	235.60	256.10	240.34	0.17	0.40	30.51	-0.23	23.05	2.36
2014-08-07	231.40	251.40	235.99	0.17	0.39	29.96	-0.23	22.86	2.36
2014-08-08	240.30	261.10	245.13	0.17	0.40	30.91	-0.24	23.18	2.36
2014-08-11	247.10	266.80	251.29	0.17	0.41	31.52	-0.24	23.38	2.35
2014-08-12	244.80	263.40	248.50	0.17	0.41	31.28	-0.24	23.30	2.35
2014-08-13	260.10	276.50	262.44	0.17	0.42	32.73	-0.24	23.75	2.35
2014-08-14	266.30	281.10	267.72	0.17	0.43	33.36	-0.24	23.93	2.34
2014-08-15	273.50	288.60	274.94	0.17	0.44	33.98	-0.24	24.11	2.34
2014-08-18	281.50	299.60	284.31	0.16	0.45	34.99	-0.24	24.37	2.33
2014-08-19	290.50	308.00	292.90	0.16	0.45	35.67	-0.25	24.53	2.33
2014-08-20	292.00	311.20	295.26	0.17	0.45	35.67	-0.25	24.53	2.33
2014-08-21	293.40	313.80	297.23	0.17	0.46	35.85	-0.25	24.57	2.32
2014-08-22	300.10	320.80	304.04	0.17	0.46	36.06	-0.25	24.61	2.32
2014-08-25	304.70	324.70	308.24	0.17	0.47	36.44	-0.25	24.69	2.31
2014-08-26	307.30	328.30	311.33	0.17	0.47	36.56	-0.25	24.71	2.31
2014-08-27	305.50	326.00	309.28	0.17	0.47	36.55	-0.25	24.70	2.31
2014-08-28	304.70	325.00	308.43	0.17	0.46	36.27	-0.25	24.63	2.30
2014-08-29	311.70	332.00	315.34	0.17	0.47	36.81	-0.26	24.75	2.30
2014-09-02	316.80	337.80	320.81	0.17	0.48	37.10	-0.26	24.79	2.29
2014-09-03	310.10	331.00	314.22	0.17	0.47	36.20	-0.26	24.59	2.29
2014-09-04	304.80	325.40	308.84	0.17	0.46	35.95	-0.26	24.53	2.28
2014-09-05	312.50	332.20	315.97	0.17	0.47	36.74	-0.26	24.70	2.28
2014-09-08	315.70	335.40	319.18	0.17	0.48	36.81	-0.26	24.70	2.27

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-09-09	302.30	322.60	306.30	0.17	0.46	35.57	-0.26	24.42	2.27
2014-09-10	316.10	338.10	320.78	0.17	0.48	36.71	-0.26	24.66	2.27
2014-09-11	315.10	335.30	318.92	0.17	0.47	36.57	-0.26	24.63	2.27
2014-09-12	306.50	327.10	310.70	0.17	0.47	35.71	-0.26	24.43	2.26
2014-09-15	291.40	311.60	295.68	0.17	0.45	34.21	-0.26	24.06	2.25
2014-09-16	308.50	328.90	312.67	0.17	0.47	35.48	-0.26	24.36	2.25
2014-09-17	309.40	329.80	313.56	0.17	0.47	35.65	-0.26	24.39	2.25
2014-09-18	325.70	345.80	329.55	0.17	0.48	36.65	-0.27	24.61	2.25
2014-09-19	324.00	344.00	327.81	0.17	0.48	36.50	-0.27	24.57	2.24
2014-09-22	305.50	325.30	309.48	0.17	0.46	35.00	-0.26	24.22	2.24
2014-09-23	303.50	318.50	305.15	0.17	0.46	34.62	-0.26	24.13	2.23
2014-09-24	323.80	338.80	325.22	0.17	0.48	36.07	-0.27	24.45	2.23
2014-09-25	291.60	306.60	293.51	0.17	0.45	33.11	-0.26	23.73	2.23
2014-09-26	308.40	323.40	310.07	0.17	0.46	34.60	-0.27	24.11	2.22
2014-09-29	303.00	324.10	307.79	0.17	0.46	34.25	-0.27	24.01	2.22
2014-09-30	315.00	330.00	316.74	0.18	0.46	34.35	-0.27	24.03	2.21
2014-10-01	289.20	304.20	291.28	0.18	0.44	32.19	-0.27	23.46	2.21
2014-10-02	288.50	303.50	292.01	0.18	0.44	32.06	-0.27	23.42	2.21
2014-10-03	304.40	319.40	307.75	0.18	0.45	33.36	-0.27	23.77	2.21
2014-10-06	296.50	311.50	299.93	0.18	0.45	32.83	-0.27	23.62	2.20
2014-10-07	271.50	292.20	278.00	0.18	0.43	30.93	-0.26	23.07	2.19
2014-10-08	297.60	318.00	303.64	0.17	0.45	33.50	-0.27	23.78	2.19
2014-10-09	274.50	294.30	279.17	0.18	0.43	31.28	-0.26	23.17	2.19
2014-10-10	239.50	256.20	243.16	0.18	0.40	28.11	-0.25	22.13	2.19
2014-10-14	235.70	255.70	241.26	0.19	0.39	26.59	-0.26	21.56	2.18
2014-10-15	211.80	232.20	217.75	0.18	0.37	25.44	-0.25	21.10	2.17
2014-10-16	202.20	226.20	210.07	0.18	0.36	24.77	-0.25	20.81	2.17
2014-10-17	227.40	249.30	233.96	0.19	0.38	26.41	-0.26	21.49	2.17
2014-10-20	243.20	262.10	248.04	0.18	0.40	27.83	-0.26	22.02	2.16
2014-10-21	277.80	298.50	283.05	0.18	0.43	30.90	-0.27	23.02	2.16
2014-10-22	275.70	295.40	280.57	0.18	0.43	30.26	-0.27	22.82	2.15
2014-10-23	302.70	317.70	304.92	0.18	0.45	32.18	-0.28	23.38	2.15
2014-10-24	313.20	328.20	315.28	0.18	0.46	33.07	-0.28	23.60	2.15
2014-10-27	313.80	332.10	317.56	0.18	0.46	33.07	-0.28	23.59	2.14
2014-10-28	332.60	352.10	336.67	0.18	0.48	34.95	-0.28	24.02	2.14

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Date	Bid	Ask	BS price	σ	Δ	ρ	θ	ν	T
2014-10-29	328.50	347.80	332.57	0.18	0.48	34.39	-0.28	23.89	2.13
2014-10-30	331.90	353.10	336.87	0.18	0.48	34.67	-0.28	23.95	2.13
2014-10-31	359.70	381.70	364.79	0.18	0.50	36.51	-0.29	24.30	2.13
2014-11-03	368.80	390.80	373.88	0.18	0.51	36.72	-0.29	24.32	2.12
2014-11-04	362.40	384.30	367.51	0.18	0.50	36.25	-0.29	24.23	2.12
2014-11-05	362.60	384.40	367.70	0.18	0.50	36.11	-0.30	24.19	2.12
2014-11-06	369.90	390.70	373.70	0.18	0.51	36.47	-0.30	24.25	2.11
2014-11-07	361.20	381.60	364.82	0.18	0.50	36.30	-0.29	24.22	2.11
2014-11-10	363.90	383.90	367.31	0.18	0.51	36.67	-0.29	24.25	2.10
2014-11-12	372.20	390.10	374.47	0.18	0.51	37.21	-0.29	24.33	2.10
2014-11-13	380.10	399.30	383.72	0.18	0.52	37.69	-0.29	24.39	2.09
2014-11-14	384.20	404.20	388.16	0.18	0.52	38.02	-0.29	24.43	2.09
2014-11-17	378.10	397.60	381.94	0.18	0.52	37.50	-0.29	24.33	2.08
2014-11-18	400.50	414.10	401.26	0.18	0.53	38.35	-0.30	24.43	2.08
2014-11-19	393.10	408.10	394.68	0.18	0.52	37.66	-0.30	24.33	2.08
2014-11-20	404.30	419.30	404.21	0.18	0.53	38.35	-0.30	24.41	2.07
2014-11-21	412.10	425.70	411.27	0.18	0.54	38.57	-0.31	24.43	2.07
2014-11-24	426.80	441.80	426.52	0.18	0.55	39.48	-0.31	24.50	2.06
2014-11-25	426.70	444.00	427.55	0.18	0.55	39.56	-0.31	24.50	2.06
2014-11-26	444.60	460.60	444.63	0.18	0.56	40.44	-0.31	24.56	2.06
2014-11-28	464.70	477.40	463.00	0.18	0.57	40.85	-0.32	24.57	2.05
2014-12-01	440.70	455.70	440.55	0.19	0.55	39.10	-0.32	24.39	2.04
2014-12-02	447.90	462.90	447.67	0.19	0.56	39.63	-0.32	24.42	2.04
2014-12-03	452.60	467.60	452.35	0.19	0.56	39.78	-0.32	24.43	2.04
2014-12-04	450.90	465.90	449.85	0.18	0.56	39.77	-0.32	24.41	2.04
2014-12-05	452.50	466.20	447.67	0.18	0.56	39.72	-0.32	24.40	2.03
2014-12-08	440.00	455.00	439.28	0.19	0.55	38.45	-0.33	24.24	2.02
2014-12-09	449.50	462.40	447.86	0.19	0.55	38.89	-0.33	24.27	2.02
2014-12-10	409.50	433.00	413.39	0.19	0.53	36.72	-0.32	24.01	2.02
2014-12-11	416.50	440.10	420.32	0.19	0.54	37.35	-0.32	24.08	2.02
2014-12-12	398.70	421.30	402.32	0.19	0.52	35.85	-0.32	23.86	2.01
2014-12-15	385.40	400.20	385.45	0.19	0.51	34.45	-0.33	23.60	2.01
2014-12-16	357.00	372.00	357.58	0.20	0.49	32.43	-0.32	23.18	2.00
2014-12-17	390.50	405.50	390.65	0.19	0.51	34.58	-0.33	23.61	2.00
2014-12-18	438.00	455.10	438.61	0.19	0.55	37.56	-0.33	24.04	2.00

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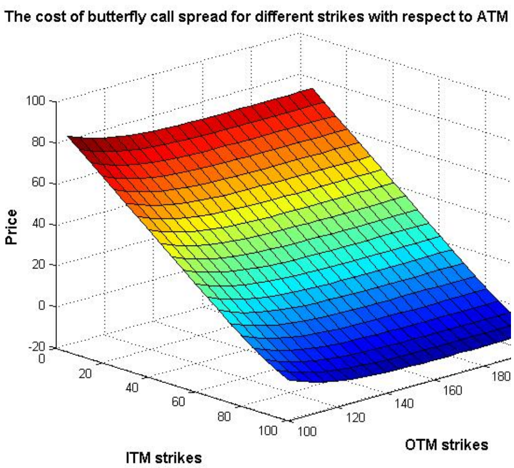
Date	**Bid**	**Ask**	**BS price**	σ	Δ	ρ	θ	ν	T
2014-12-19	451.10	465.60	448.16	0.19	0.55	37.90	-0.33	24.06	1.99
2014-12-22	459.90	474.90	459.38	0.20	0.56	38.04	-0.34	24.04	1.99
2014-12-23	458.40	472.60	457.63	0.20	0.55	37.52	-0.34	23.98	1.98
2014-12-24	460.10	475.10	459.73	0.20	0.55	37.57	-0.34	23.97	1.98
2014-12-26	475.60	490.60	475.08	0.20	0.56	38.36	-0.35	24.03	1.98
2014-12-29	472.20	487.20	471.75	0.20	0.56	38.18	-0.35	23.98	1.97
2014-12-30	454.00	469.00	448.41	0.19	0.55	37.33	-0.34	23.88	1.96
2014-12-31	429.60	444.60	429.63	0.20	0.54	35.92	-0.34	23.70	1.96

Appendix B

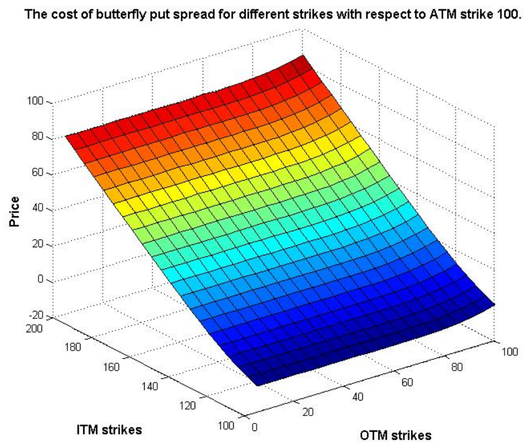
Monte Carlo Prices for Option Portfolios

Name	c_{ATM}	p_{ATM}	c_{OTM}	p_{OTM}	c_{ITM}	p_{ITM}
Short Call	-1	0	0	0	0	0
Long Call	1	0	0	0	0	0
Long Put	0	1	0	0	0	0
Short Put	0	-1	0	0	0	0
Bull Call Spread	0	0	-1	0	1	0
Bull Put Spread	0	0	0	1	0	-1
Bear Call Spread	0	0	1	0	-1	0
Bear Put Spread	0	0	0	-1	0	1
Butterfly Call Spread	-2	0	1	0	1	0
Butterfly Put Spread	0	-2	0	1	0	1
Long Call Ladder	-1	0	-1	0	1	0
Short Call Ladder	1	0	1	0	-1	0
Long Put Ladder	0	-1	0	-1	0	1
Short Put Ladder	0	1	0	1	0	-1
Iron Butterfly	-1	-1	1	1	0	0
Long Straddle	1	1	0	0	0	0
Short Straddle	-1	-1	0	0	0	0
Long Strangle	0	0	1	1	0	0
Short Strangle	0	0	-1	-1	0	0
Strip	1	2	0	0	0	0
Strap	2	1	0	0	0	0

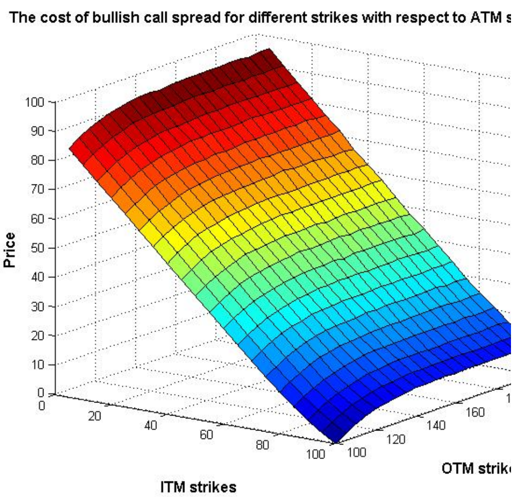
Table B.1: Option Portfolios (a.k.a. Option Trading Strategies)



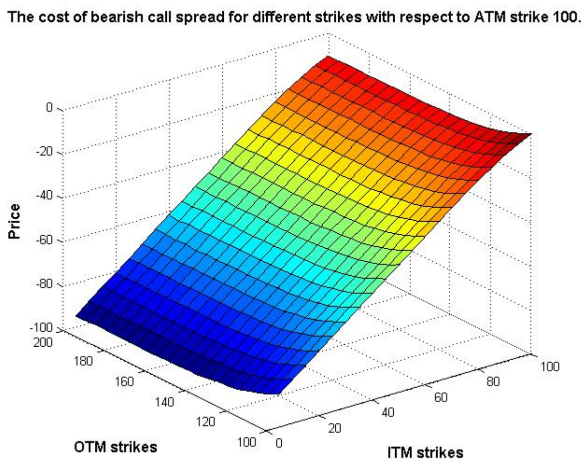
(a) Butterfly Call Spread prices for different OTM and ITM strikes. ATM strike is 100.



(b) Butterfly Put Spread prices for different OTM and ITM strikes. ATM strike is 100.

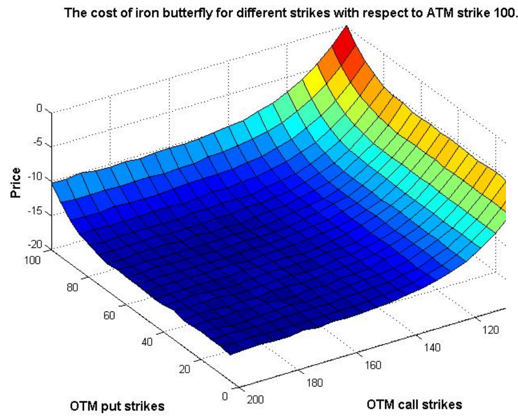


(c) Bullish Call Spread prices for different OTM and ITM strikes. ATM strike is 100.

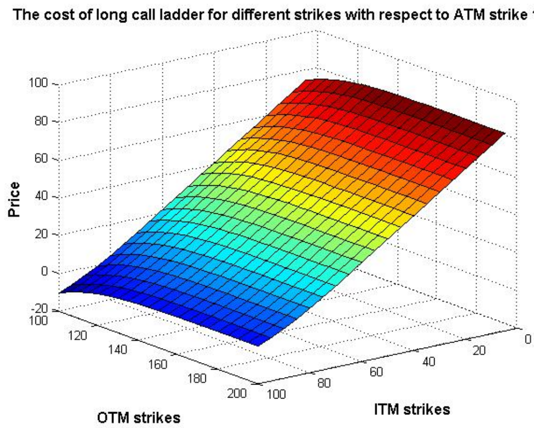


(d) Bearish Call Spread prices for different OTM and ITM strikes. ATM strike is 100.

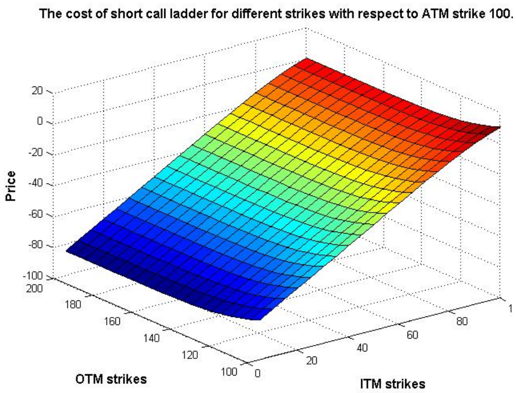
Figure B.1: Option Portfolio prices for different strikes



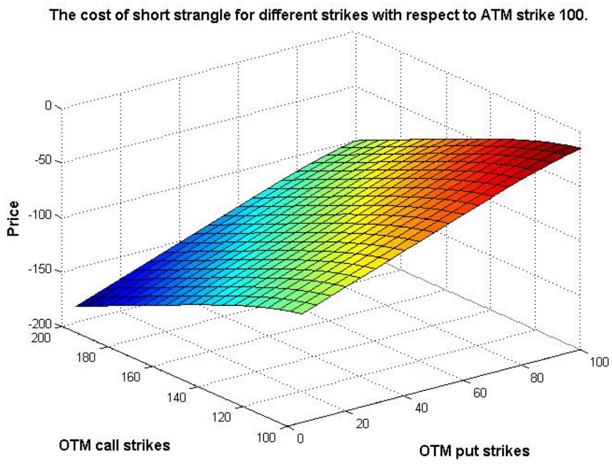
(a) Iron Butterfly prices for different OTM put and OTM call strikes. ATM strike is 100.



(b) Long Call Ladder prices for different OTM and ITM strikes. ATM strike is 100.



(c) Short Call Ladder prices for different OTM and ITM strikes. ATM strike is 100.



(d) Short Strangle prices for different OTM and ITM strikes. ATM strike is 100.

Figure B.2: Option Portfolio prices for different strikes

Appendix C

Bids and asks of LMSR traders holding different option portfolios

Below we list bids and asks of LMSR trader holding different option portfolios in comparison with Black-Scholes prices. We set the liquidity parameter to $b = 100$.

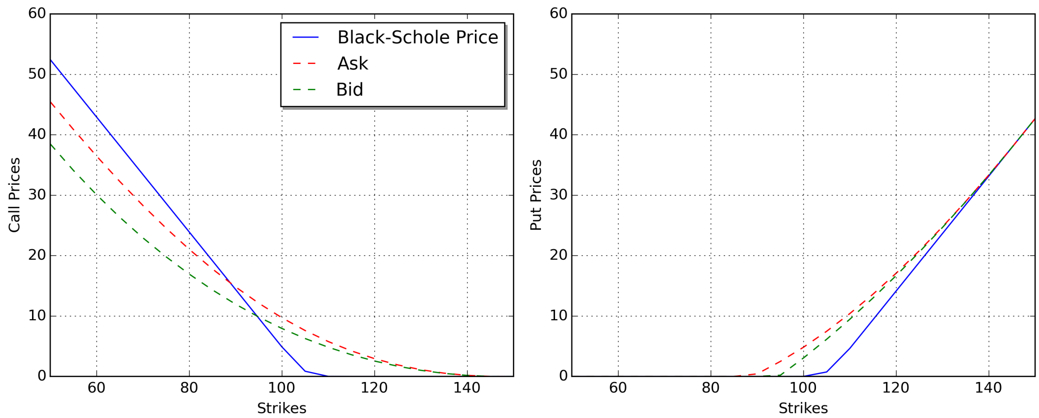


Figure C.1: Short Call

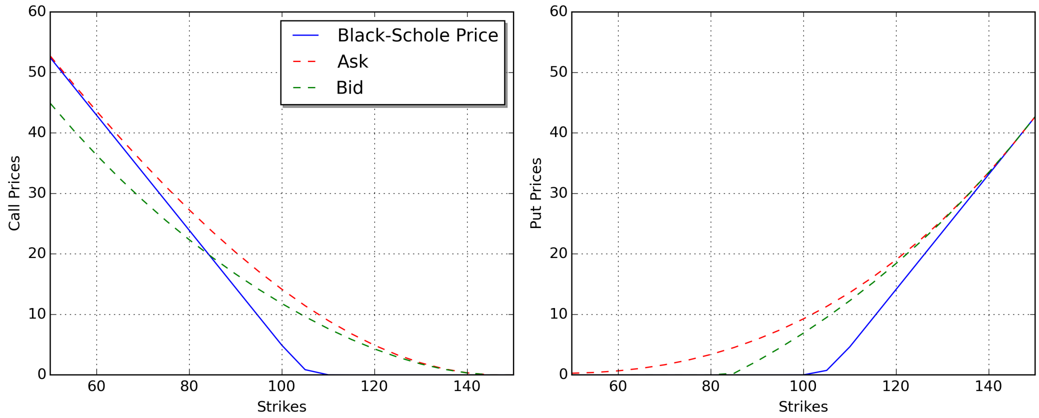


Figure C.2: Long Call

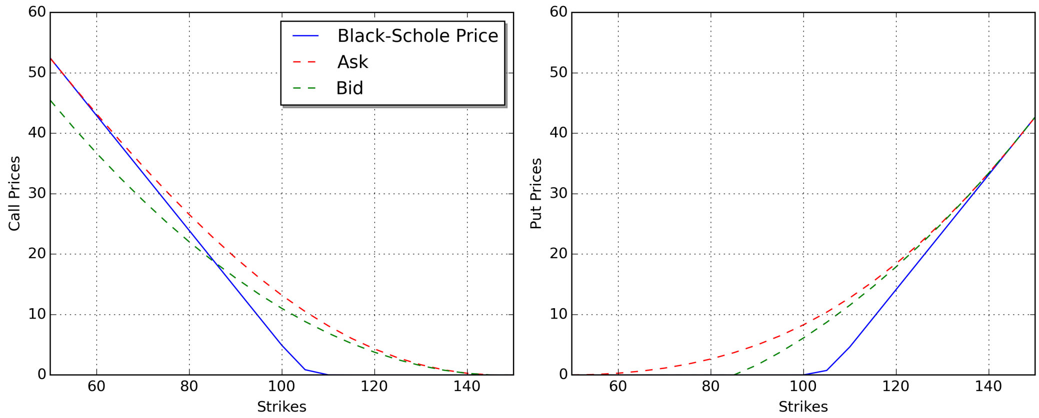


Figure C.3: Short Put

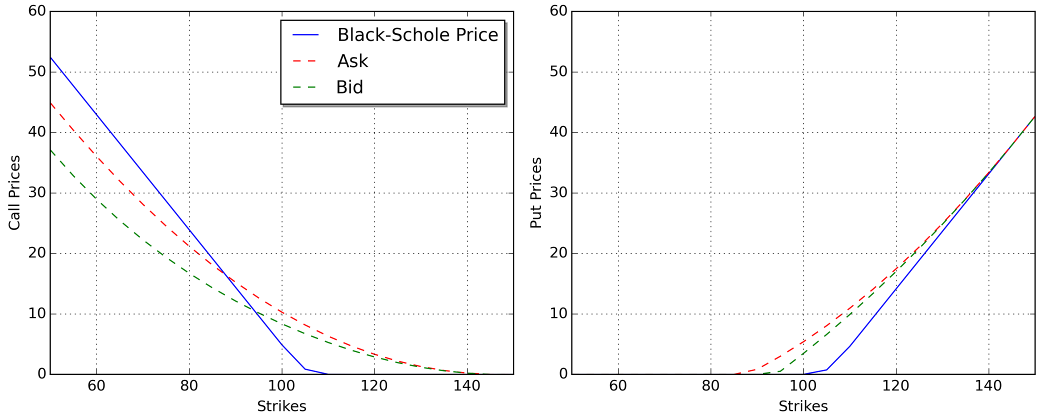


Figure C.4: Long Put

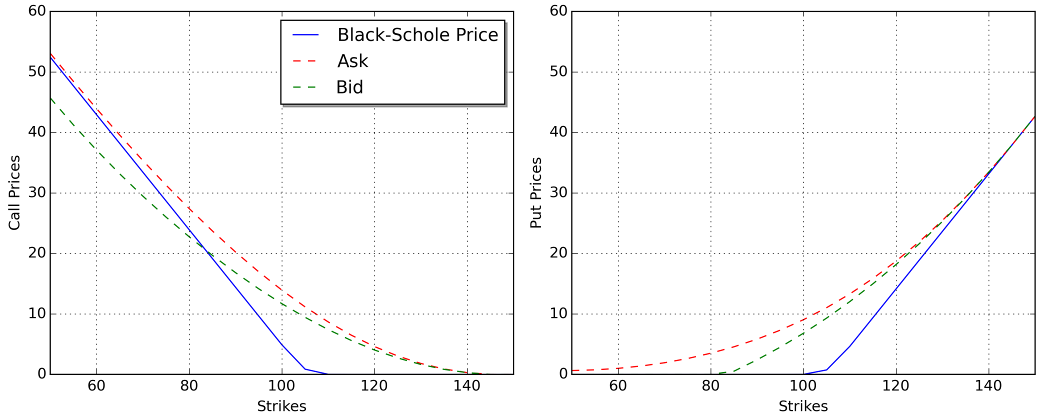


Figure C.5: Bullish Call/Put Spread

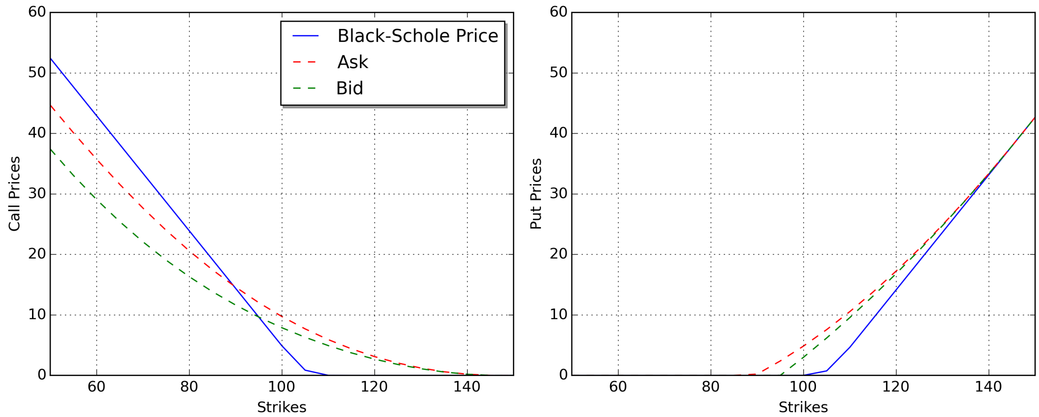


Figure C.6: Bearish Call/Put Spread

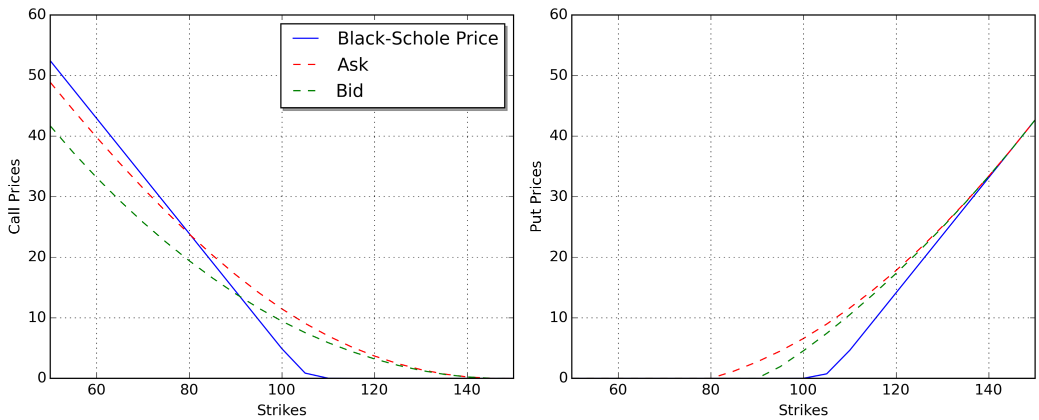


Figure C.7: Butterfly Call/Put Spread

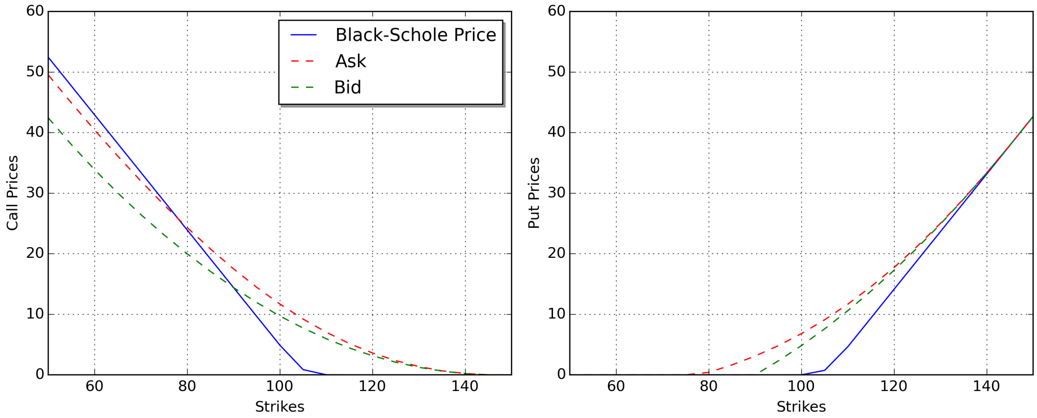


Figure C.8: Long Call/Put Ladder

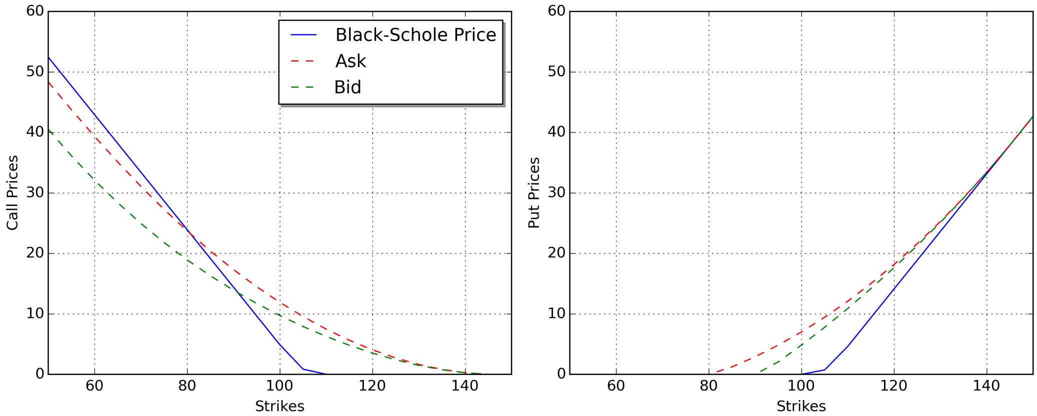


Figure C.9: Short Call/Put Ladder

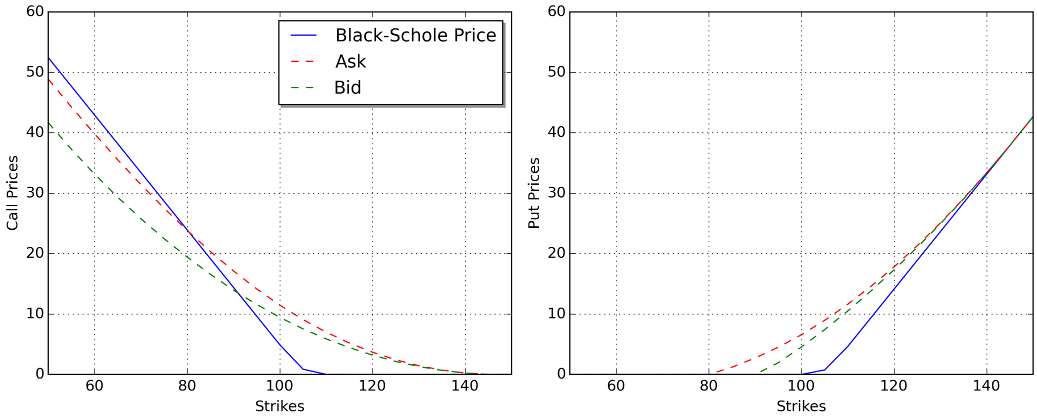


Figure C.10: Iron Butterfly

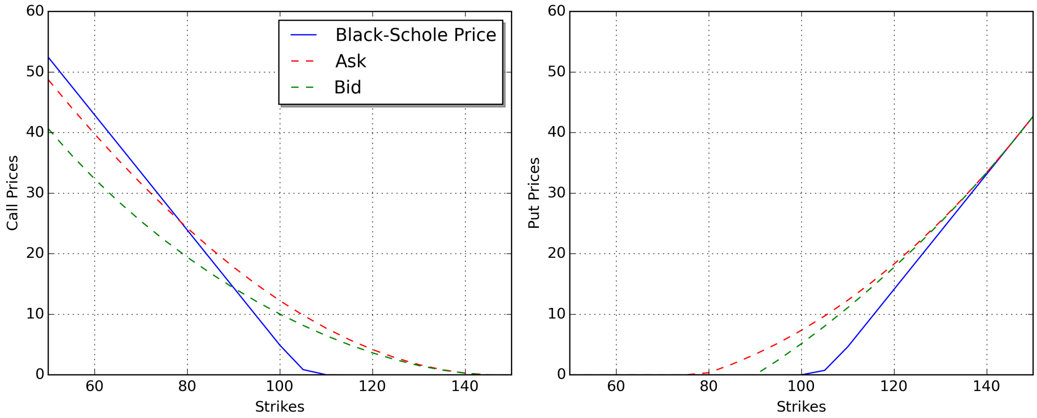


Figure C.11: Long Strangle

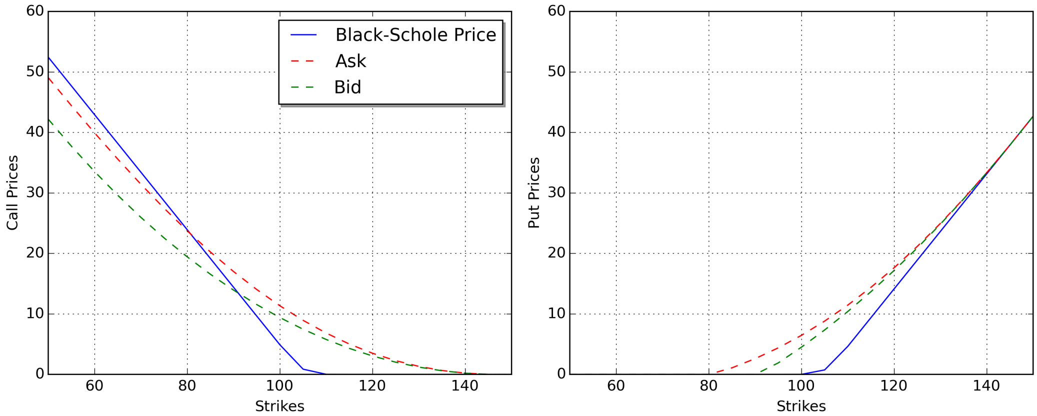


Figure C.12: Short Strangle

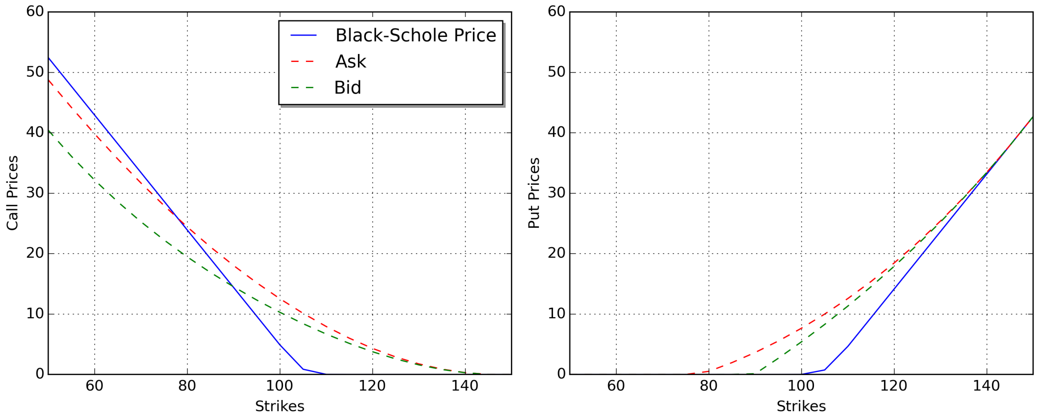


Figure C.13: Long Straddle

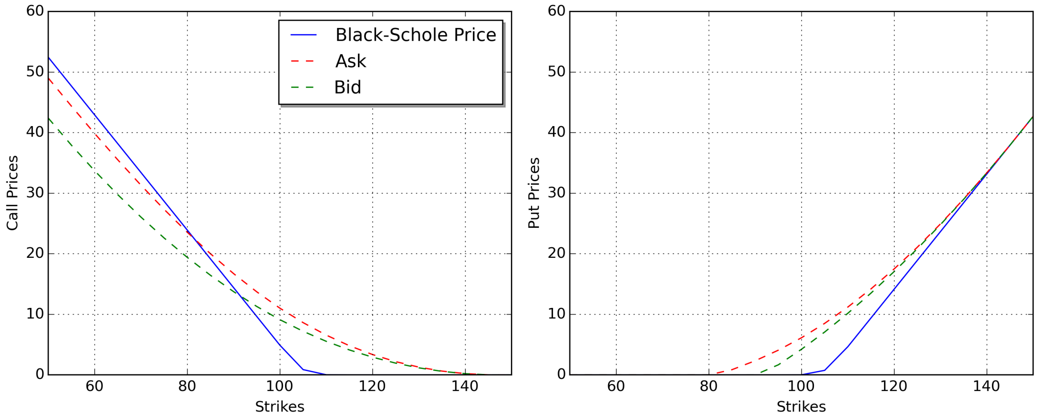


Figure C.14: Short Straddle

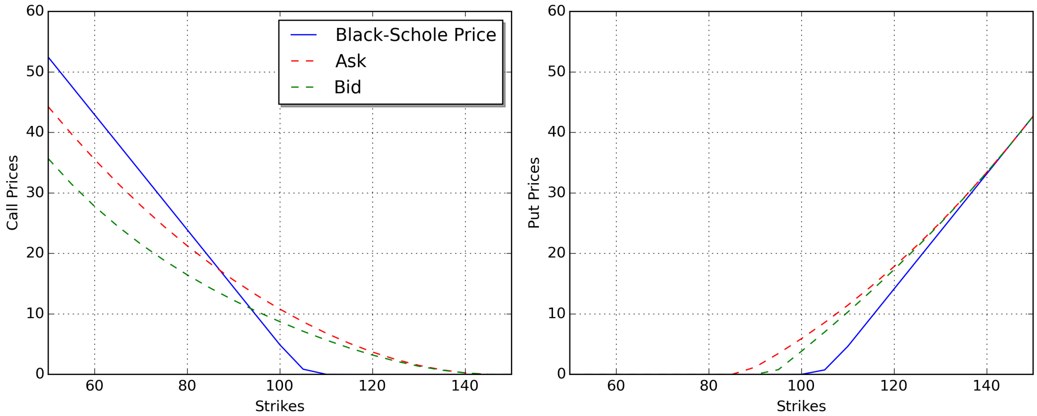


Figure C.15: Strip

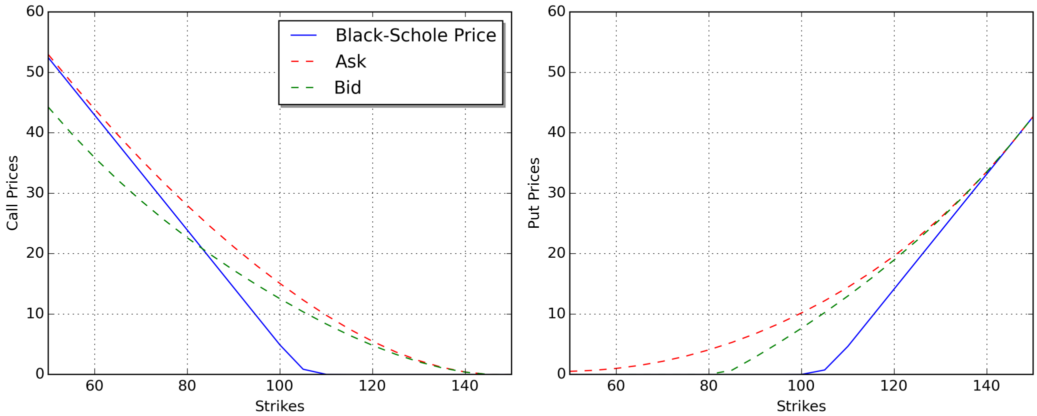


Figure C.16: Strap

Appendix D

Direct DA Simulation Results

Option Pricing	Choosing Quantity
ZI - Zero-Intelligence	RND-Random Integer
BS - Black-Scholes	LIN- Linear Quantity
MC - Monte Carlo using GBM Model	
MC* - Monte Carlo using JD Model	
VOL - Volatility Surface	
EXP - Exponential Utility	
LMSR - Log Market Scoring Rule	

Table D.1: Nomenclature for naming agents

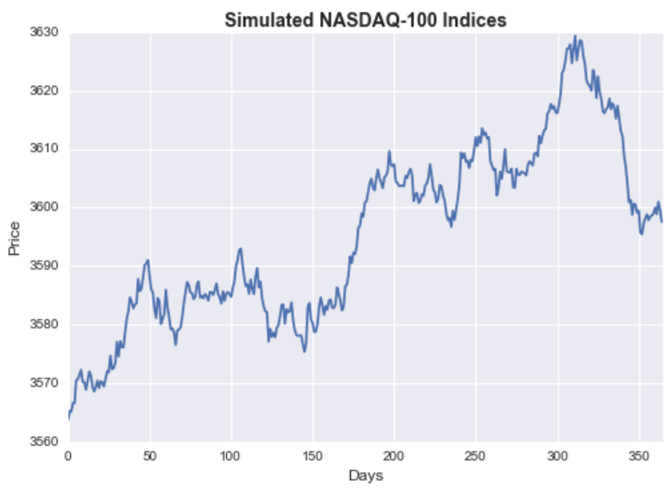


Figure D.1: Simulated NASDAQ-100 Indices

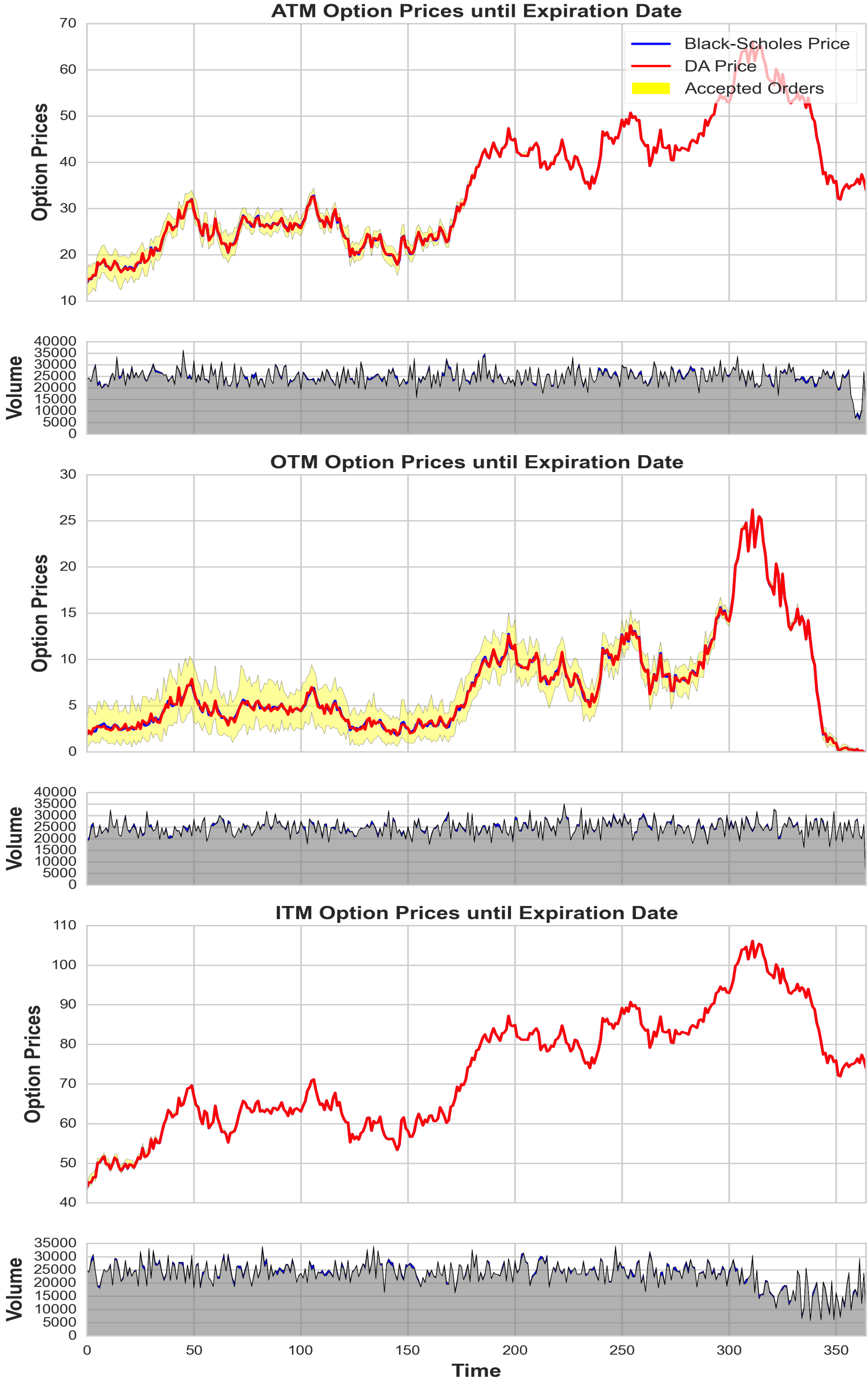


Figure D.2: DA prices of ATM, OTM and ITM options for VOL-RND traders

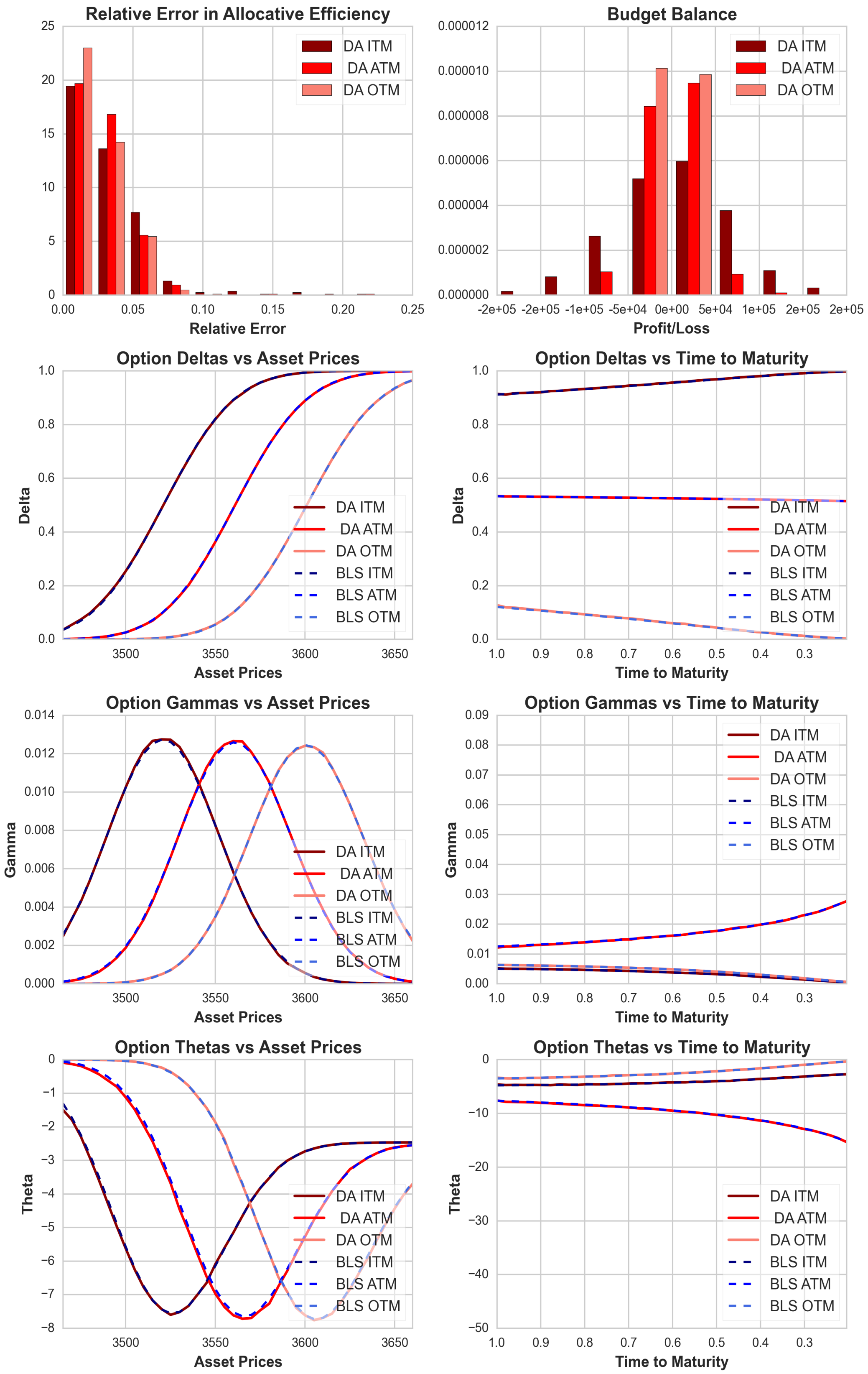


Figure D.3: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for VOL-RND traders

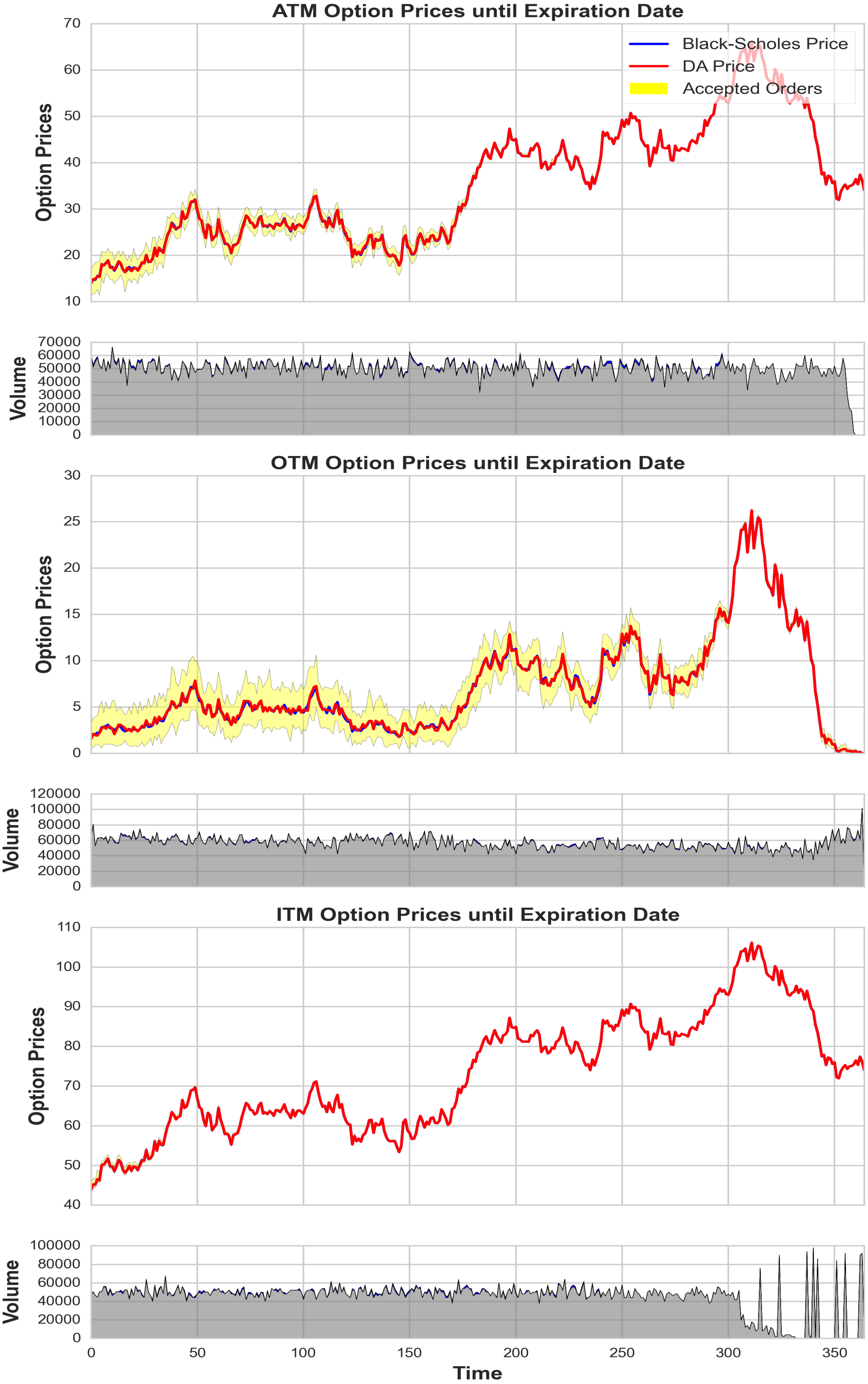


Figure D.4: DA prices of ATM, OTM and ITM options for VOL-LIN traders

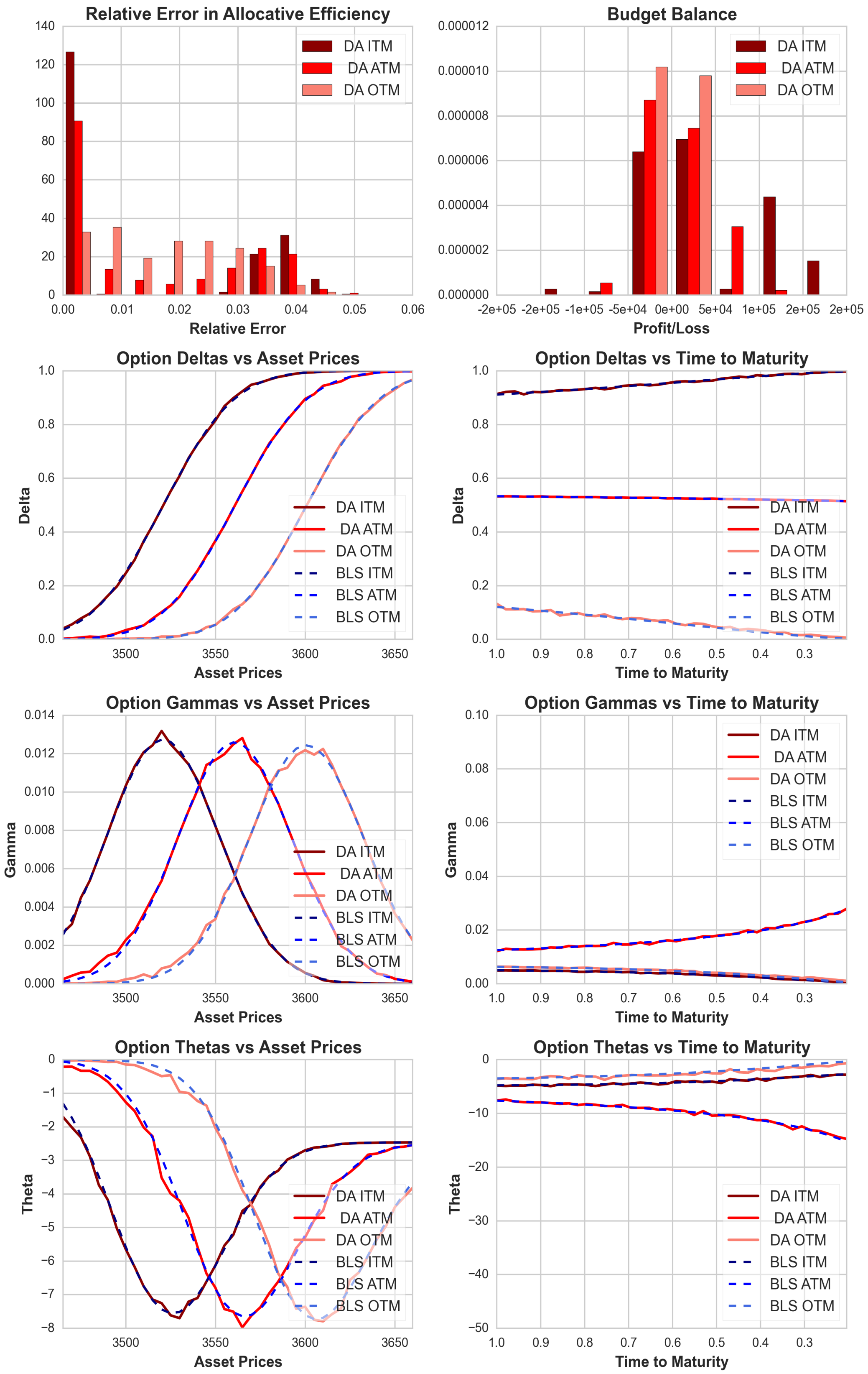


Figure D.5: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for VOL-LIN traders

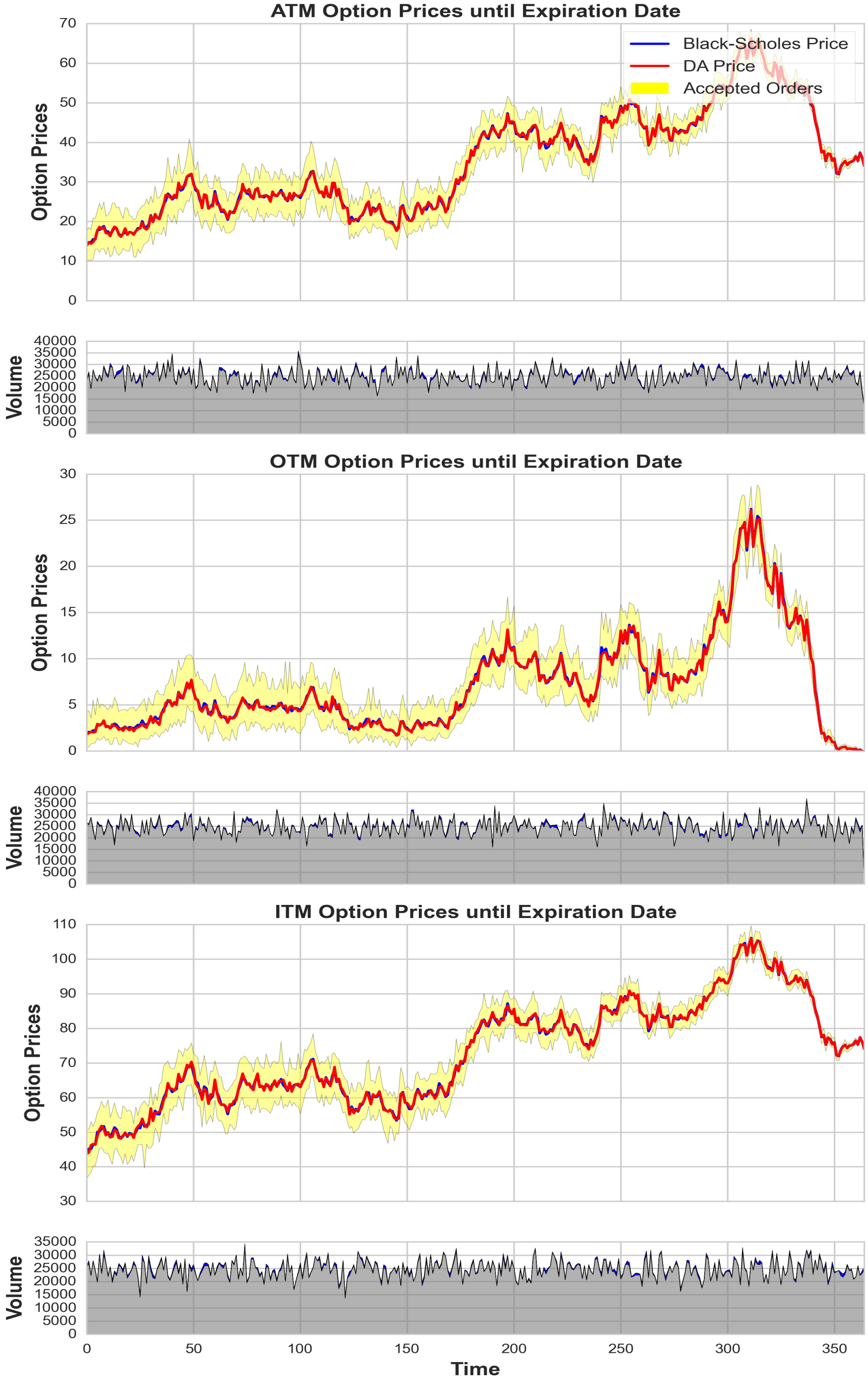


Figure D.6: DA prices of ATM, OTM and ITM options for MC-RND traders

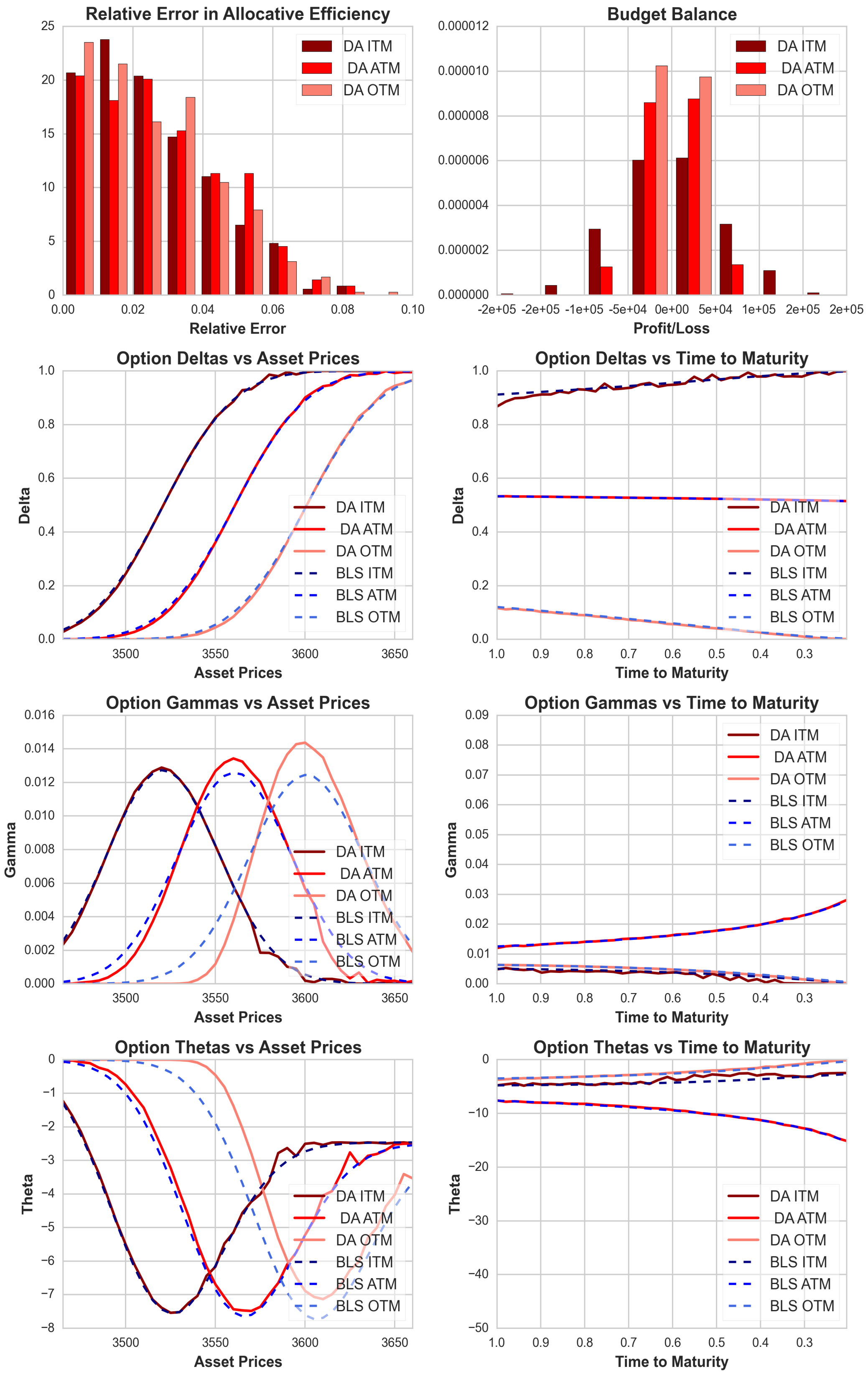


Figure D.7: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for MC-RND traders

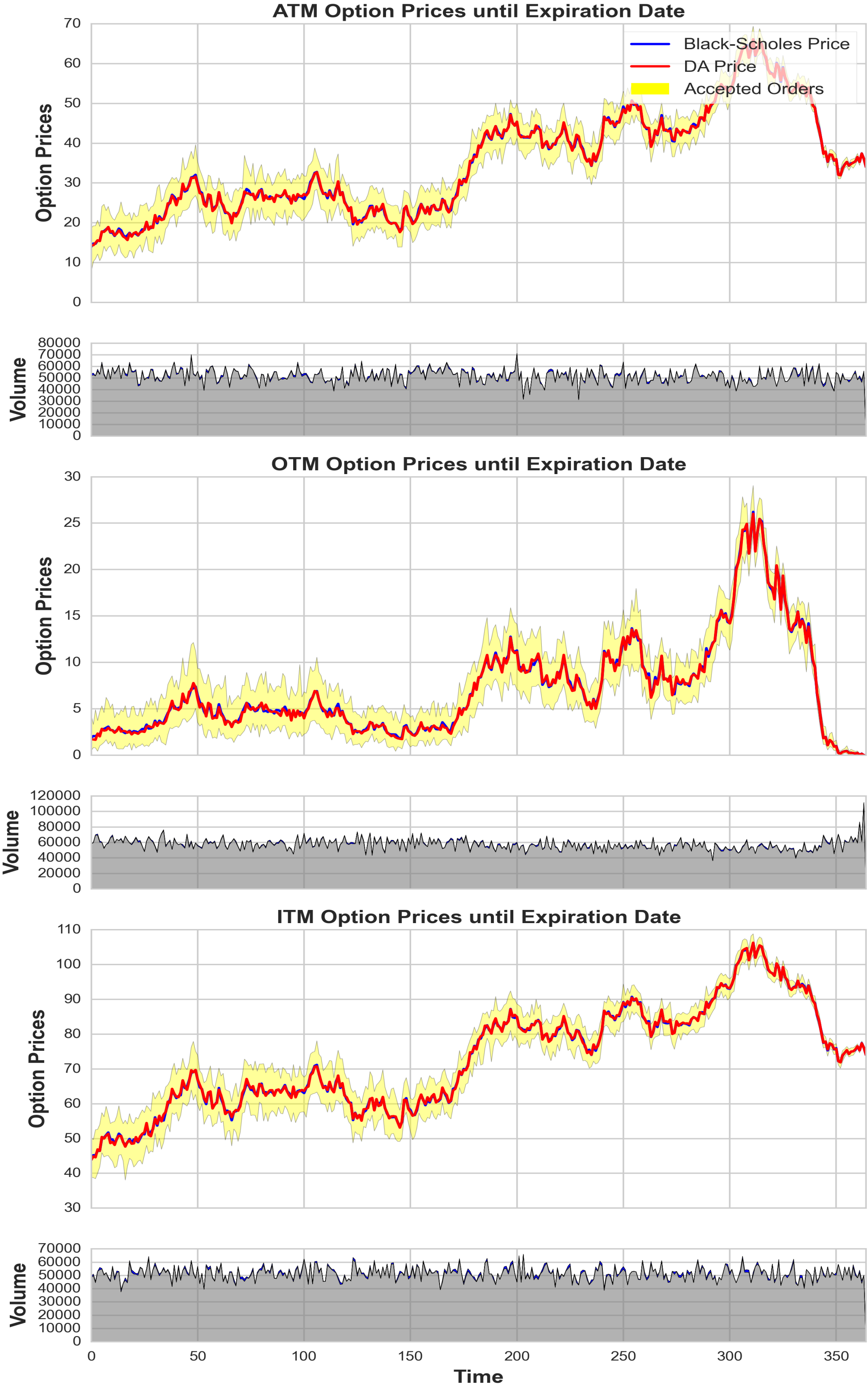


Figure D.8: DA prices of ATM, OTM and ITM options for MC-LIN traders

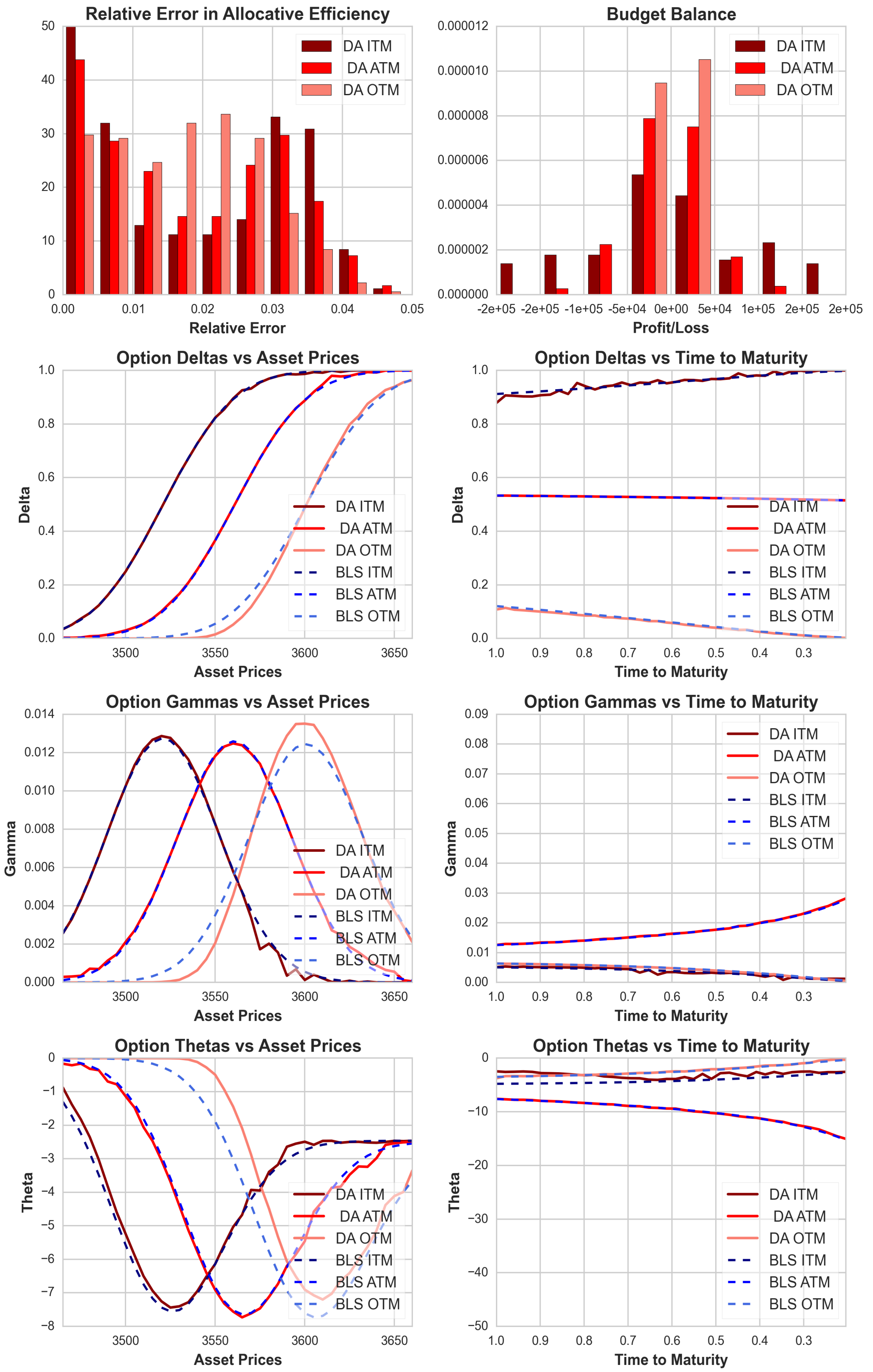


Figure D.9: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for MC-LIN traders

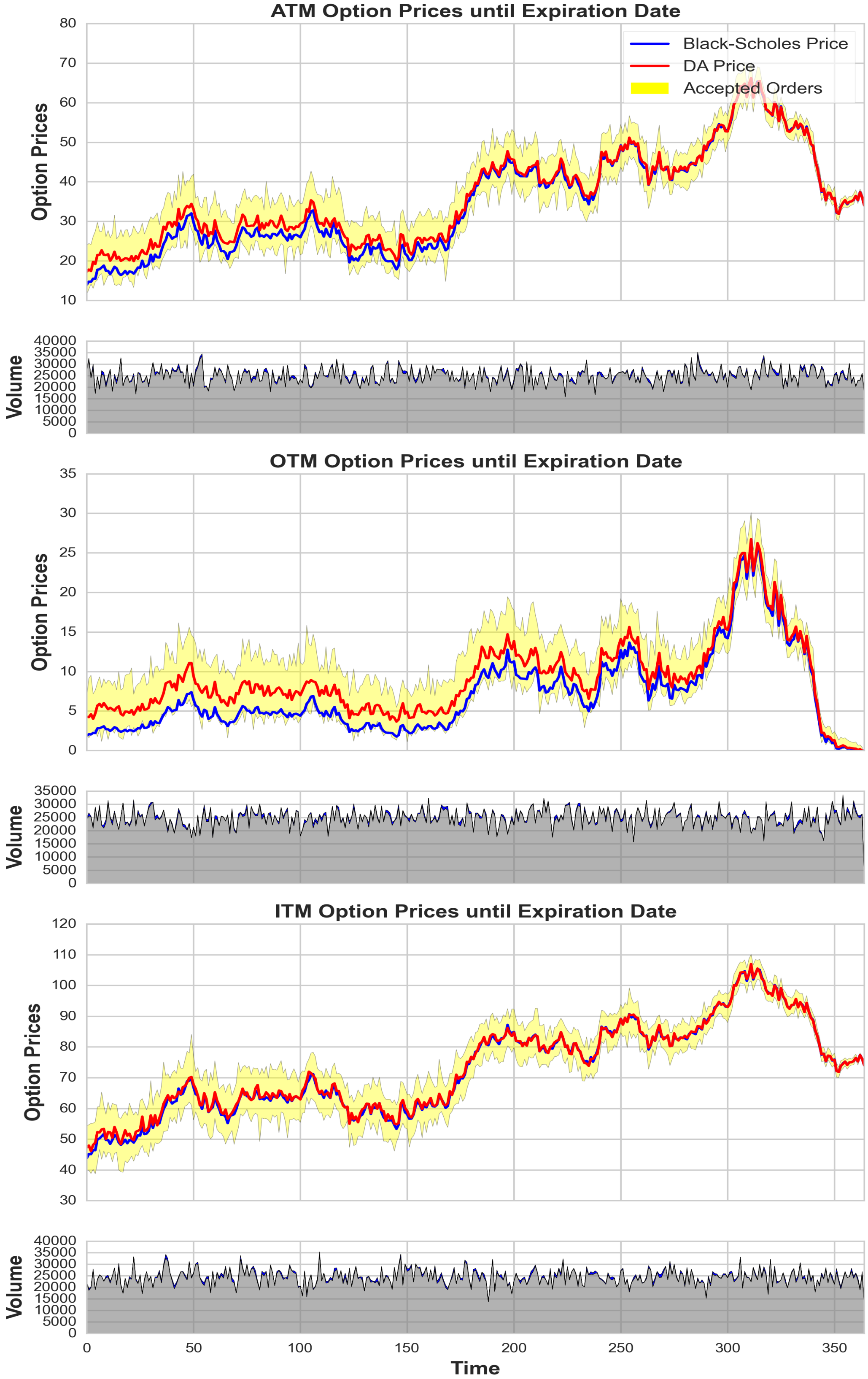


Figure D.10: DA prices of ATM, OTM and ITM options for MC*-RND traders

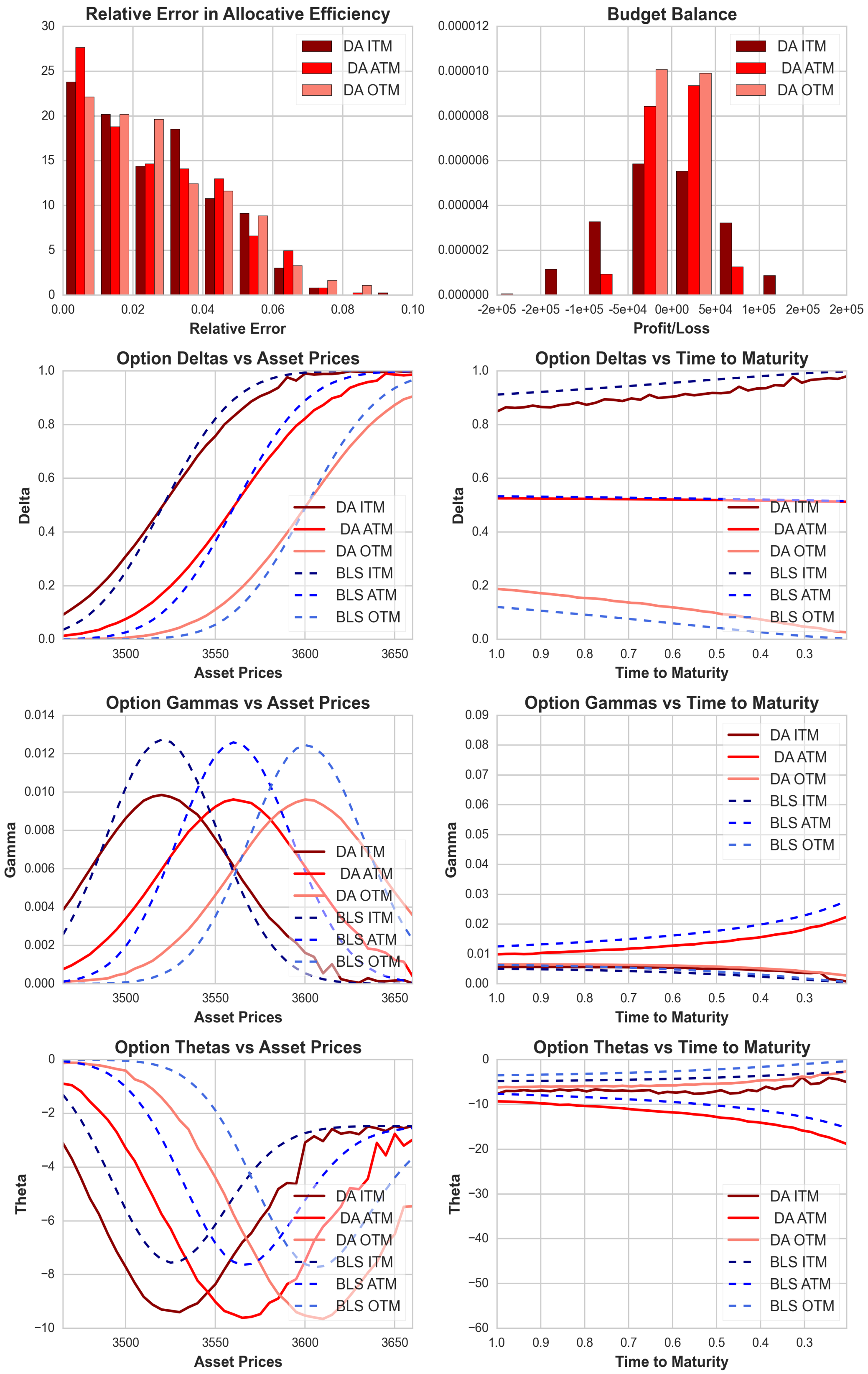


Figure D.11: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for MC*-RND traders

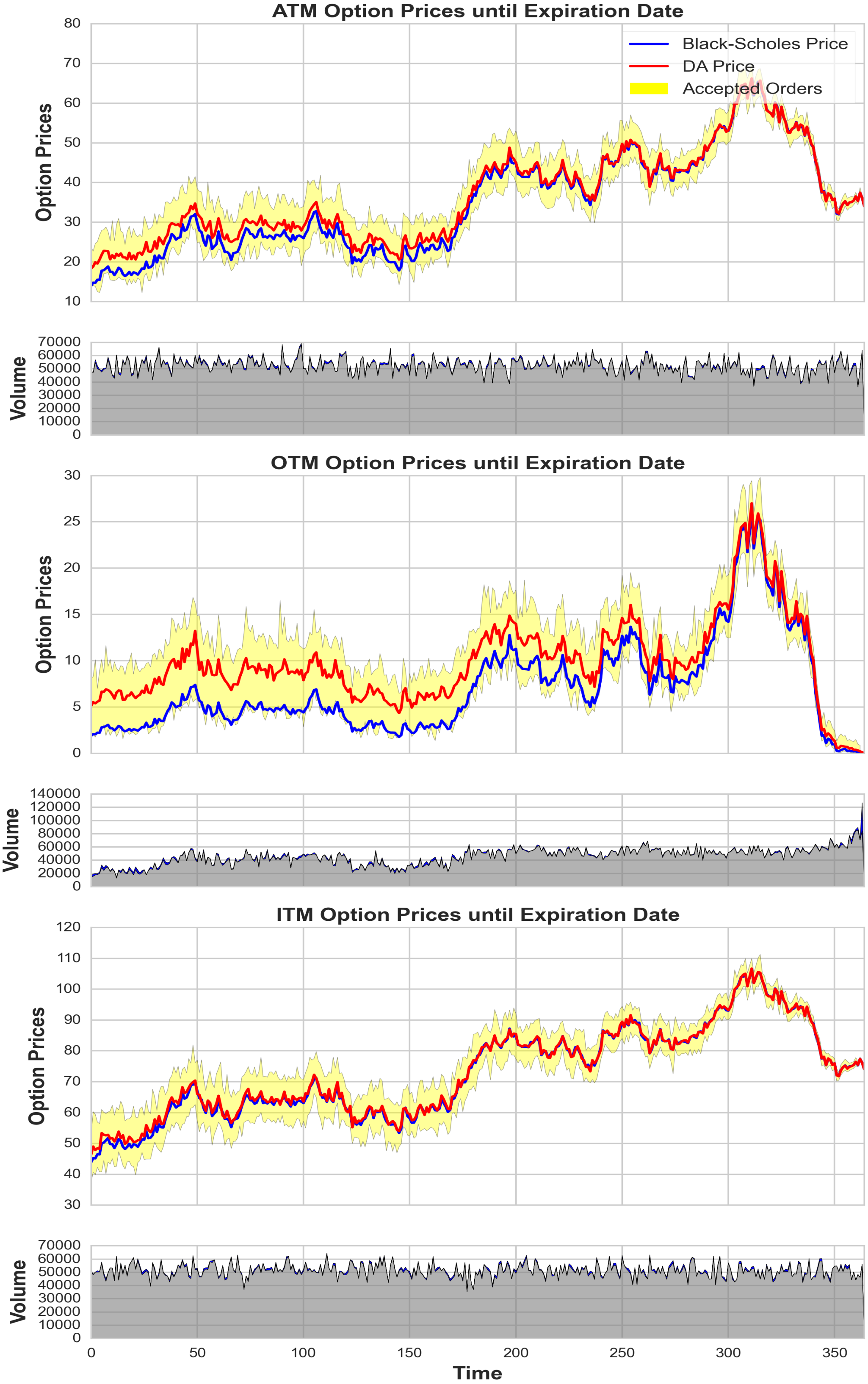


Figure D.12: DA prices of ATM, OTM and ITM options for MC*-LIN traders

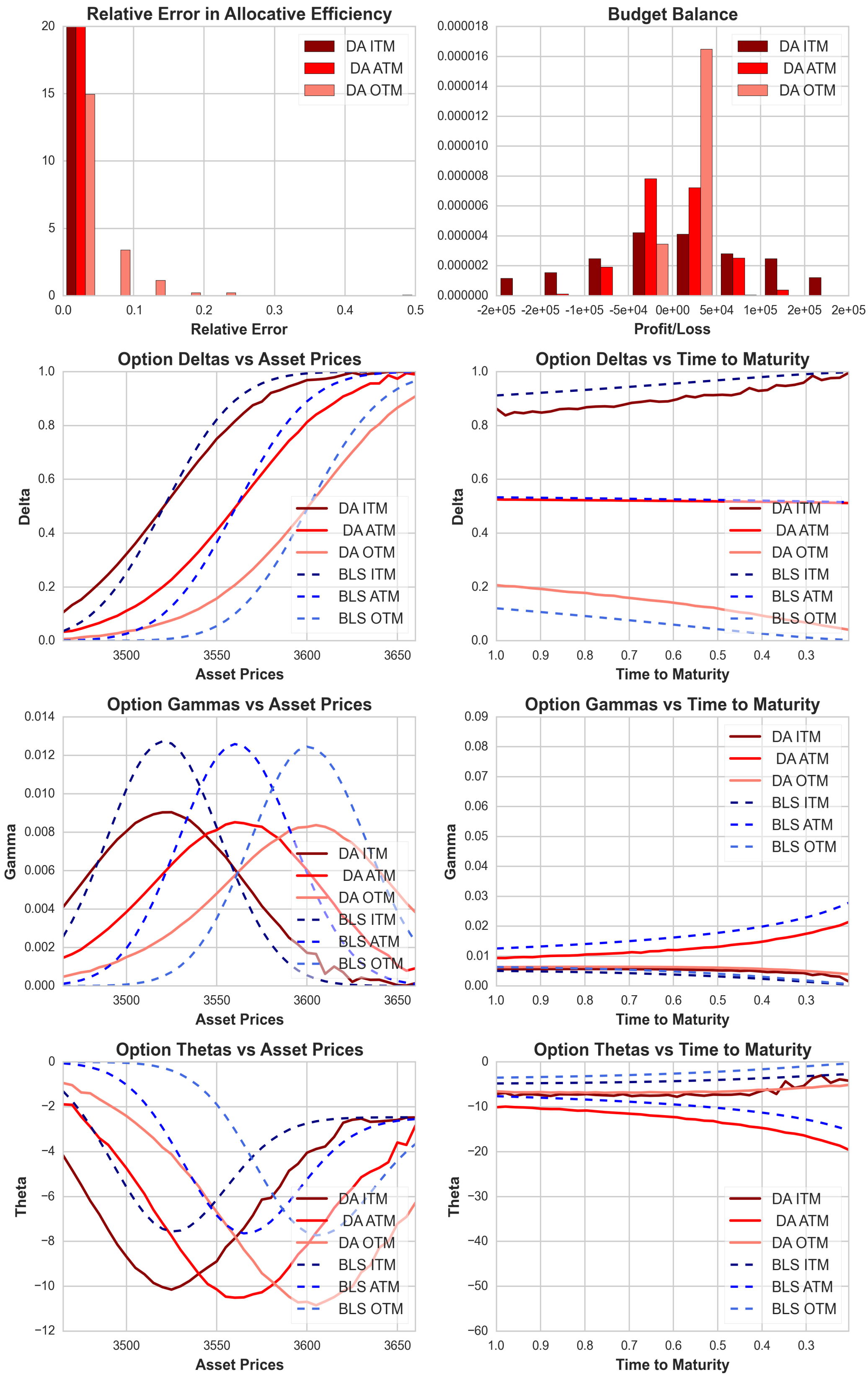


Figure D.13: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for MC*-LIN traders

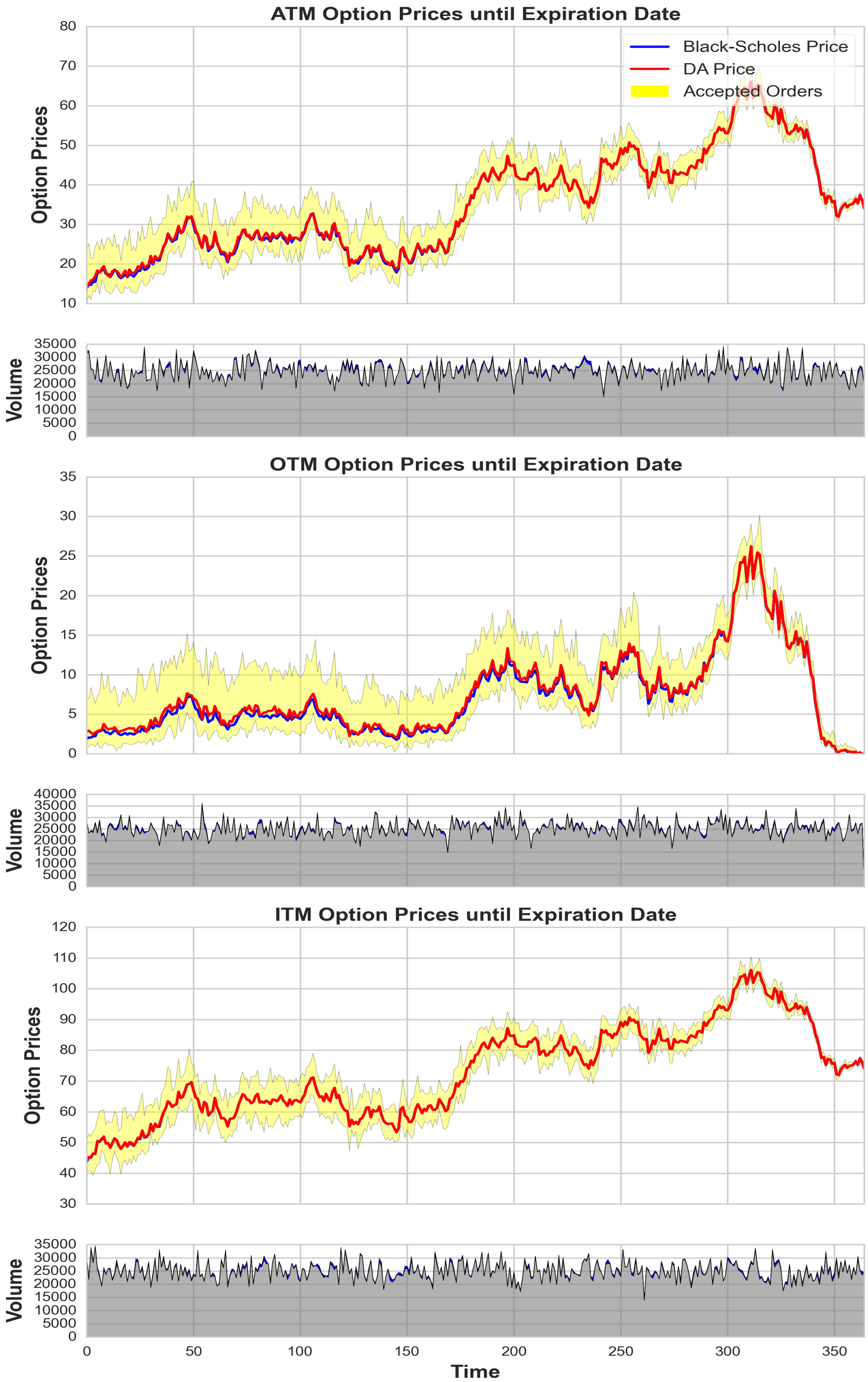


Figure D.14: DA prices of ATM, OTM and ITM options for Mixed Risk Neutral

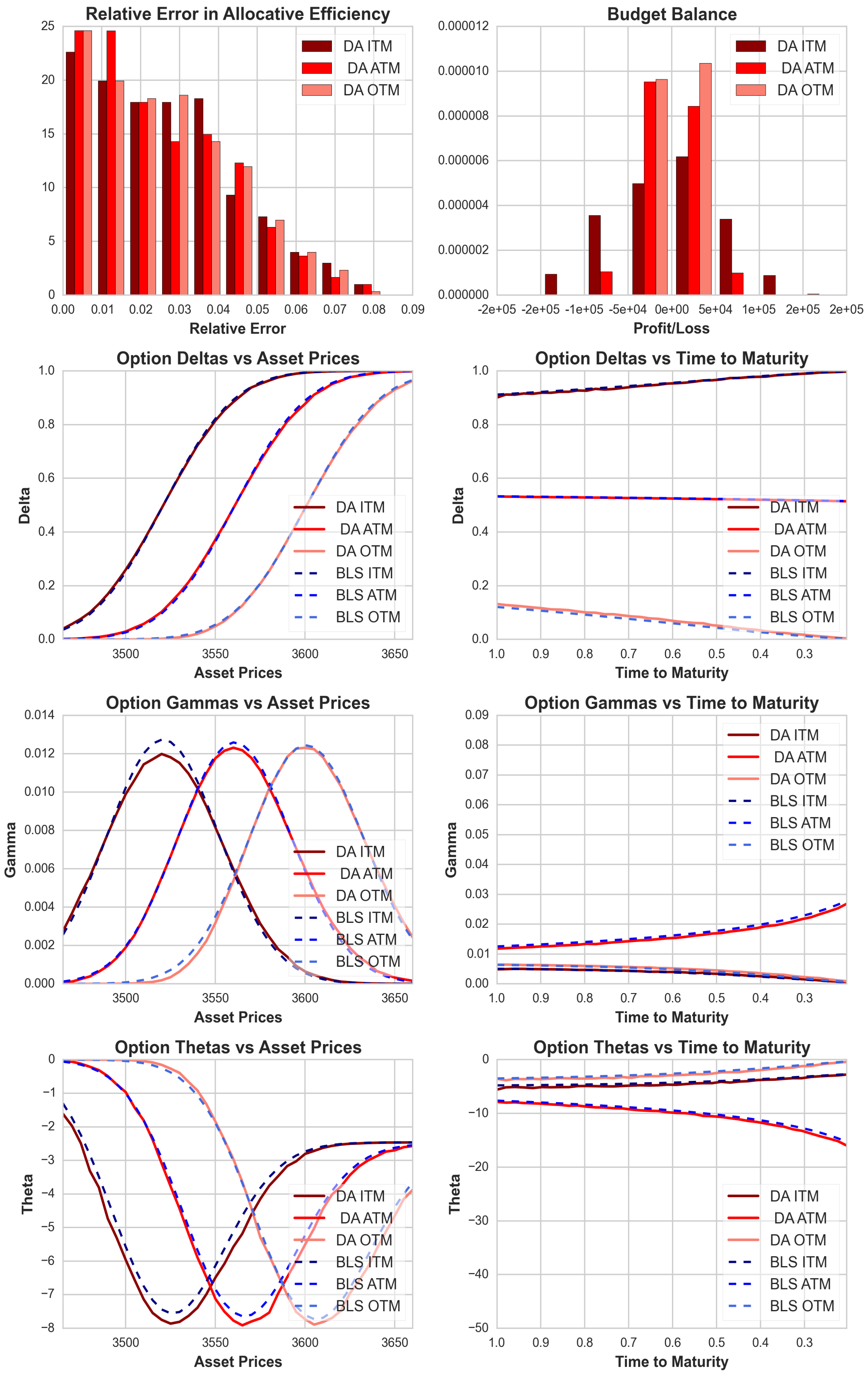


Figure D.15: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for Mixed Risk Neutral

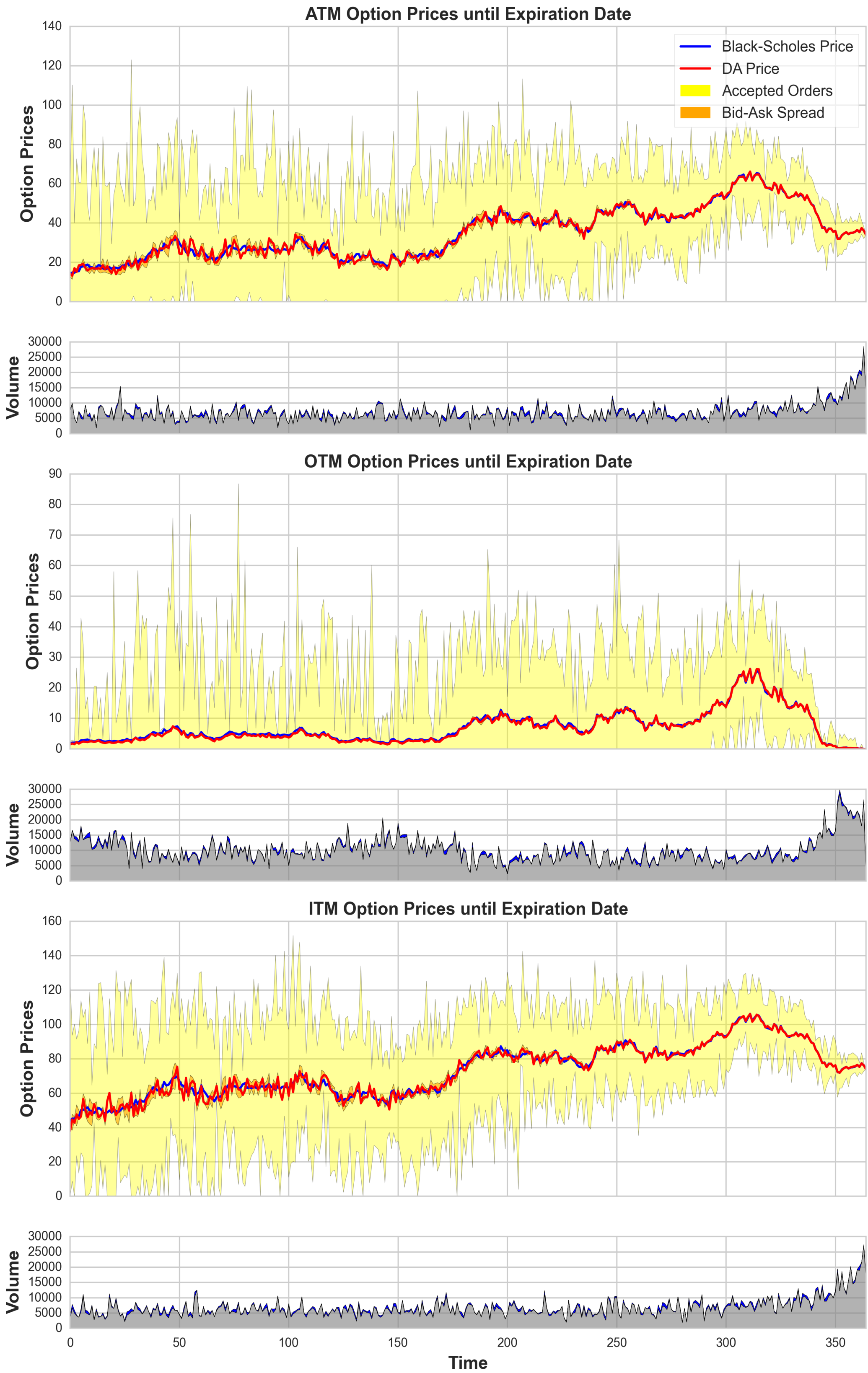


Figure D.16: DA prices of ATM, OTM and ITM options for EXP-RND and ZI-RND traders

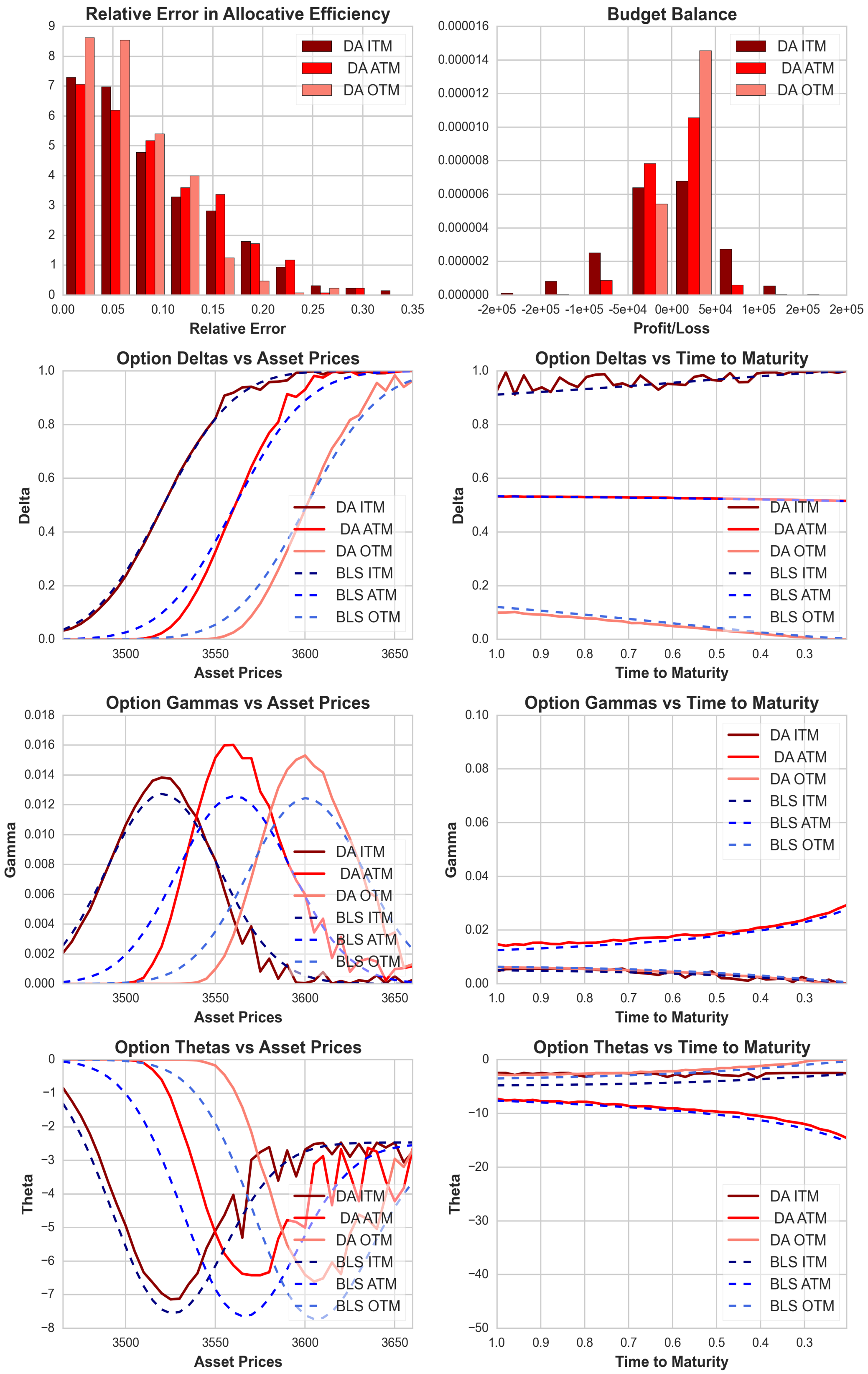


Figure D.17: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for EXP-RND and ZI-RND traders

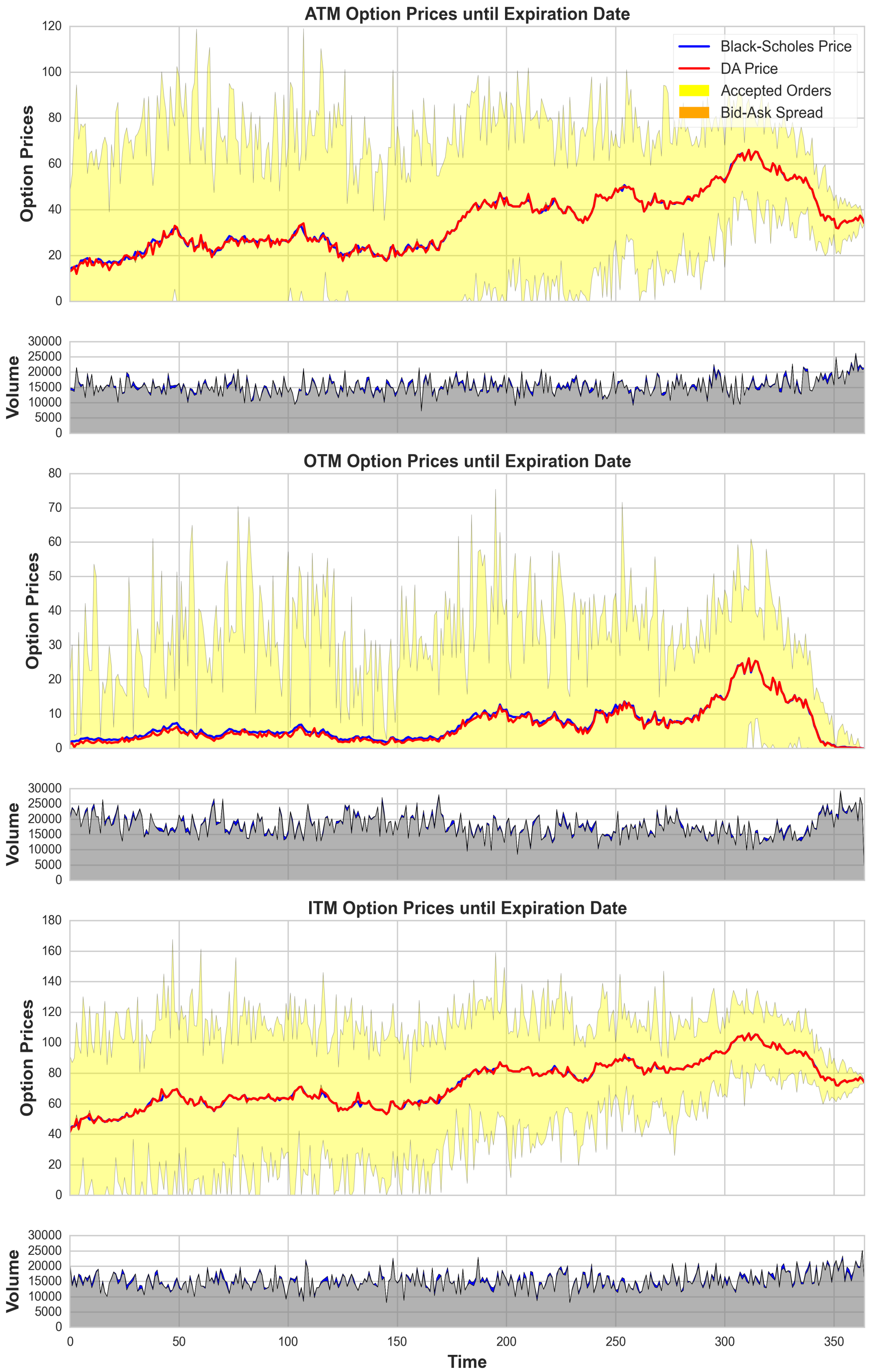


Figure D.18: DA prices of ATM, OTM and ITM options for EXP-RND, ZI-RND and VOL-RND traders

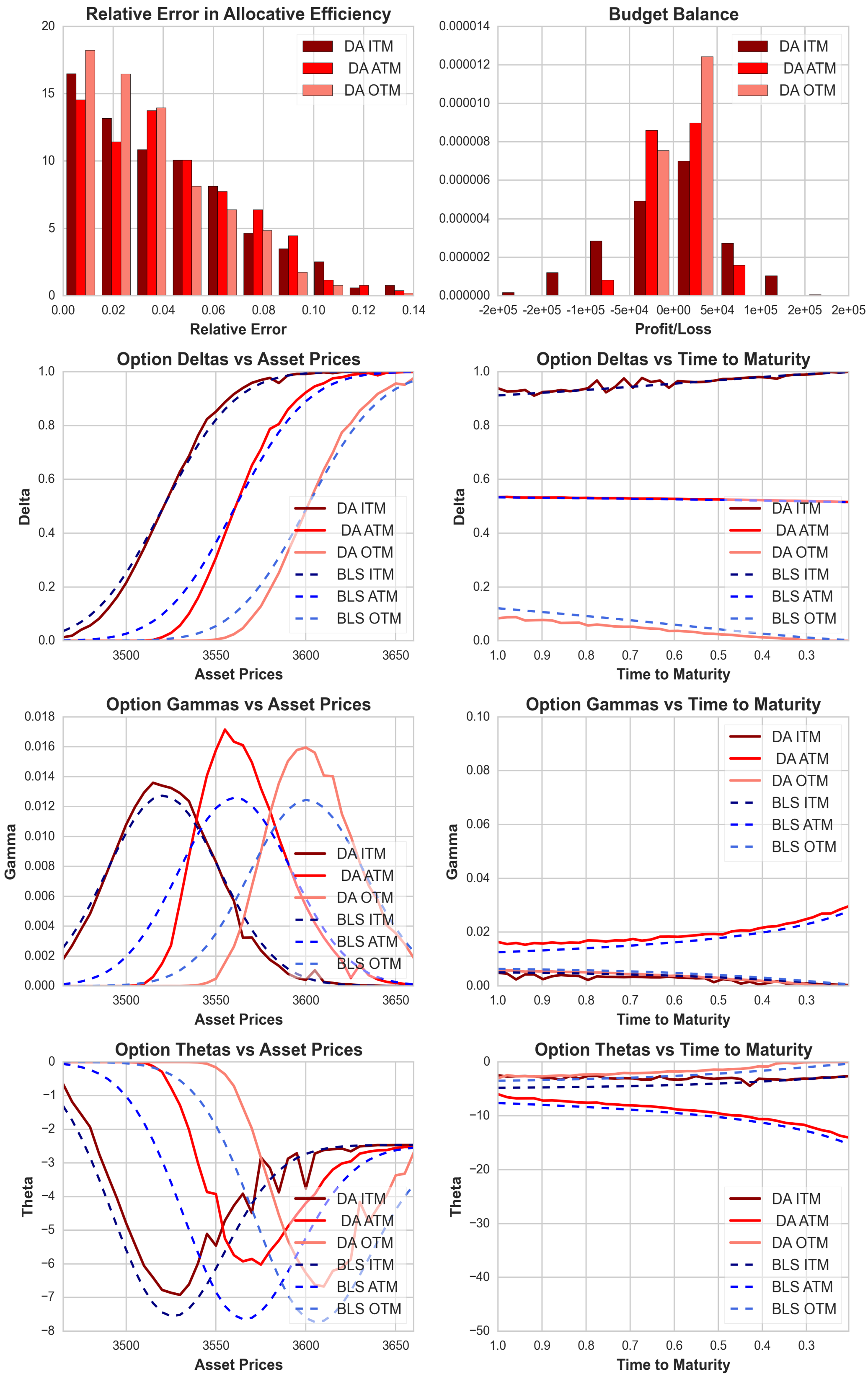


Figure D.19: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for EXP-RND, ZI-RND and VOL-RND traders

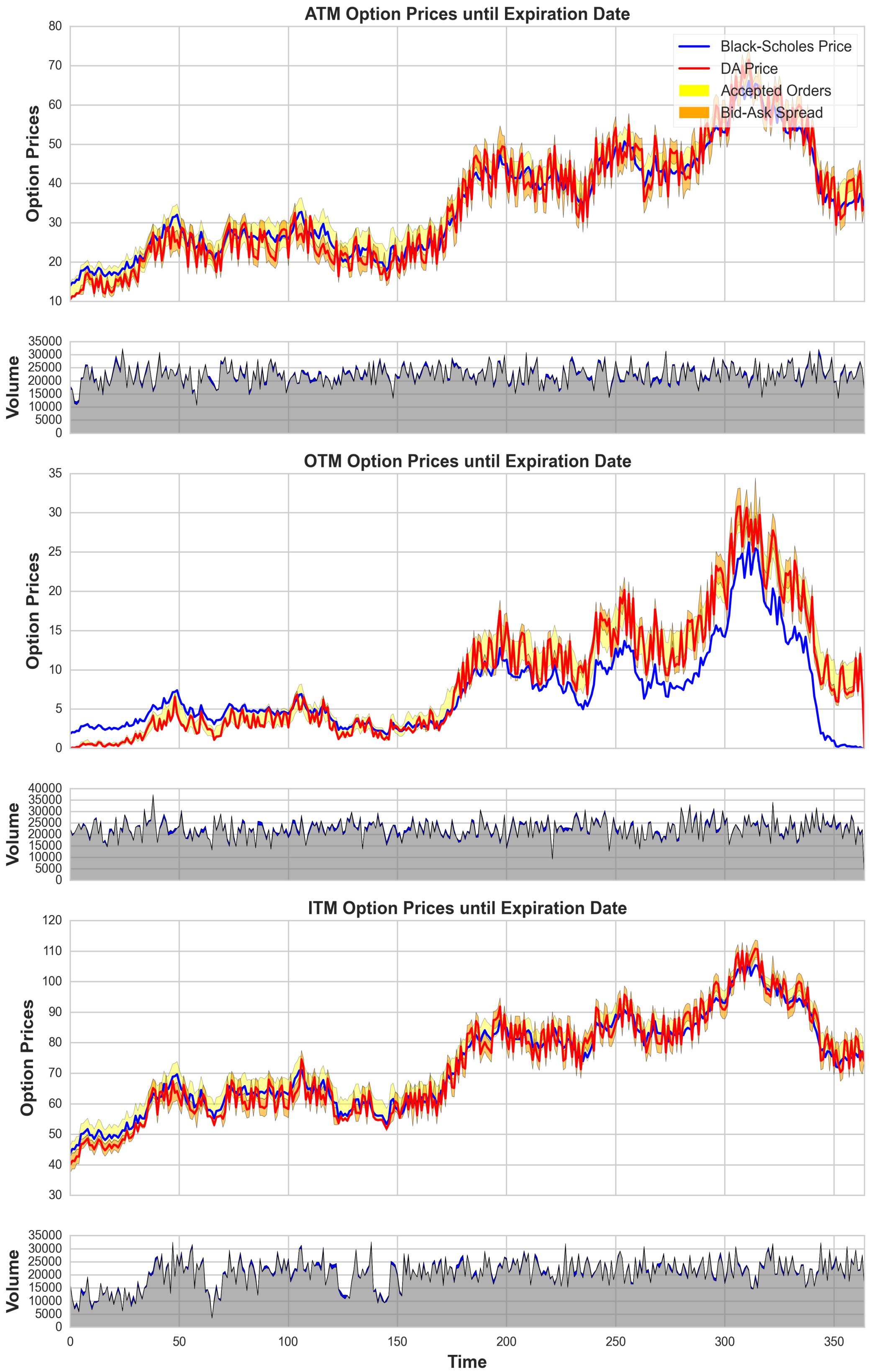


Figure D.20: DA prices of ATM, OTM and ITM options for LMSR Neutral and Non-Neutral traders

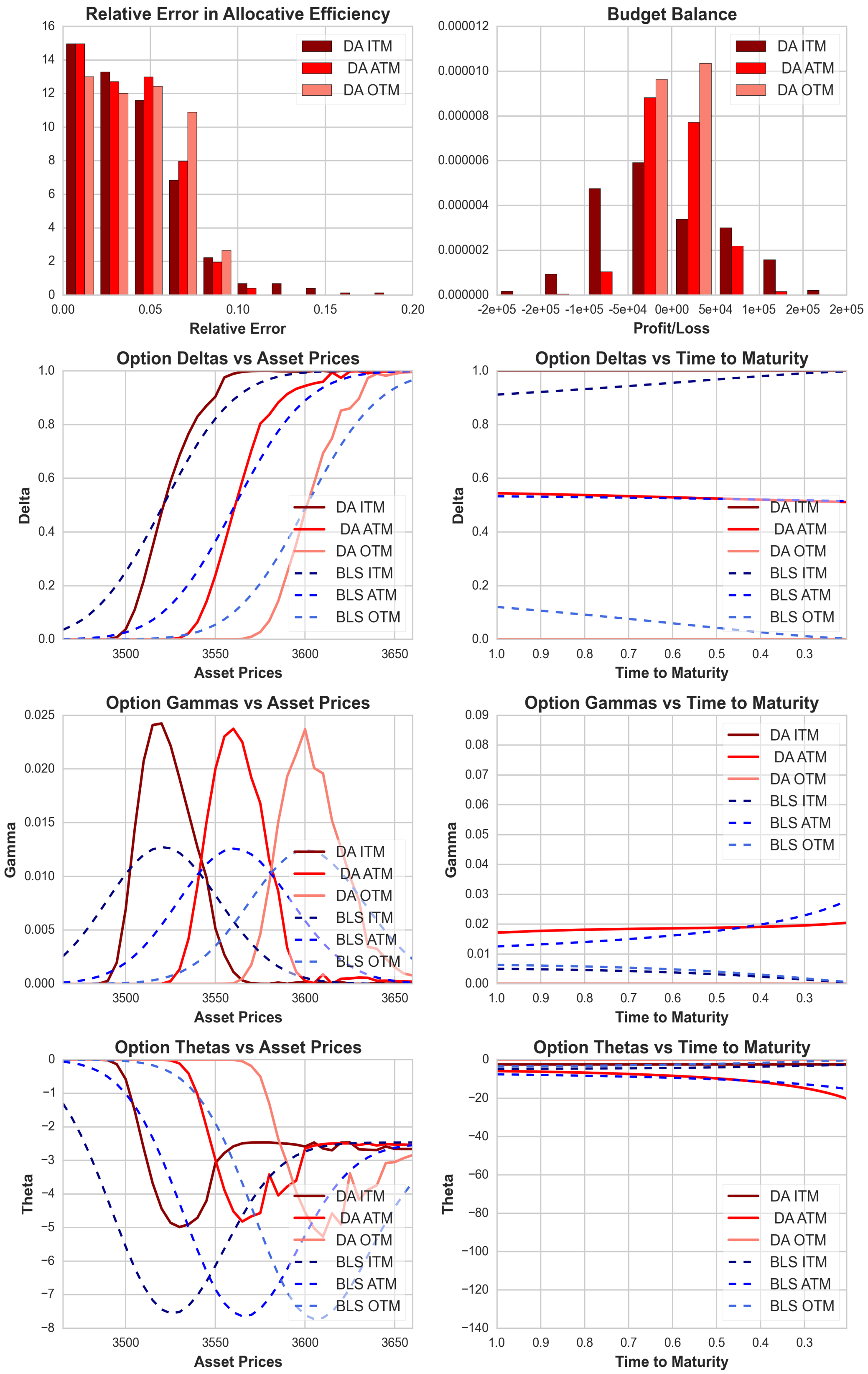


Figure D.21: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for LMSR Neutral and Non-Neutral traders

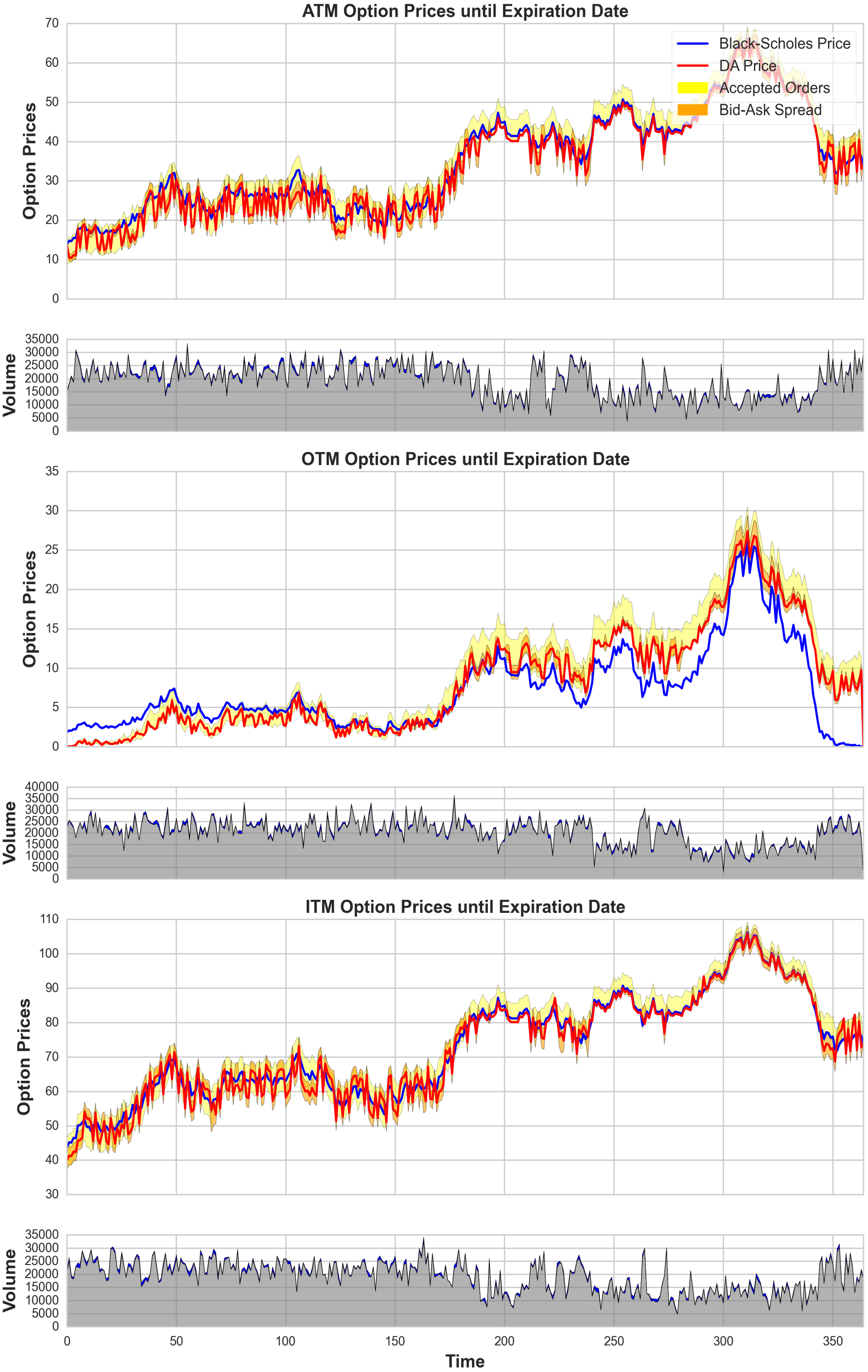


Figure D.22: DA prices of ATM, OTM and ITM options for LMSR All traders

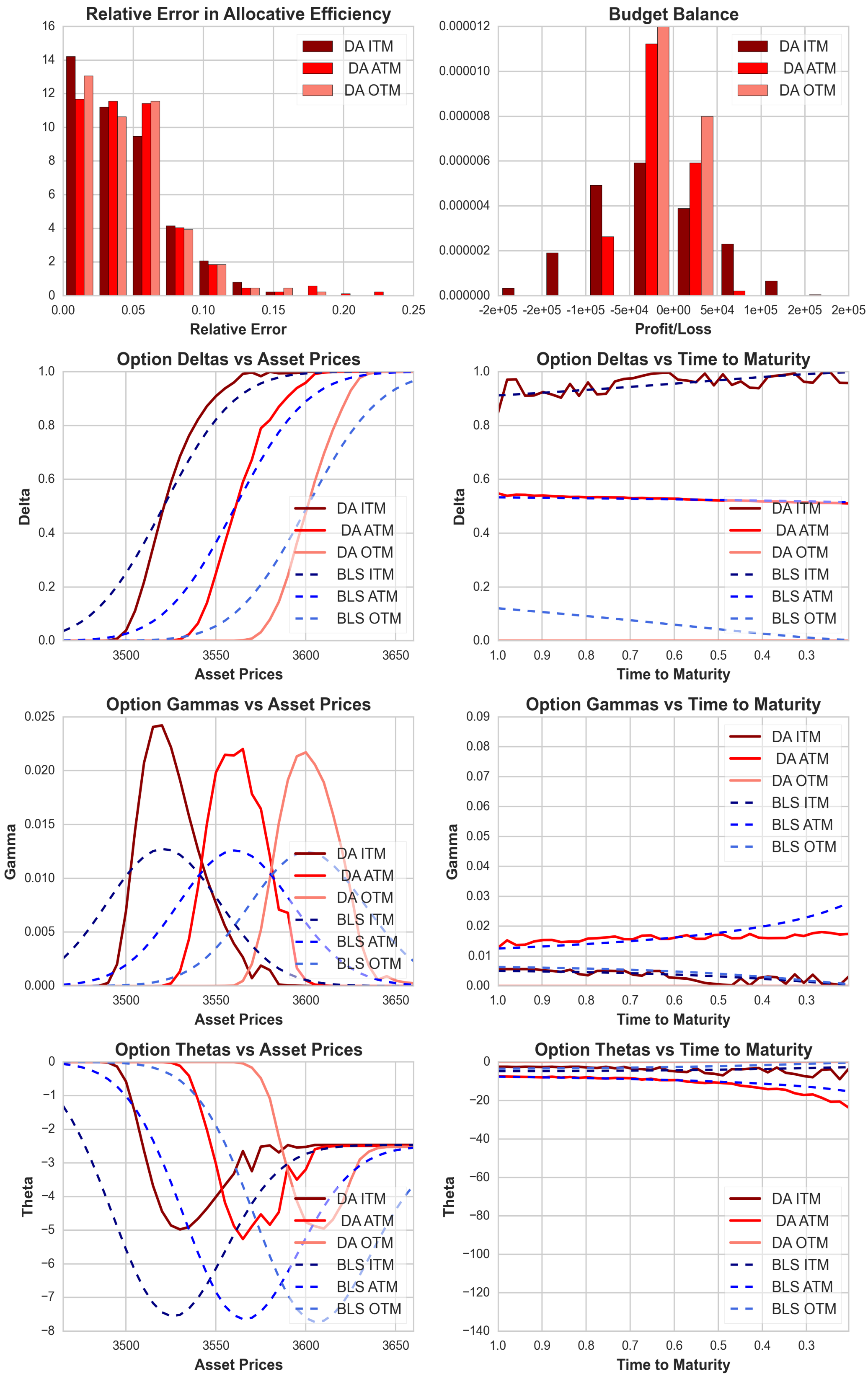


Figure D.23: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for LMSR All traders

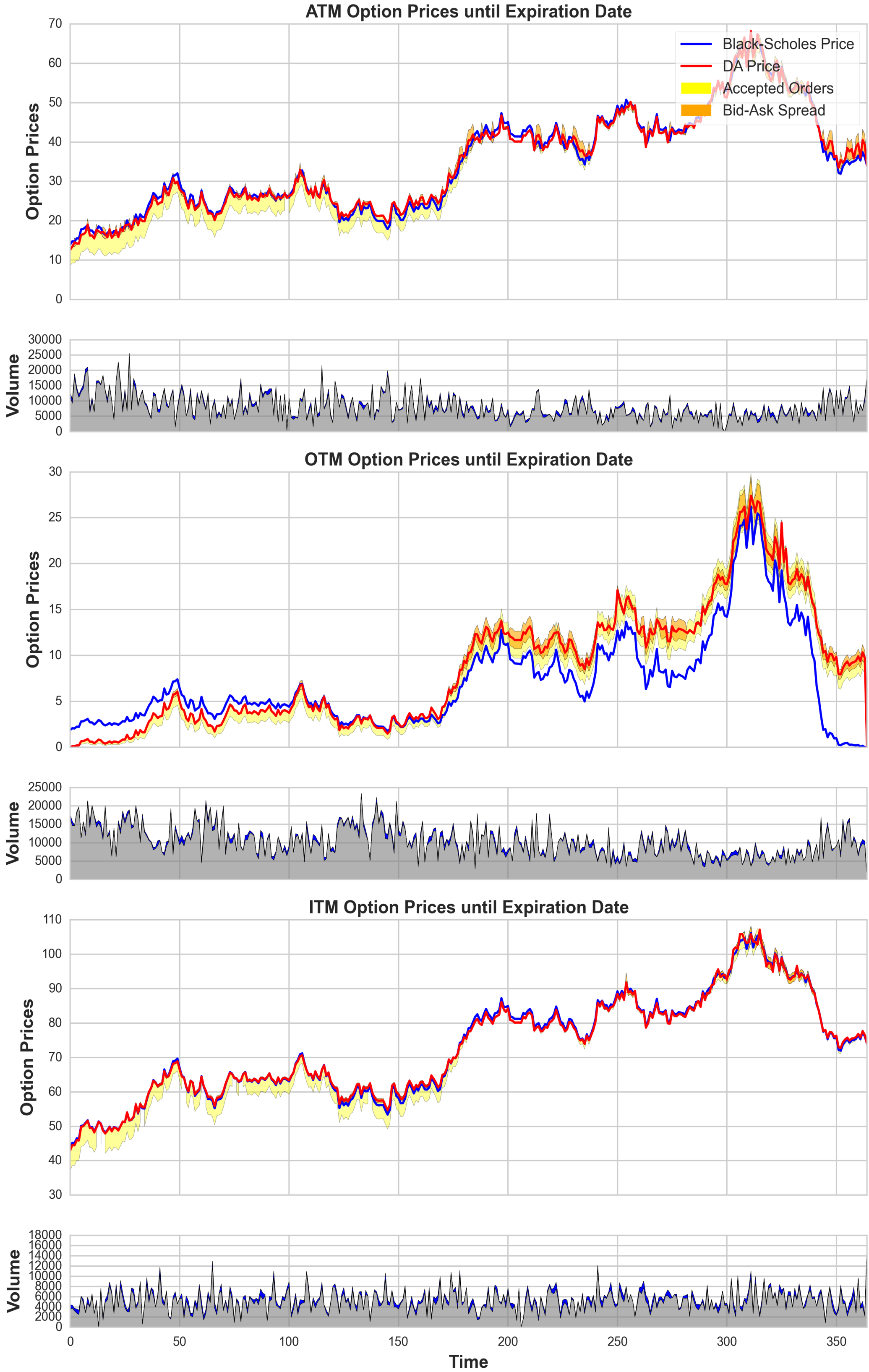


Figure D.24: DA prices of ATM, OTM and ITM options for LMSR More Bull traders

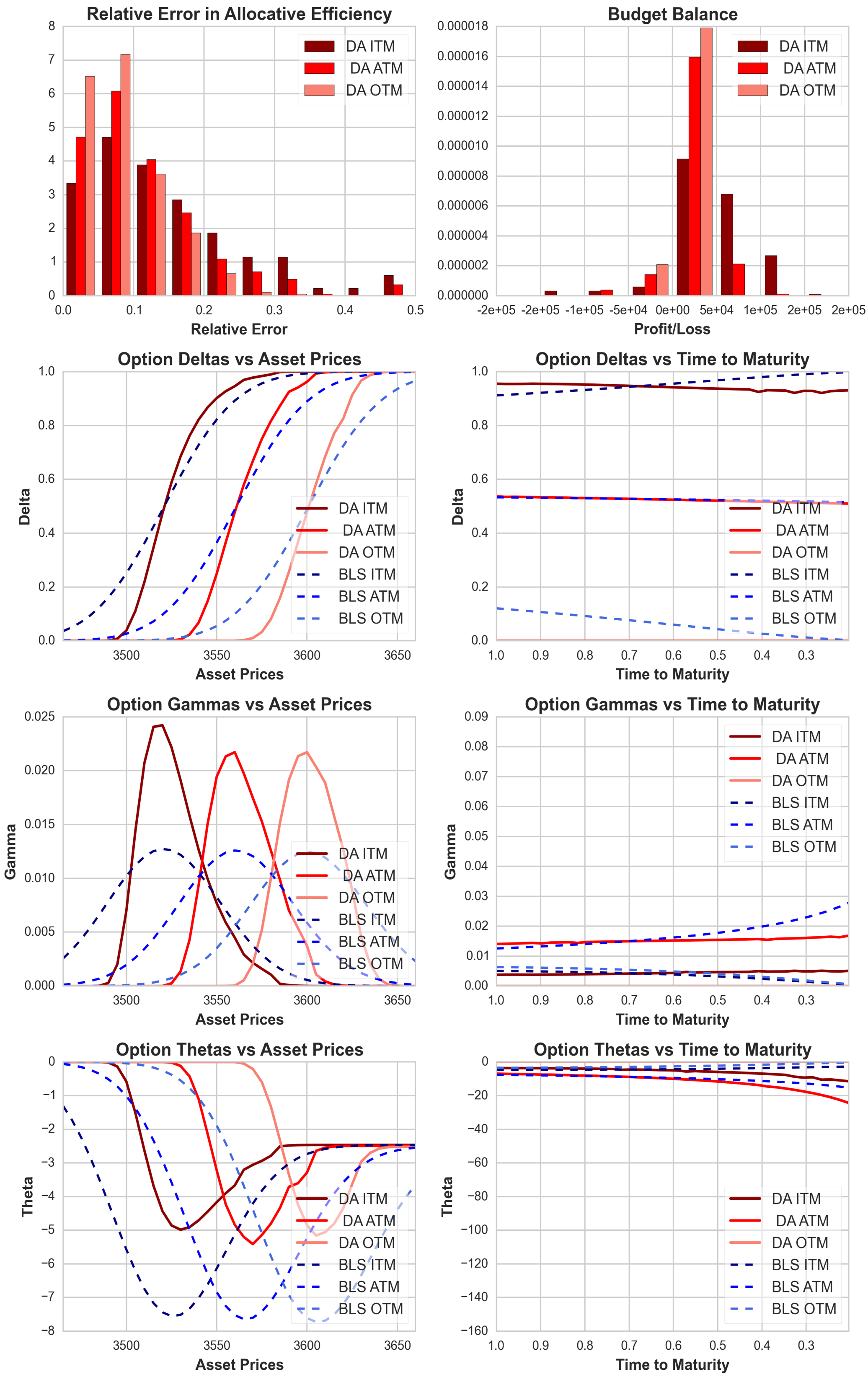


Figure D.25: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for LMSR More Bull traders

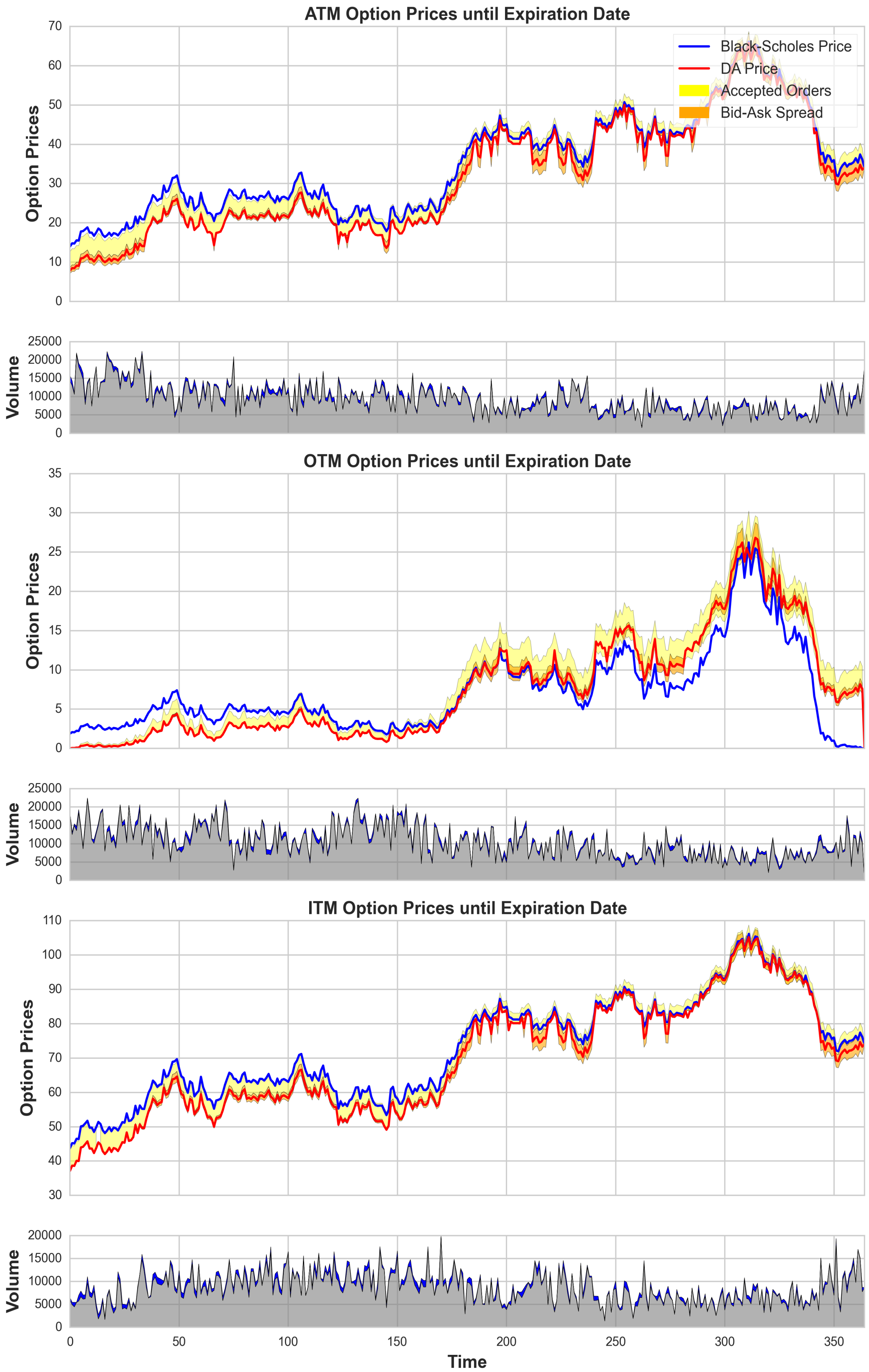


Figure D.26: DA prices of ATM, OTM and ITM options for LMSR More Bear traders

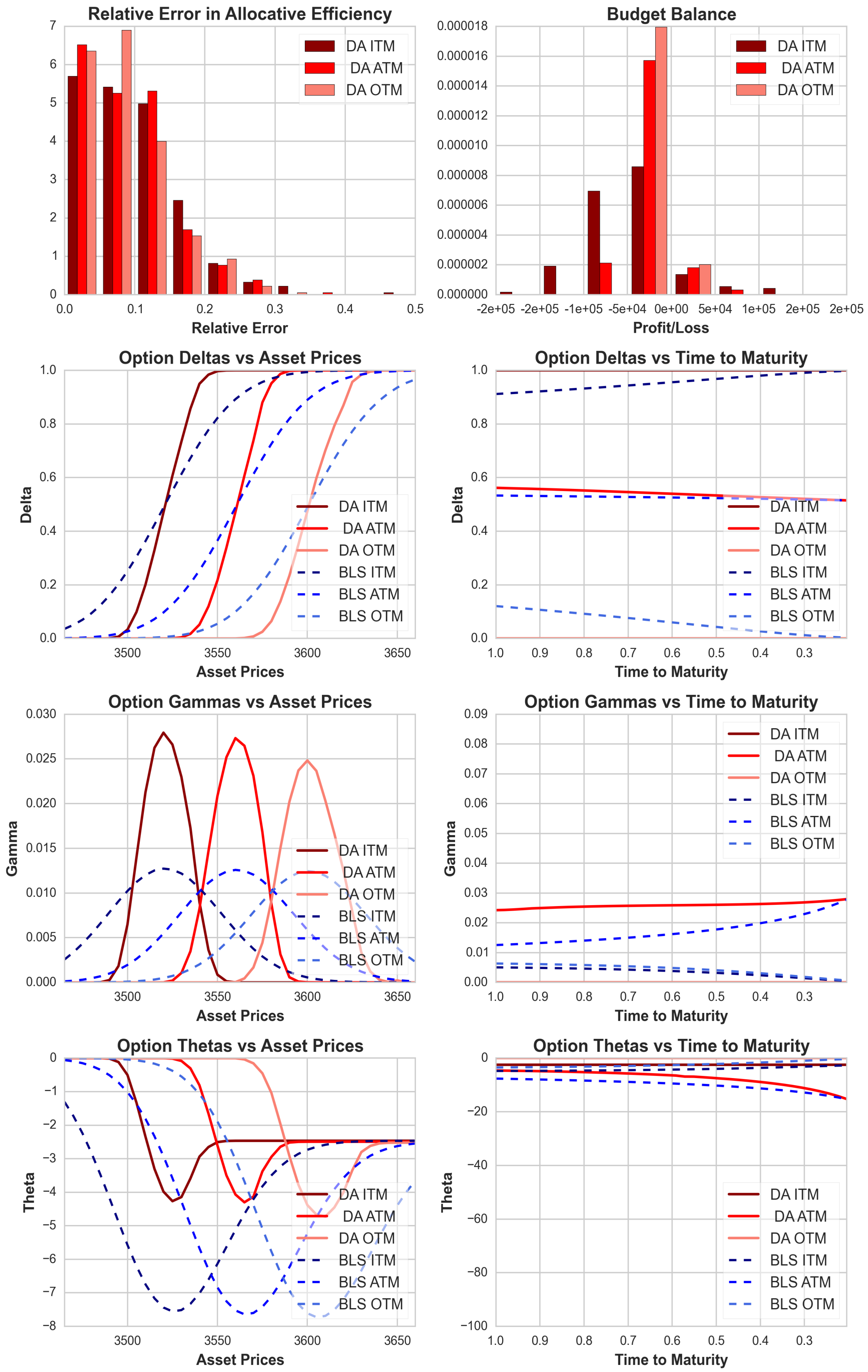


Figure D.27: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for LMSR More Bear traders

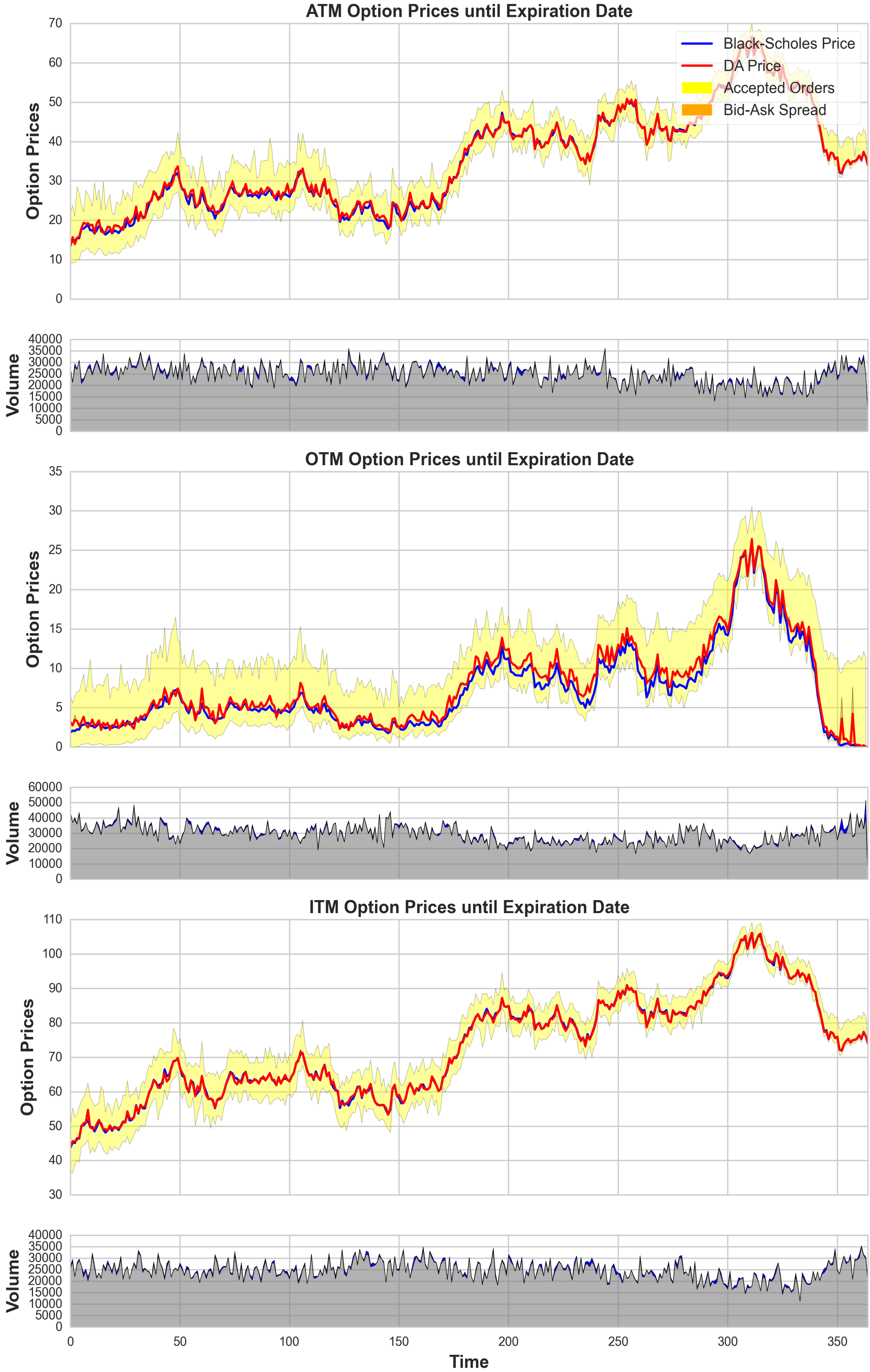


Figure D.28: DA prices of ATM, OTM and ITM options for Mixed All traders

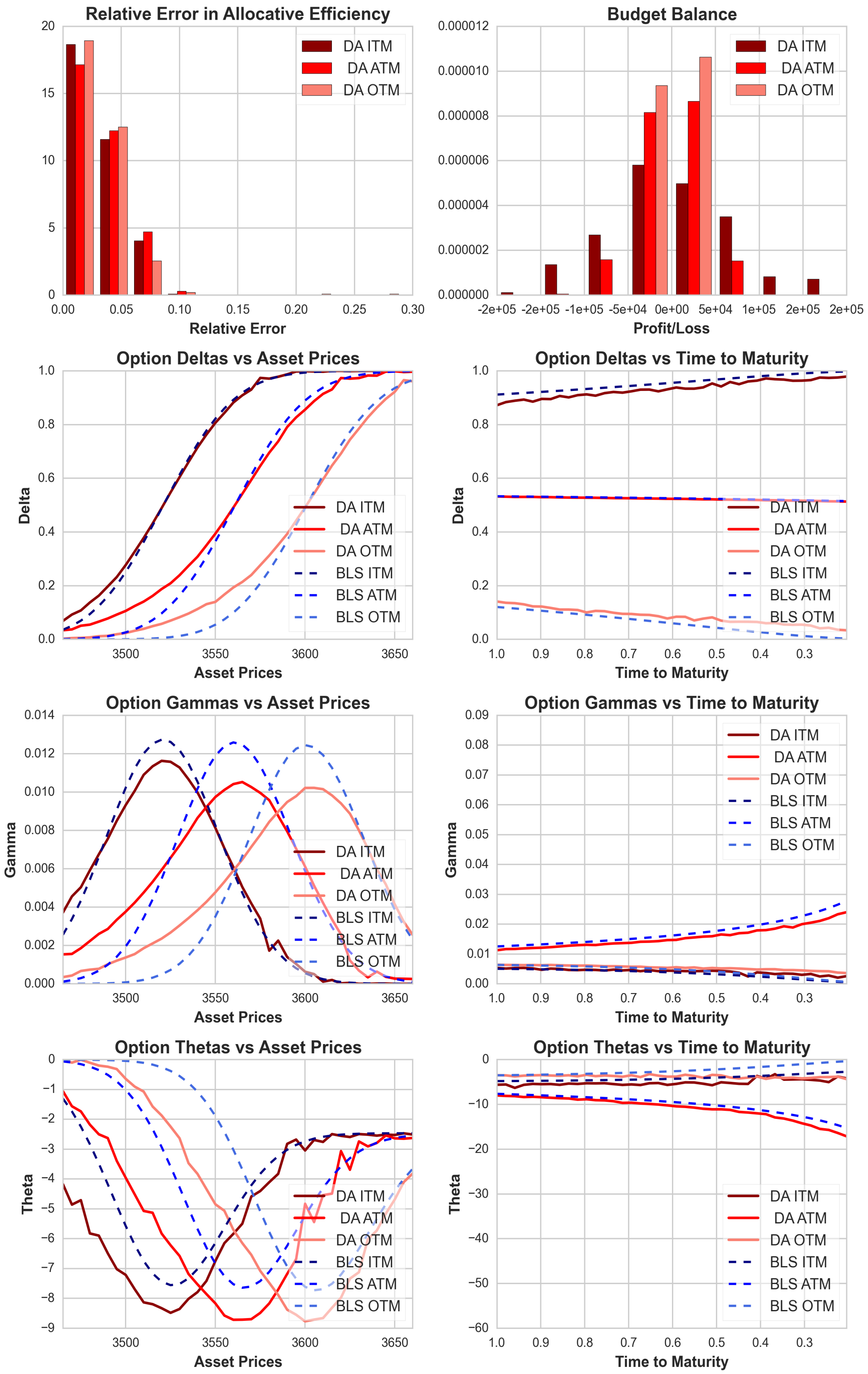


Figure D.29: DA efficiency error, budget-balance and Greeks of ATM, OTM and ITM options for Mixed All traders

Appendix E

Online DA Simulation Results

Trading Algorithms
ZIP - Zero Intelligence Plus
GD - Gjerstad-Dickhaut
GAR - Garman's Inventory-based Model
COP - Copeland-Galai Information-based Model
INF - Informed Trader
MC - Monte-Carlo trader

Table E.1: Nomenclature for naming agents

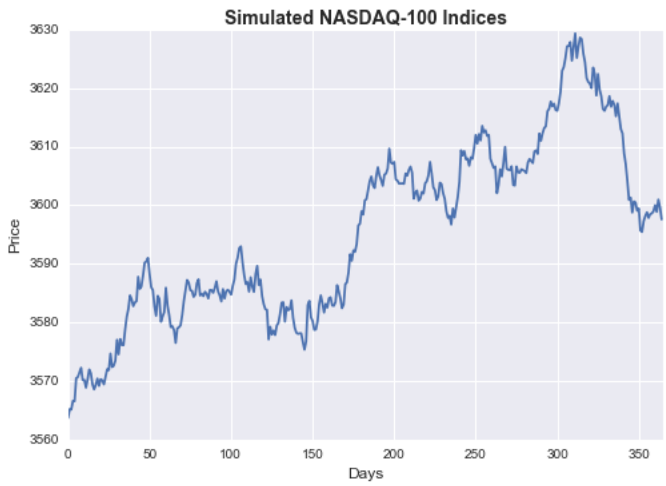


Figure E.1: Simulated NASDAQ-100 Indices

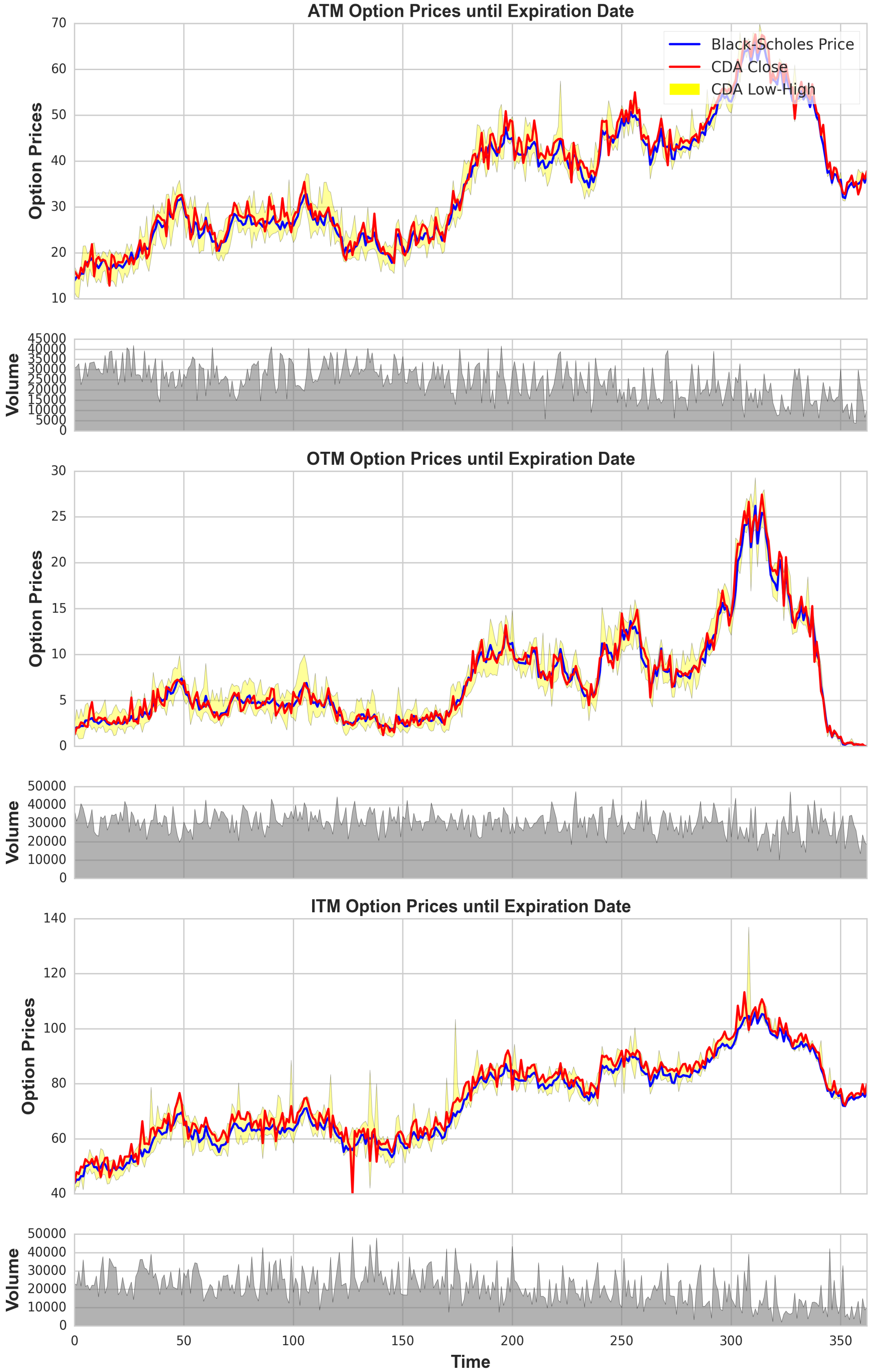


Figure E.2: CDA prices of ATM, OTM and ITM options for ZIP traders

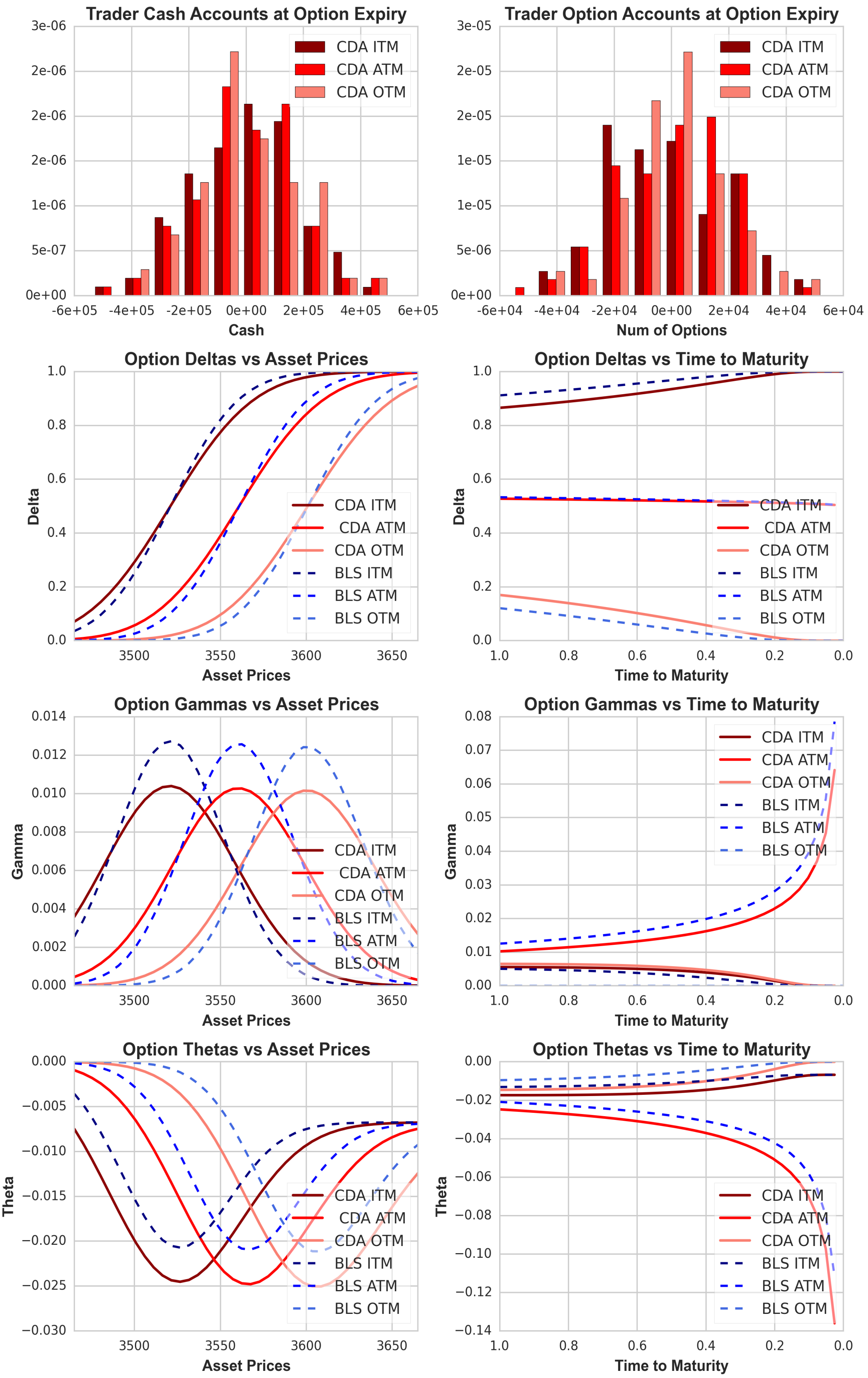


Figure E.3: CDA inventory distribution and Greeks of ATM, OTM and ITM options for ZIP traders

Figure E.4: CDA prices of ATM, OTM and ITM options for ZIP and GD traders

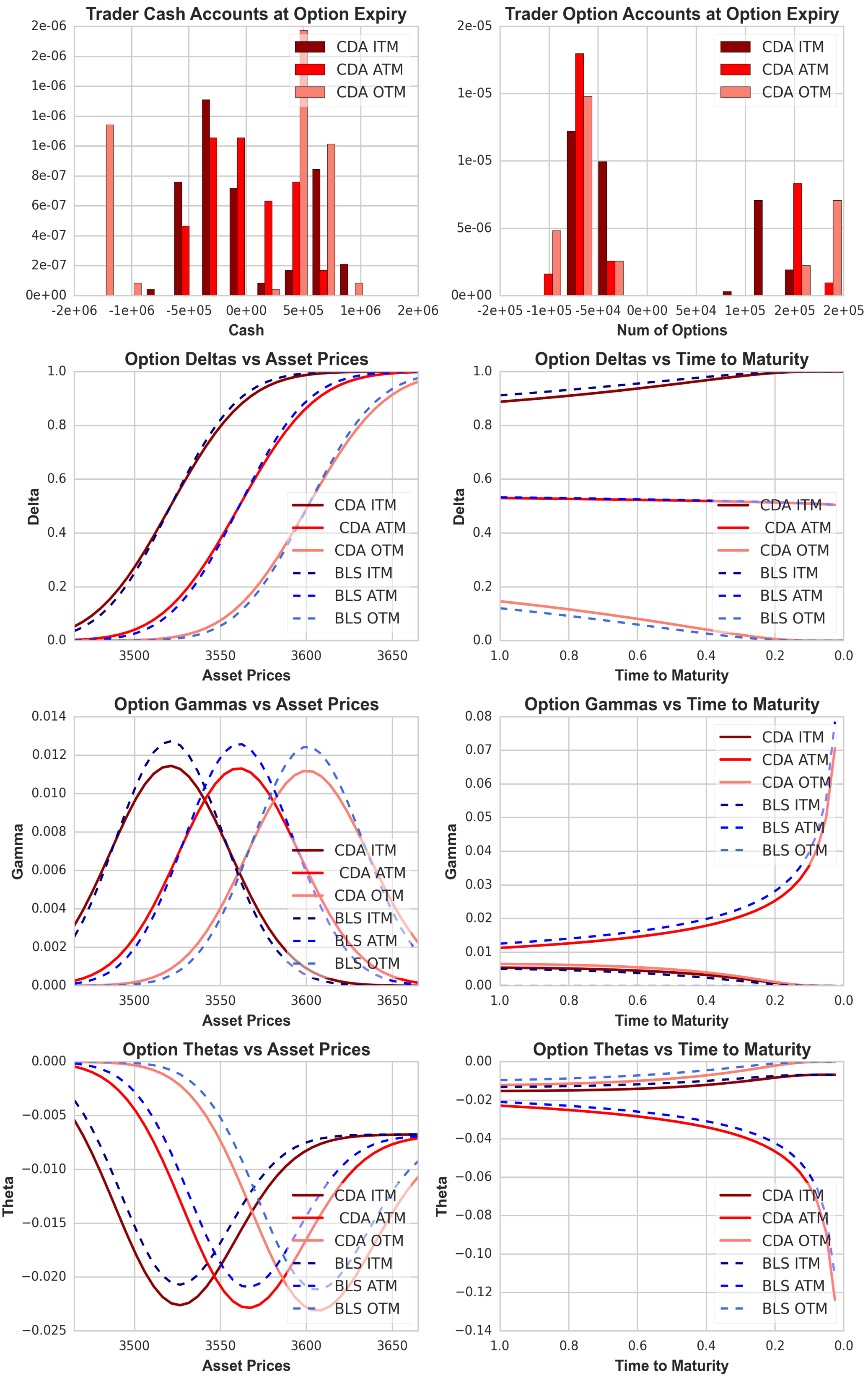


Figure E.5: CDA inventory distribution and Greeks of ATM, OTM and ITM options for ZIP and GD traders

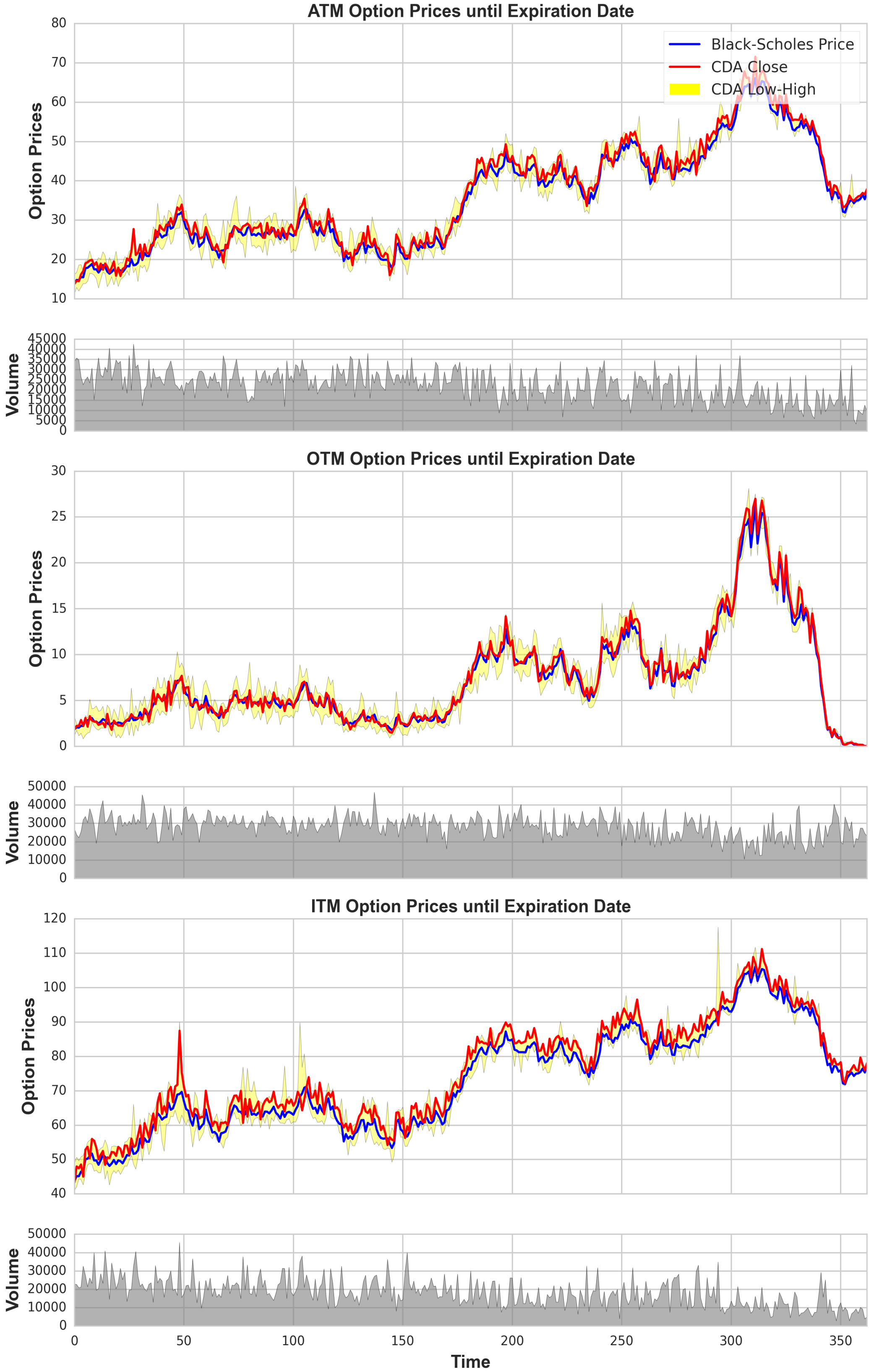


Figure E.6: CDA prices of ATM, OTM and ITM options for GAR and ZIP traders

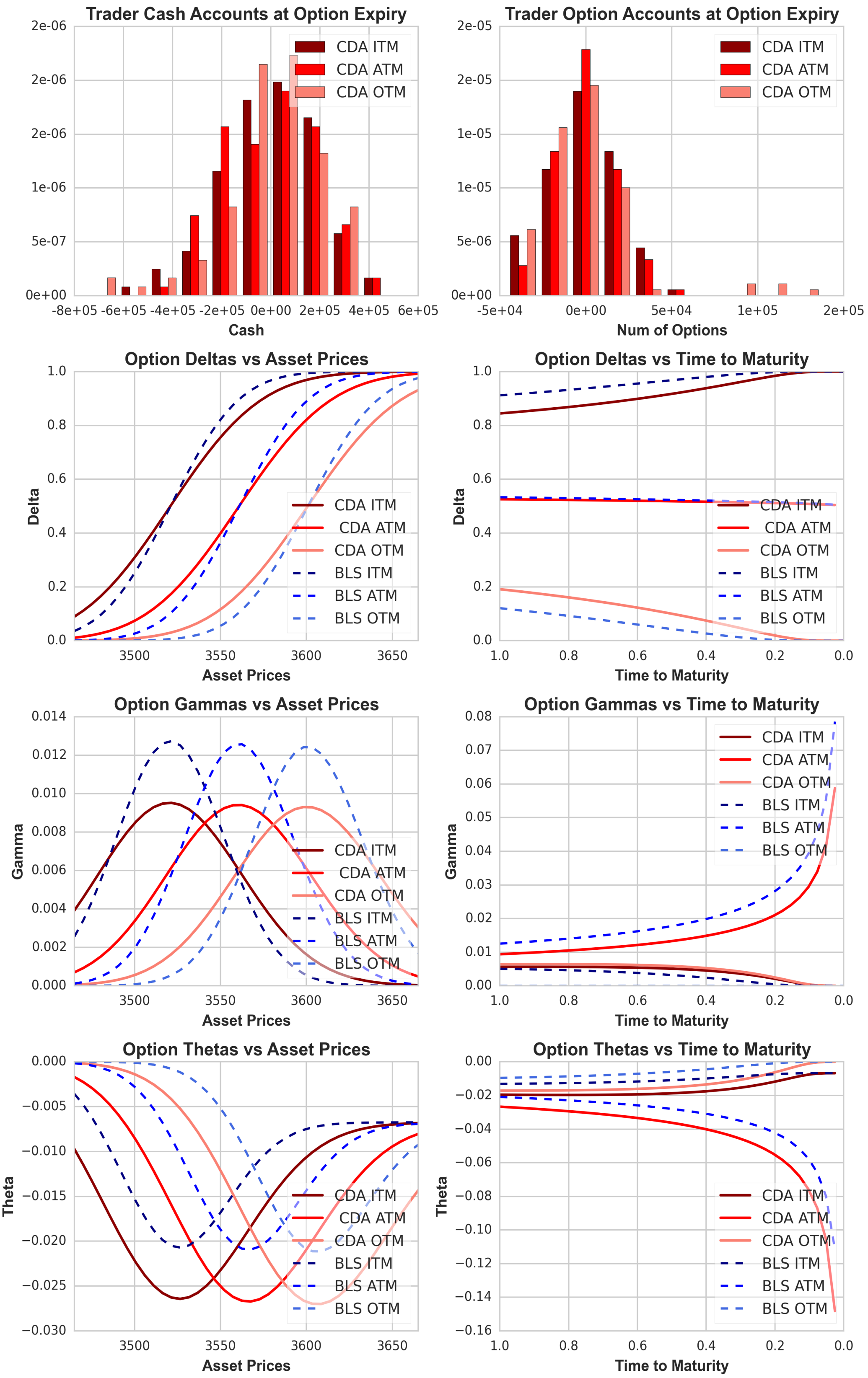


Figure E.7: CDA inventory distribution and Greeks of ATM, OTM and ITM options for GAR and ZIP traders

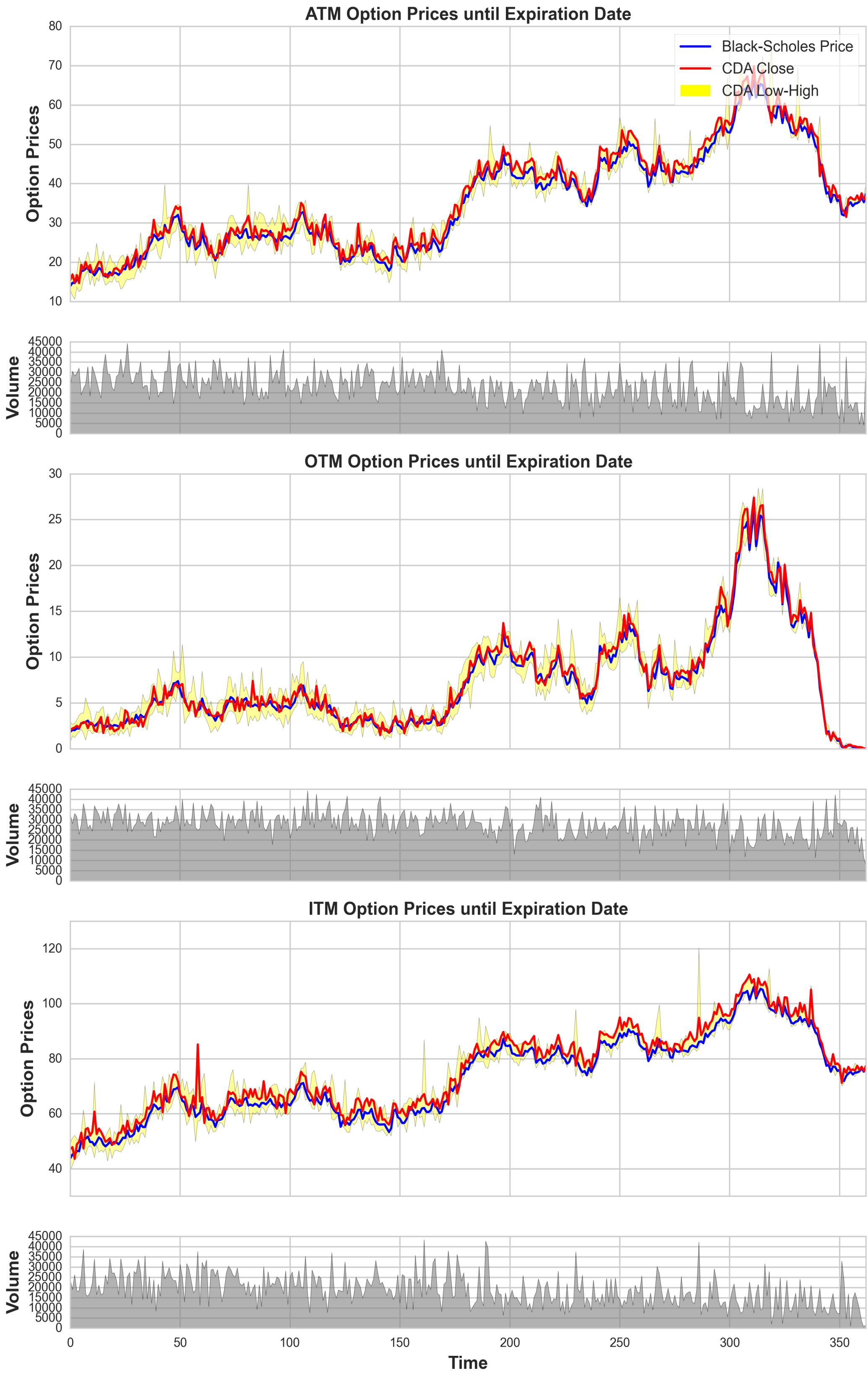


Figure E.8: CDA prices of ATM, OTM and ITM options for COP, INF and ZIP traders

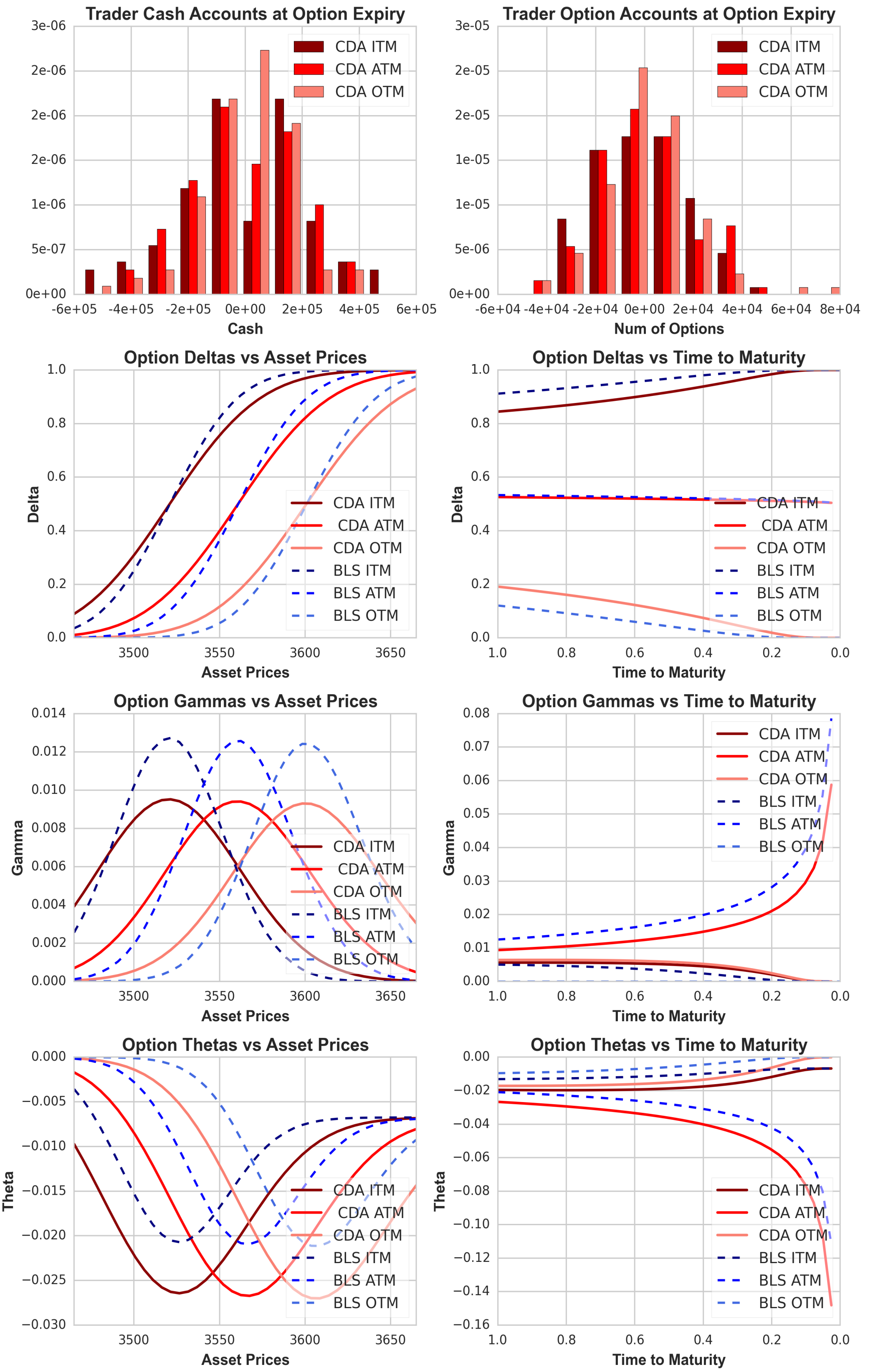


Figure E.9: CDA inventory distribution and Greeks of ATM, OTM and ITM options for COP, INF and ZIP traders

Figure E.10: CDA prices of ATM, OTM and ITM options for mixed traders

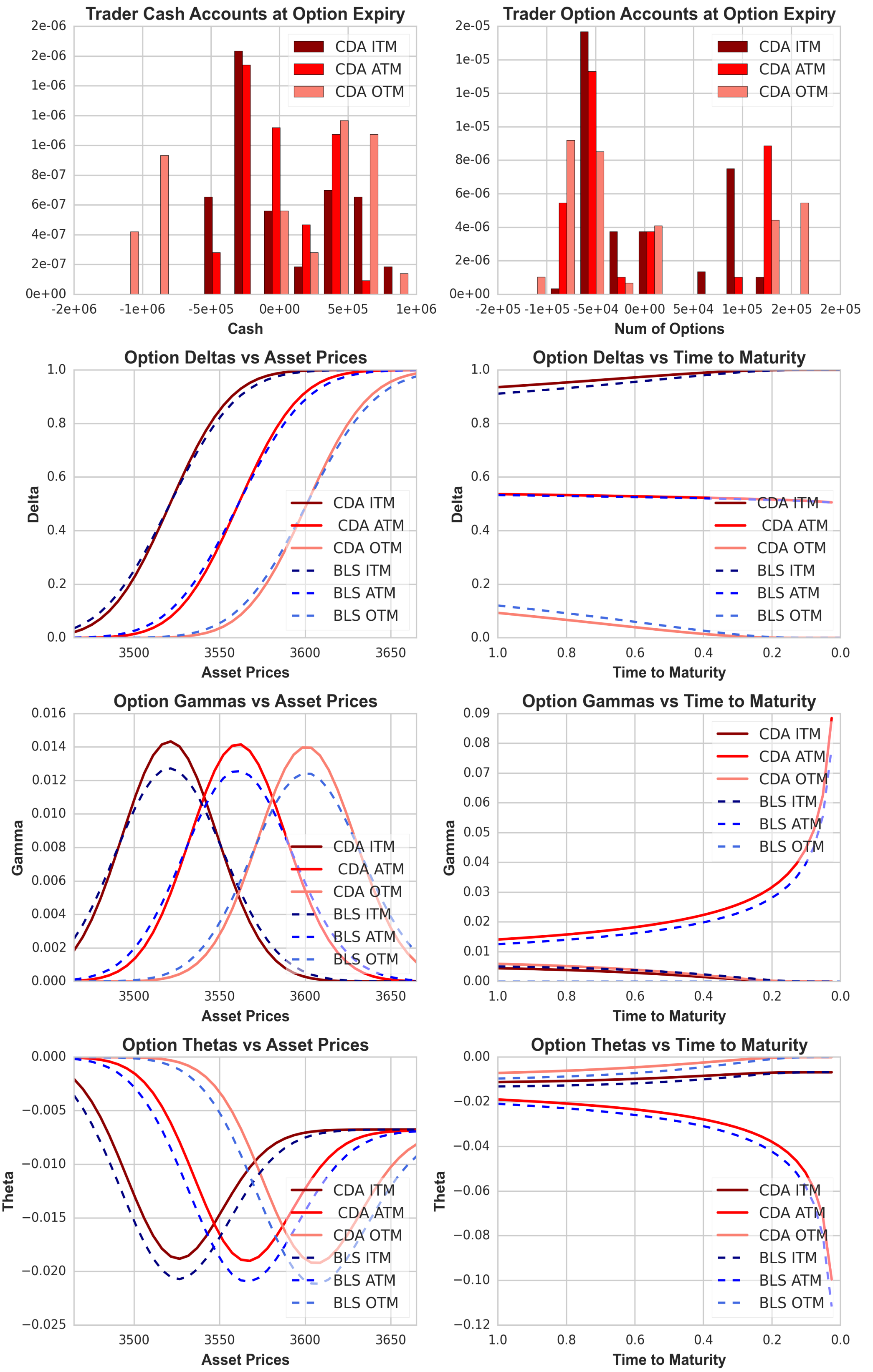


Figure E.11: CDA inventory distribution and Greeks of ATM, OTM and ITM options for mixed traders

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